Neutrino Theory Workshop, Melbourne, June 3 2008

Naturalness Bounds on Neutrino Magnetic Moments

Vincenzo Cirigliano

Theoretical Division, Los Alamos National Laboratory

Los Alamos NATIONAL LABORATORY EST.1943

Outline

- Introduction: basic facts on neutrino magnetic moments

- "Naturalness bounds"

  - Dirac neutrino magnetic moments
  - Majorana neutrino transition moments

  } Quite different bounds !!

- Understanding the difference on the basis of symmetry

- N.F. Bell, VC, M.J. Ramsey-Musolf, P.Vogel, M. Wise, PRL 95 151802, 2005
- S. Davidson, M Gorbahn, A. Santamaria PLB 626 151, 2005
- N.F. Bell, M. Gorchtein, M.J. Ramsey-Musolf, P.Vogel, P.Wang, PLB 642 377, 2006

D

M

M

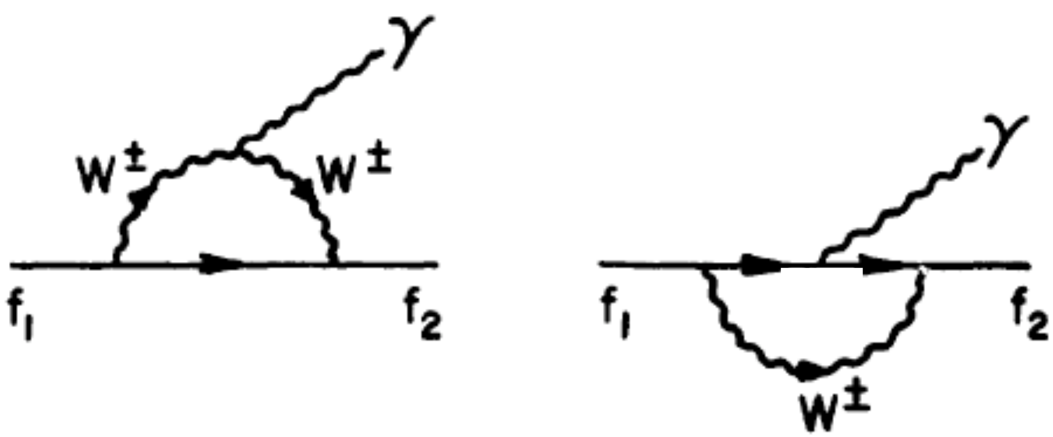
Neutrino magnetic moments: facts

- Massive neutrinos have non-zero magnetic moment.
  In minimally extended SM

$$\frac{\mu_\nu}{\mu_B} = \frac{3\, G_F\, m_e\, m_\nu}{4\sqrt{2}\,\pi^2} \approx 3 \times 10^{-19} \left[\frac{m_\nu}{1\,\mathrm{eV}}\right]$$

Lee-Shrock '77
Marciano-Sanda '77

$$\mu_B = e/(2\,m_e)$$



Neutrino magnetic moments: facts

- Massive neutrinos have non-zero magnetic moment
  In minimally extended SM

$$\frac{\mu_\nu}{\mu_B} = \frac{3\,G_F\,m_e\,m_\nu}{4\sqrt{2}\,\pi^2} \approx 3 \times 10^{-19} \left[\frac{m_\nu}{1\,\mathrm{eV}}\right]$$

Lee-Shrock '77
Marciano-Sanda '77

$$\mu_B = e/(2\,m_e)$$

- Most stringent phenomenological bounds come from:

