

Lecture 11	Krane	Enge	Cohen	Williams
Isospin	11.3	6.7	6.3	8.10
Beta decay`	Ch 9	Ch 11	Ch 11	5.3/4

Problems Lecture 11

- 1 Discuss the experimental evidence for the existence of the neutrino.
- 2 The nuclide ^{21}Na decays by positron emission to ^{22}Na . Knowing that the radius of a nucleus is $r_0 A^{1/3}$, estimate the maximum energy the positron can have.
- 3 Compare this with a calculation from known nuclear masses.
- 4 Using a table of nuclear masses in one of the text books, calculate the total energy (E_0) and the KE (T_0) for the beta decay of ^{56}Mn .

Lecture 11

How do we find the value of T?

We take the configuration of the nucleons, and make the most “neutron rich” configuration of these A nucleons, that is consistent with the Pauli principle.

For the configuration we have, this most neutron-rich nucleus is ^{14}O , and for this nucleus the T of this configuration is Σt for all 14 nucleons. Note this is a vector sum, **so the value is $T = 1$** (we could also say that T is the value of T_z for this most-neutron-rich configuration)

If $T=1$, it must have 3 projections $T_z = 1, 0, -1$.

$T_z = +1$ means a nucleus with 2 nett neutrons out of 14 nucleons.... ^{14}C

$T_z = 0$ means $Z=N$ ^{14}N

$T_z = -1$ means 2 nett protons..... ^{14}O

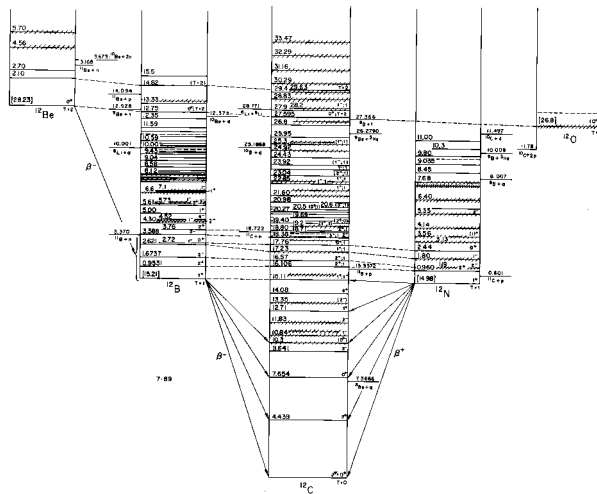
What is the T of the GS of ^{14}N ?

GS has $I=1$. ie p and n are parallel, and there is no way I can change a p to an n without violating Pauli principle. So T is the value of T_z for the most neutron-rich form of this configuration. This is it!!

The GS and the remaining unmatched states all have $T=0$. There is only one projection of this $T_z = 0$.

All states that can be generated by successive $T+$ (or $-$) operations are called ISOBARIC analogues, and the entire set is called an ISOSPIN multiplet.

Look at ^{12}C as a further example .



Beta decay

We have already mentioned that only a limited number of nuclei are stable. Most combinations of N and Z lead to nuclides that must shed mass and/or charge to gain values of N and Z that are stable.