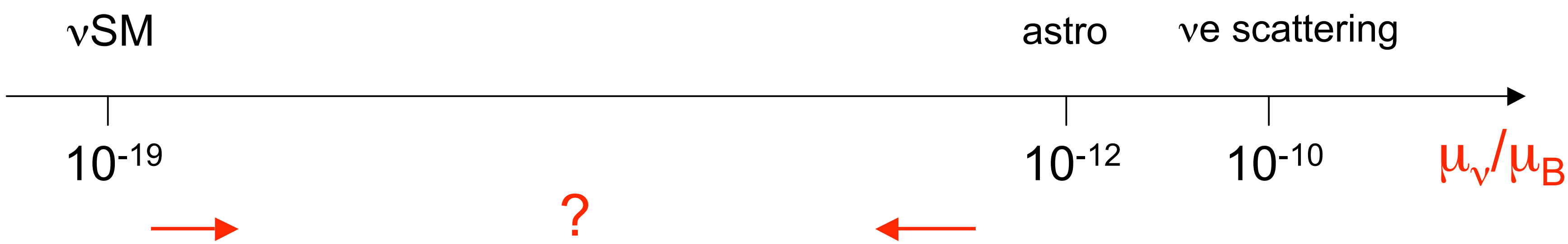
$\nu e \to \nu e$

Energy loss in stars via $\gamma^* \to \nu\bar{\nu}$

$$\frac{|\mu_\nu^{\rm exp}|}{\mu_B} \simeq \frac{G_F\,m_e}{\sqrt{2}\pi\alpha}\sqrt{m_e T_{\rm kin}} \sim 10^{-10}$$

$$\frac{|\mu_\nu^{\rm astro}|}{\mu_B} \simeq \frac{G_F\,m_e}{\sqrt{2}\pi\alpha}\omega_P \sim 3 \times 10^{-12}$$

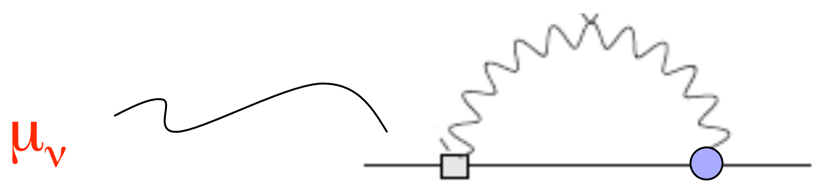
Theoretical considerations



How large a magnetic moment can we expect ?

Theoretical considerations

- We expect BSM physics to generate $\mu_\nu/\mu_B > 10^{-19}$
- Several models can make $\mu_\nu/\mu_B >> 10^{-19}$ despite $m_\nu < 1$ eV (based on *symmetries that are broken by SM interactions)*
- SM radiative corrections induce $\delta m_\nu \propto \mu_\nu$ $\Rightarrow$ get "naturalness" upper bounds on μ_ν by requiring $\delta m_\nu < m_\nu^{obs}$



Naturalness bounds: Dirac ν

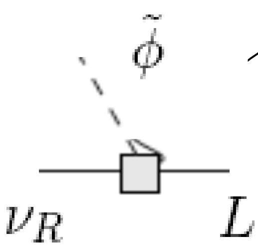
- *Assume* new physics responsible for μ_ν at scale $\Lambda > v_{ew}$
- Work within EFT at $E < \Lambda$ that respects SU(2) x U(1) and L

Naturalness bounds: Dirac ν

- *Assume* new physics responsible for μ_ν at scale $\Lambda > v_{ew}$
- Work within EFT at $E < \Lambda$ that respects SU(2) x U(1) and L

$$\mathcal{L}_{\rm eff} \supset C_M^{(4)}\, \mathcal{O}_M^{(4)} + \frac{C_M^{(6)}}{\Lambda^2}\, \mathcal{O}_M^{(6)} + \frac{C_B}{\Lambda^2}\, \mathcal{O}_B^{(6)} + \frac{C_W}{\Lambda^2}\, \mathcal{O}_W^{(6)}$$

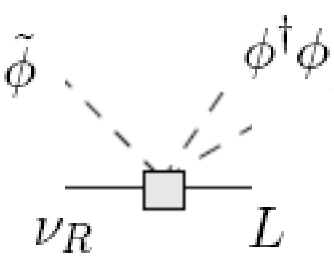
$\bar{L}\tilde{\phi}\,\nu_R$



$$\tilde{\phi} = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}$$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$\bar{L}\tilde{\phi}\,\nu_R\,(\phi^\dagger\phi)$



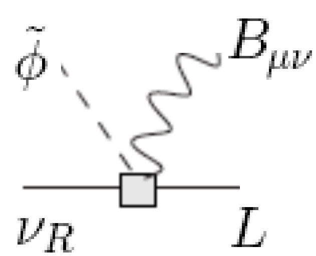
Naturalness bounds: Dirac ν

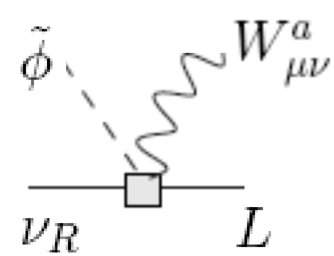
- *Assume* new physics responsible for μ_ν at scale $\Lambda > v_{ew}$
- Work within EFT at E < Λ that respects SU(2) x U(1) and L

$$\mathcal{L}_{\rm eff} \supset C_M^{(4)}\,\mathcal{O}_M^{(4)} + \frac{C_M^{(6)}}{\Lambda^2}\,\mathcal{O}_M^{(6)} + \frac{C_B}{\Lambda^2}\,\mathcal{O}_B^{(6)} + \frac{C_W}{\Lambda^2}\,\mathcal{O}_W^{(6)}$$

$$g_1\ \bar{L}\tilde{\phi}\,\sigma^{\mu\nu}\,\nu_R\,B_{\mu\nu}$$

$$g_2\ \bar{L}\tau^a\tilde{\phi}\,\sigma^{\mu\nu}\,\nu_R\,W^a_{\mu\nu}$$





Naturalness bounds: Dirac ν

- *Assume* new physics responsible for μ_ν at scale $\Lambda > v_{ew}$
- Work within EFT at E < Λ that respects SU(2) x U(1) and L

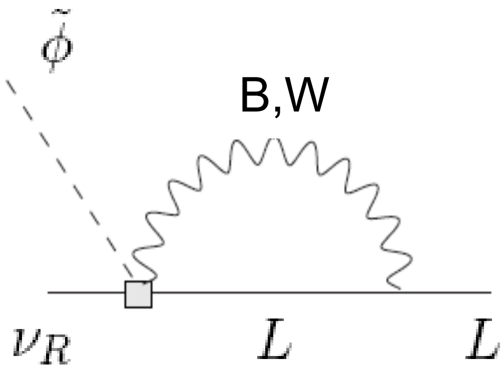
$$\mathcal{L}_{\rm eff} \supset C_M^{(4)}\, \mathcal{O}_M^{(4)} + \frac{C_M^{(6)}}{\Lambda^2}\, \mathcal{O}_M^{(6)} + \frac{C_B}{\Lambda^2}\, \mathcal{O}_B^{(6)} + \frac{C_W}{\Lambda^2}\, \mathcal{O}_W^{(6)}$$

$$\phi \to \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$m_\nu = -C_M^{(4)}(v)\frac{v}{\sqrt{2}} - C_M^{(6)}(v)\frac{v^3}{2\sqrt{2}\Lambda^2}$$

$$\frac{\mu_\nu}{\mu_B} = -4\sqrt{2}\left(\frac{m_e v}{\Lambda^2}\right)[C_B(v) + C_W(v)]$$

- $\mathcal{O}^{(6)}_{B,W} \;\longrightarrow\; \mathcal{O}^{(4)}_{M}$ “mixing”

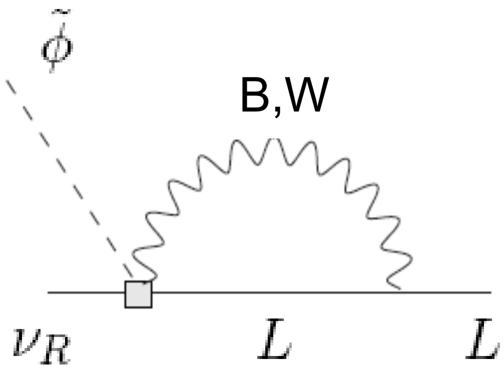


$$\delta C^{(4)}_M(\Lambda) \sim \frac{\alpha_{1,2}}{4\pi} C_{B,W}(\Lambda)$$

↔

$$\delta m_\nu \sim \frac{\alpha}{4\pi}\frac{\Lambda^2}{m_e}\frac{\mu_\nu}{\mu_B}$$