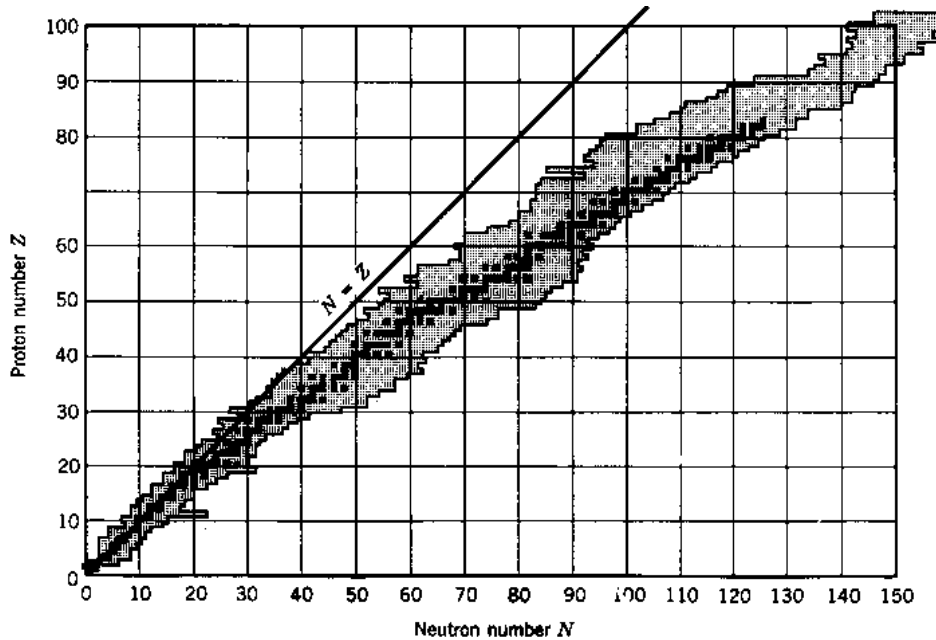


Figure 1.1 Stable nuclei are shown in dark shading and known radioactive nuclei are in light shading.

Reminder:

Decay constant and half life

For a sample of N_0 nuclei with a decay constant of $\lambda \text{ sec}^{-1}$, the number N that will remain after a time t is given by: $N = N_0 e^{-\lambda t} \Rightarrow N = N_0 e^{\frac{0.693t}{T_{1/2}}}$ where the half-life $T_{1/2} = \ln 2 / \lambda$.

For heavy nuclei (above about Pb) the preferred method is to eject an α particle. For lighter nuclei the preferred method is β -decay. In principle this means that for a nucleus with too many neutrons, a neutron will undergo a change to a proton plus an electron. This requires a force! In practice emission of another particle necessary to conserve angular momentum and energy, a neutrino (actually an antineutrino). For proton rich nuclei, a proton converts to a neutron, a positron, and a neutrino.

$n \rightarrow p + e^- + \bar{\nu}$ in so doing $N \rightarrow N-1$, $Z \rightarrow Z+1$, and A is unchanged.
 $p \rightarrow n + e^+ + \nu$ in so doing $N \rightarrow N+1$, $Z \rightarrow Z-1$, and A is unchanged.

Hence β -decay moves the unstable nucleus at 45 deg towards the stability line.

Energy considerations

For any particle decay to occur, energy must be conserved. We have already seen in our discussion of the Semi-empirical Mass Formula, that for a nucleus with a given value of A , the masses of these isotones fall on a parabola (mass as a function of Z) such as shown.

In this example ^{125}Te is the stable nucleus, and the energy available for β -decay between the successive nuclei involved, is the mass difference.

When the early researchers found this emission from naturally occurring radioactive material (e.g. Ra), they quickly identified β -particles as electrons.

The confusion arose because when they measured the energy of β 's from a particular decay, they got a spectrum of energies. This was quite unlike the case of α -emission where the energy of an α group was very well defined (as we shall see next lecture).

Later research work found the energy spectrum of β s to look something like that shown.

How did they measure the spectrum?

The explanation of this spectrum had to wait many years until Pauli, in 1930 proposed that the β was not emitted alone, but a neutral spin- $1/2$ particle was emitted simultaneously. This particle was called the neutrino, and you have already met this in the 1st half of the course. It interacts very weakly with matter, and is very hard to detect because it is neutral, and interacts only via the weak interaction). Its mass is very small (\sim few eV) but probably not zero, since there are many current theories that suggest that a major part of the mass of the universe is due to these neutrinos.

So the reactions are

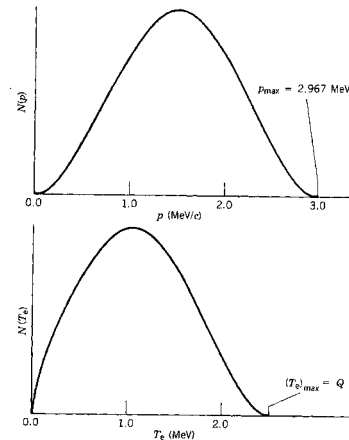
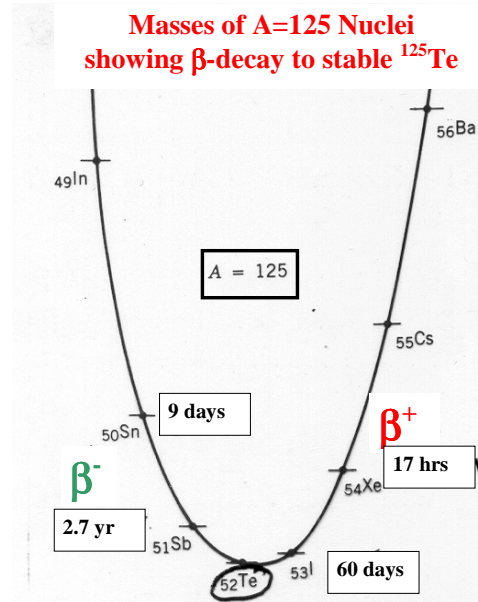
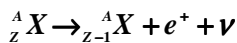
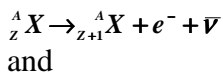
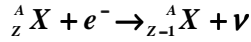


Figure 9.2 Expected electron energy and momentum distributions, from Equations 9.24 and 9.25. These distributions are drawn for $Q = 2.5$ MeV.