- $\mathcal{O}^{(6)}_{B,W} \rightarrow \mathcal{O}^{(4)}_{M}$ "mixing" (*matching* at scale Λ)



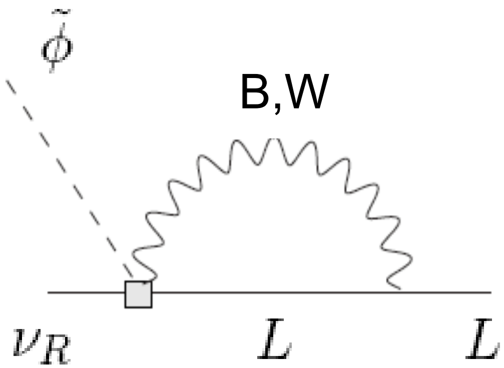
$$\delta C^{(4)}_M(\Lambda) \sim \frac{\alpha_{1,2}}{4\pi} C_{B,W}(\Lambda)$$

$\longleftrightarrow$

$$\delta m_\nu \sim \frac{\alpha}{4\pi} \frac{\Lambda^2}{m_e} \frac{\mu_\nu}{\mu_B}$$

coefficient is model-dependent

- $\mathcal{O}^{(6)}_{B,W} \;\rightarrow\; \mathcal{O}^{(4)}_{M}$ "mixing" (*matching* at scale Λ)

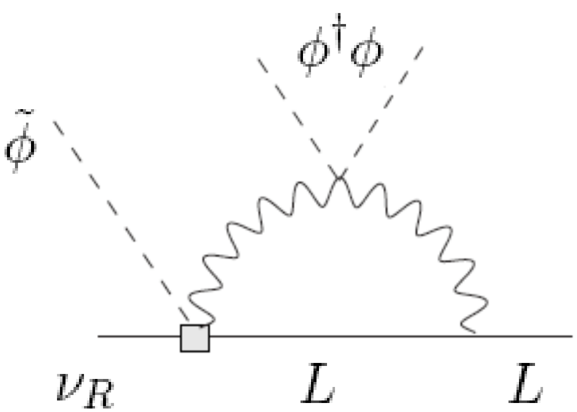


$$\delta C^{(4)}_{M}(\Lambda) \sim \frac{\alpha_{1,2}}{4\pi} C_{B,W}(\Lambda)$$

$$\delta m_\nu \sim \frac{\alpha}{4\pi} \frac{\Lambda^2}{m_e} \frac{\mu_\nu}{\mu_B}$$

coefficient is model-dependent

- $\mathcal{O}^{(6)}_{B,W} \;\rightarrow\; \mathcal{O}^{(6)}_{M}$ *mixing*



$+ \;\cdots$

$$\delta m_\nu \sim \frac{\alpha}{4\pi} \frac{v^2}{m_e} \left(\alpha \log \frac{\Lambda}{M_Z}\right) \frac{\mu_\nu}{\mu_B}$$

coefficient is model independent

- Resulting bounds on Dirac ν magnetic moment:

Dim-4 "matching"

$$\frac{|\mu_\nu|}{\mu_B} \lesssim 3 \times 10^{-15} \times \left(\frac{\delta m_\nu}{1 \text{ eV}}\right) \times \left(\frac{1 \text{ TeV}}{\Lambda}\right)^2$$

Strong dependence on Λ

Dim-6 "mixing"

$$\frac{|\mu_\nu|}{\mu_B} \lesssim 8 \times 10^{-15} \times \left(\frac{\delta m_\nu}{1 \text{ eV}}\right)$$