Particularly for nuclei with large Z, β^+ decay is inhibited because of Coulomb effects, and an inner atomic electron is captured by the proton to effect the transition. This is called **electron capture (EC)**



What is the energy of a β^- -particle emitted from nucleus A,Z to form nucleus A, Z+1?

In terms of the neutral atom masses we can write:

$${}^A_Z X \rightarrow {}^A_{Z+1} X + e^-$$

$$Q_{\beta^-} = \{[m({}^A_Z X) - Zm_e] - [m({}^A_{Z+1} X) - (Z+1)m_e] - m_e\}c^2$$

$$Q_{\beta^-} = [m({}^A_Z X) - m({}^A_{Z+1} X)]c^2$$

Q_{β^-} represents the energy shared by the e^- and the antineutrino

$$Q_{\beta^-} = T_e + E_\nu$$

And since when one is max the other is zero

$$(T_{e^-})_{\max} = (E_\nu)_{\max} = Q_{\beta^-}$$

For positron emission, a similar calculation gives that:

$${}^A_Z X \rightarrow {}^A_{Z-1} X + e^+$$

$$Q_{\beta^+} = [m({}^A_Z X) - m({}^A_{Z-1} X) - 2m_e]c^2$$

The orthodox explanation as to how a nucleon can decay with the emission of a β and neutrino is based on field theory in a similar way to the strong nuclear force deriving from field particles.

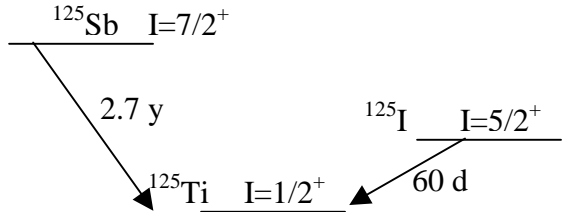
In this case the weak beta-neutrino field consists of an infinite density of negative-energy, unobservable electrons and neutrinos pervading all of space. A β^- decay is an exchange of energy between a nucleus and that field, in which the nucleus contributes energy by changing a neutron into a proton. This energy excites the field by changing a **neutrino** (in the negative energy sea) into an **electron**, which is emitted (the β); the missing neutrino behaves as an **anti neutrino** (antimatter).

Conversely in a β^+ decay the nucleus changes a proton into a neutron, and the energy given to the field converts an **electron** (in the negative-energy sea) into a **neutrino**, with the missing electron behaving as a **positron** (antimatter). Now if this sounds a bit far-fetched, blame Dirac.

Returning to the Mass diagram for A=125

To zero order you can see that the half-life depends on the energy available. It is shorter when the energy is greater.

But not entirely so! $\text{Sb} \rightarrow \text{Sn}$ has more available energy than the transition $\text{I} \rightarrow \text{Te}$, yet its half-life is much longer. Why?



The β -particle has to carry away more AM in one case than in the other. So the half-life depends on l as well as the energy.

The angular momentum must be carried off by the electron and neutrino, and the shortest half-lives (largest decay constants) are when $\Delta L=0$. The electron and neutrino have intrinsic spin $s = 1/2$, and in this case they couple to 0. Such transitions are termed “allowed β decays”. If $\Delta L = 1, 2$ etc they are termed 1st forbidden, 2nd forbidden etc.

The transition rates depend strongly on Δl . As an example

β energy 1 MeV	Rel probability
$\Delta l=0$	1
$\Delta l=1$	1/80
$\Delta l=2$	1/8000

$$\lambda = \frac{g^2 |M_{fi}|^2}{2\pi^3 \hbar^7 c^3} \int_0^{p_{\max}} F(Z', p) p^2 (Q - T_e)^2 dp \quad (9.28)$$

The integral will ultimately depend only on Z' and on the maximum electron total energy E_0 (since $cp_{\max} = \sqrt{E_0^2 - m_e^2 c^4}$), and we therefore represent it as

$$f(Z', E_0) = \frac{1}{(m_e c)^3 (m_e c^2)^2} \int_0^{p_{\max}} F(Z', p) p^2 (E_0 - E_e)^2 dp \quad (9.29)$$

where the constants have been included to make f dimensionless. The function $f(Z', E_0)$ is known as the *Fermi integral* and has been tabulated for values of Z' and E_0 .

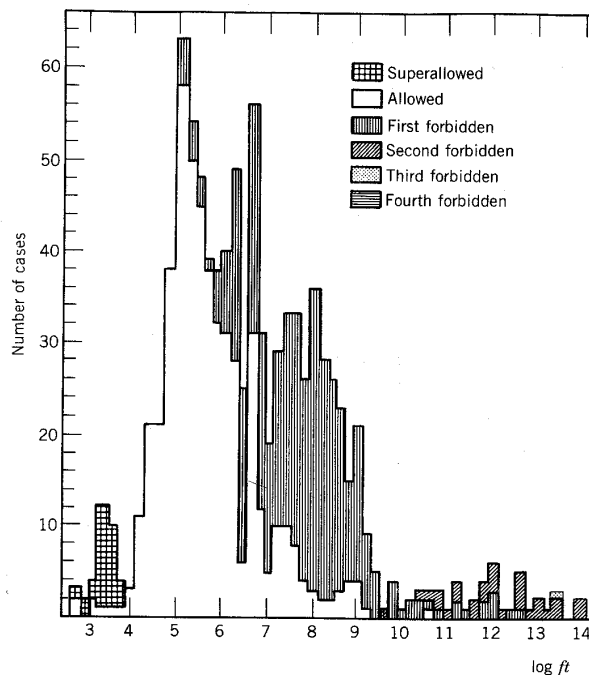
With $\lambda = 0.693/t_{1/2}$, we have

$$ft_{1/2} = 0.693 \frac{2\pi^3 \hbar^7}{g^2 m_e^5 c^4 |M_{fi}|^2} \quad (9.30)$$

This is known as **the comparative life time** or ft value. It gives a way to compare lifetimes for different nuclei. And because E_0 and Z are included in f , the differences in the ft values must be due to **differences in the nuclear matrix element**, and thus to differences in the nuclear wave function. Since the range of lifetimes for β -decay range from 10^3 to 10^{22} sec. one usually refers to the “log ft ”.

What this means is that the more similar the initial and final wavefunctions are, the smaller will be the ft value. The limit for M_{fi}^2 is 1. In this case the initial and final states have identical configurations: except one n is now a proton or vice versa. These are beta decays between **mirror nuclei** such as we discussed last lecture.

The initial theory by Fermi, considered that the emitted electron and neutrino, each having a spin of $\frac{1}{2}$, coupled these antiparallel to $S=0$. So that beta decays only occurred between nuclei with the same spin (and parity): i.e. $\Delta l=0$. However Gamow and Teller observed that other beta decays occurred, albeit less strongly, and concluded that if the electron and neutron coupled their spins to $S=1$, this would allow transitions with $\Delta l=0$ and $\Delta l=\pm 1$, and hence between nuclei with different GS spins and parity. Indeed they argued that the e - ν pair may also have $AM l$, allowing an even large range of possible transitions. These transitions are less likely (have larger ft values), because the overlap of the initial and final wavefunctions is much smaller M_{fi}^2 is much smaller.



The effect of the overlap of the initial and final nuclear wave functions is evident when one plots the frequency distribution of $\log ft$. The broadness of the peaks indicates the extreme variation of the effect of the nuclear wavefunction overlaps

Figure 9.9 Systematics of experimental $\log ft$ moments of Nuclear Physics (New York: McGraw-Hill)

There is one group of β -decays that are classified as “super allowed”. In this case the nuclear wavefunction is identical in both nuclei, and M_{fi} can be put to a constant ($2^{1/2}$), so that the log ft values should be the same. The table shows that this is approximately true.