Mild logarithmic dependence on Λ

At Λ ~ 1 TeV both bounds are 10^2 (10^4) stronger than astrophysical (laboratory) limits

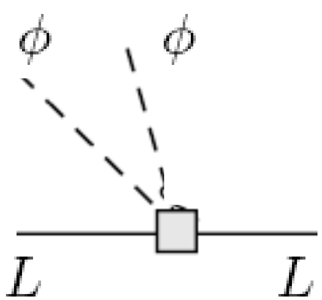
Naturalness bounds: Majorana ν

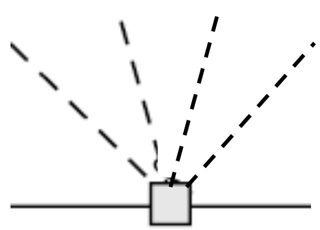
- SU(2) x U(1) invariant EFT at E < Λ [but L is not conserved]

$$\mathcal{L}_{\rm eff} \supset \frac{C_M^{(5)}}{\Lambda}\mathcal{O}_M^{(5)} + \frac{C_M^{(7)}}{\Lambda^3}\mathcal{O}_M^{(7)} + \frac{C_B}{\Lambda^3}\mathcal{O}_B^{(7)} + \frac{C_W}{\Lambda^3}\mathcal{O}_W^{(7)}$$

$$(\overline{L^c_\alpha}\epsilon\phi)\,(\phi^T\epsilon L_\beta)$$

$$(\overline{L^c_\alpha}\epsilon\phi)\,(\phi^T\epsilon L_\beta)\,(\phi^\dagger\phi)$$





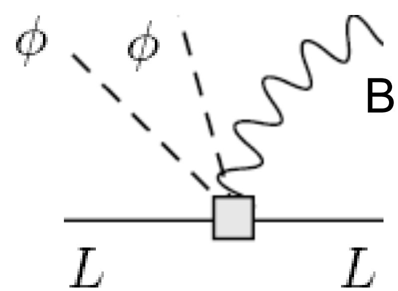
Naturalness bounds: Majorana ν

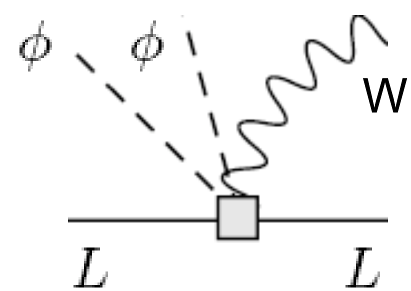
- SU(2) x U(1) invariant EFT at E < Λ [but L is not conserved]

$$\mathcal{L}_{\rm eff} \supset \frac{C_M^{(5)}}{\Lambda}\mathcal{O}_M^{(5)} + \frac{C_M^{(7)}}{\Lambda^3}\mathcal{O}_M^{(7)} + \frac{C_B}{\Lambda^3}\mathcal{O}_B^{(7)} + \frac{C_W}{\Lambda^3}\mathcal{O}_W^{(7)}$$

$$g_1\left(\overline{L^c}_\alpha \epsilon\phi\right)\sigma^{\mu\nu}\left(\phi^T\epsilon L_\beta\right)B_{\mu\nu}$$

$$g_2\left(\overline{L^c_\alpha}\epsilon\phi\right)\sigma^{\mu\nu}\left(\phi^T\epsilon\tau^a L_\beta\right)W^a_{\mu\nu}$$





Naturalness bounds: Majorana ν

- SU(2) x U(1) invariant EFT at E < Λ [but L is not conserved]

$$\mathcal{L}_{\rm eff} \supset \frac{C_M^{(5)}}{\Lambda}\mathcal{O}_M^{(5)} + \frac{C_M^{(7)}}{\Lambda^3}\mathcal{O}_M^{(7)} + \frac{C_B}{\Lambda^3}\mathcal{O}_B^{(7)} + \frac{C_W}{\Lambda^3}\mathcal{O}_W^{(7)}$$

$$\phi \to \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

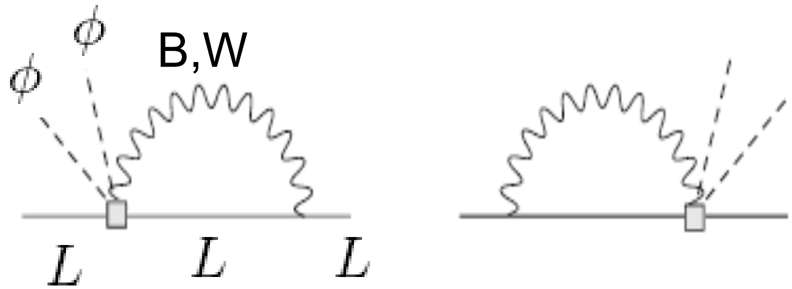
$$[m_\nu]_{\alpha\beta} = [m_\nu]_{\beta\alpha}$$

$$\frac{1}{2}[m_\nu]_{\alpha\beta} = \frac{v^2}{2\Lambda}\left[C_M^{(5)}(v)\right]_{\alpha\beta} + \frac{v^4}{4\Lambda^3}\left[C_M^{(7)}(v)\right]_{\alpha\beta}$$

$$[\mu_\nu]_{\alpha\beta} = -[\mu_\nu]_{\beta\alpha}$$

$$\frac{[\mu_\nu]_{\alpha\beta}}{\mu_B} = \frac{2m_e v^2}{\Lambda^3}\left([C_B(v)]_{\alpha\beta} + [C_W(v)]_{\alpha\beta}\right)$$

- $\mathcal{O}^{(7)}_{B,W} \to \mathcal{O}^{(5)}_{M}$ "mixing" (*matching* at scale Λ)

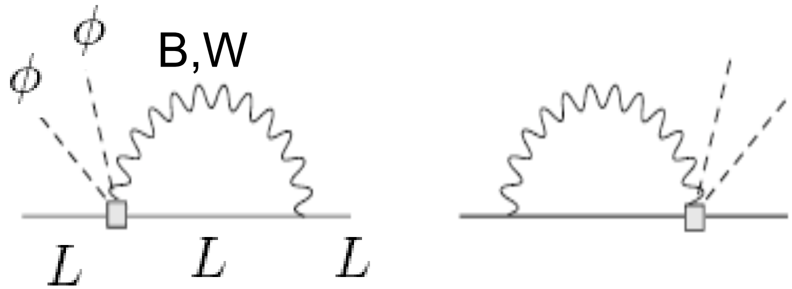


$\longleftrightarrow$

The sum vanishes, as expected !

$$m^T = m \qquad \mu^T = -\mu$$

- $\mathcal{O}^{(7)}_{B,W} \longrightarrow \mathcal{O}^{(5)}_{M}$ "mixing" (*matching* at scale Λ)



$\longleftrightarrow$

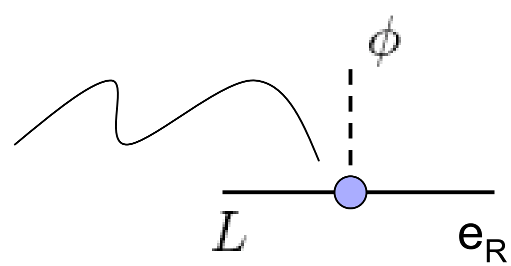
The sum vanishes, as expected !