Table 9.2 ft Values for $0^+ \rightarrow 0^+$ Superallowed Decays

Decay	ft (s)
$^{10}\text{C} \rightarrow ^{10}\text{B}$	3100 ± 31
$^{14}\text{O} \rightarrow ^{14}\text{N}$	3092 ± 4
$^{18}\text{Ne} \rightarrow ^{18}\text{F}$	3084 ± 76
$^{22}\text{Mg} \rightarrow ^{22}\text{Na}$	3014 ± 78
$^{26}\text{Al} \rightarrow ^{26}\text{Mg}$	3081 ± 4
$^{26}\text{Si} \rightarrow ^{26}\text{Al}$	3052 ± 51
$^{30}\text{S} \rightarrow ^{30}\text{P}$	3120 ± 82
$^{34}\text{Cl} \rightarrow ^{34}\text{S}$	3087 ± 9
$^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$	3101 ± 20
$^{38}\text{K} \rightarrow ^{38}\text{Ar}$	3102 ± 8
$^{38}\text{Ca} \rightarrow ^{38}\text{K}$	3145 ± 138
$^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$	3091 ± 7
$^{42}\text{Ti} \rightarrow ^{42}\text{Sc}$	3275 ± 1039
$^{46}\text{V} \rightarrow ^{46}\text{Ti}$	3082 ± 13
$^{46}\text{Cr} \rightarrow ^{46}\text{V}$	2834 ± 657
$^{50}\text{Mn} \rightarrow ^{50}\text{Cr}$	3086 ± 8
$^{54}\text{Co} \rightarrow ^{54}\text{Fe}$	3091 ± 5
$^{62}\text{Ga} \rightarrow ^{62}\text{Zn}$	2549 ± 1280

With this constant value of M_{fi}^2 , one can also find the value of “g” in the equation above.

$$g = 0.88 \times 10^{-4} \text{ MeV fm}^3.$$

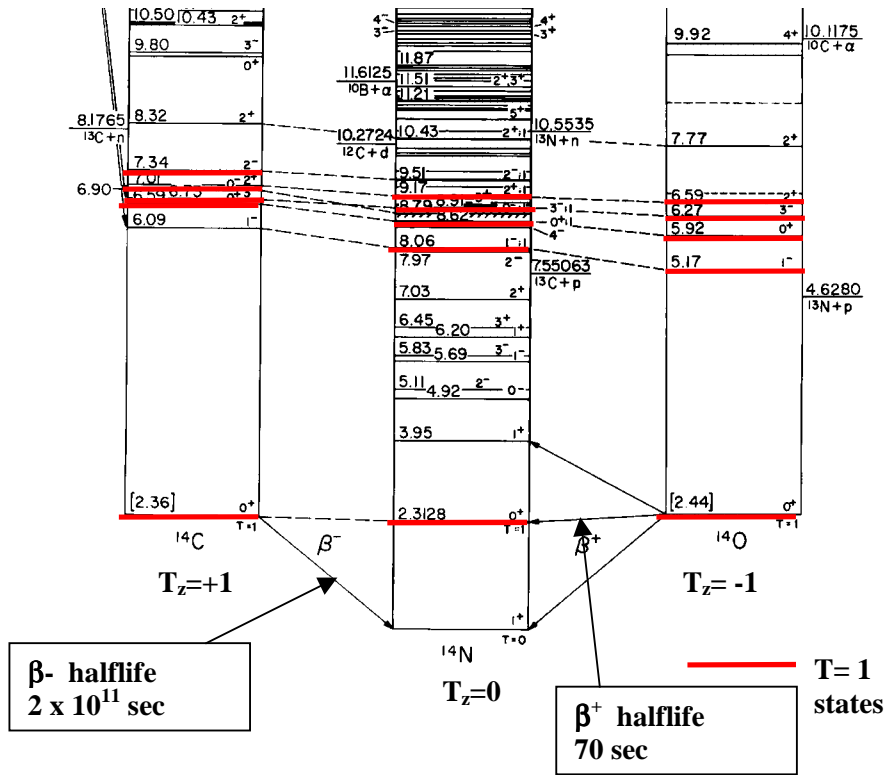
When compared in a dimensionless form the following relative strengths of the 4 fundamental strength constants can be shown.

Force	Rel Strength
Strong Nuclear	1
electromagnetic	10^{-2}
β-decay “weak”	10^{-15}
gravitational	10^{-35}

Now what is so special about these super-allowed β -decays? As I said the reason they are super-allowed is because the wave functions of the parent and daughter nuclei are the same.

This builds on our discussion of isospin earlier.

They involve isobaric analogue states



Why this incredible difference?

Now I am asking Beck Scott to tell you something of her PhD project where measuring the half-life of ^{26}Al may provide a test for the Standard Model.