$m^T = m \qquad \mu^T = -\mu$

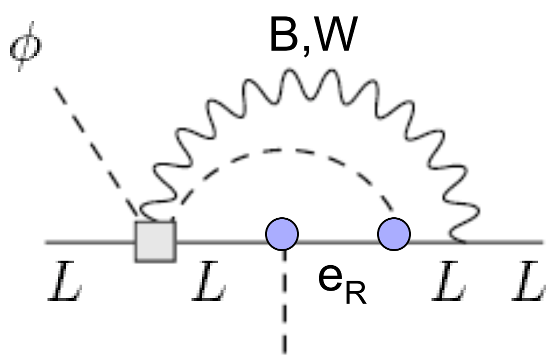
Need to insert non-trivial flavor structure:

provided by charged lepton Yukawa couplings

$$Y_\alpha = \frac{m_\alpha}{v}$$



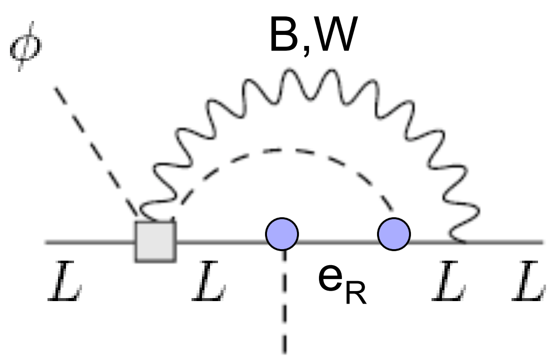
- $\mathcal{O}_{B,W}^{(7)} \rightarrow \mathcal{O}_{M}^{(5)}$ "mixing" (*matching* at scale Λ)



$$\delta[m_\nu]_{\alpha\beta} \sim \frac{\alpha}{4\pi}\,\frac{\Lambda^2}{m_e}\,\frac{[\mu_\nu]_{\alpha\beta}}{\mu_B}\cdot\left[\frac{Y_\alpha^2 - Y_\beta^2}{(4\pi)^2}\right]$$

coefficient is model-dependent

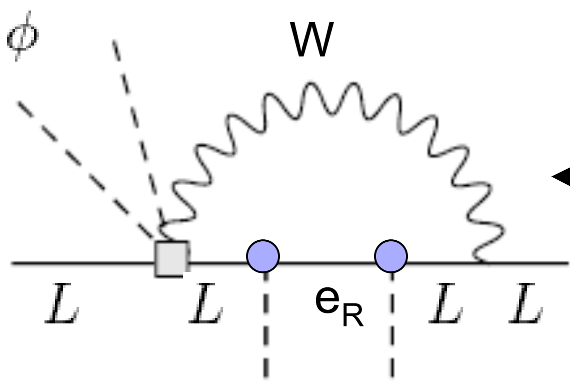
- $\mathcal{O}^{(7)}_{B,W} \rightarrow \mathcal{O}^{(5)}_{M}$ "mixing" (*matching* at scale Λ)



$$\delta[m_\nu]_{\alpha\beta} \sim \frac{\alpha}{4\pi}\,\frac{\Lambda^2}{m_e}\,\frac{[\mu_\nu]_{\alpha\beta}}{\mu_B}\cdot\left[\frac{Y_\alpha^2 - Y_\beta^2}{(4\pi)^2}\right]$$

coefficient is model-dependent

- $\mathcal{O}^{(7)}_{B,W} \rightarrow \mathcal{O}^{(7)}_{M}$ *mixing*



$$\delta[m_\nu]_{\alpha\beta} \sim \frac{\alpha}{4\pi}\,\frac{v^2}{m_e}\,\log\frac{\Lambda}{M_Z}\,\frac{[\mu_\nu]_{\alpha\beta}}{\mu_B}\cdot\left[Y_\alpha^2 - Y_\beta^2\right]$$

coefficient is model independent

- Resulting bounds on Majorana ν transition moments:

Dim-5 "matching"

$$\frac{[\mu_\nu]_{\alpha\beta}}{\mu_B} \leq 4 \times 10^{-9} \left(\frac{[m_\nu]_{\alpha\beta}}{1 \text{ eV}} \right) \left(\frac{1 \text{ TeV}}{\Lambda} \right)^2 \left| \frac{m_\tau^2}{m_\alpha^2 - m_\beta^2} \right|$$

Strong dependence on Λ

Dim-7 "mixing"

$$\frac{[\mu_\nu]_{\alpha\beta}}{\mu_B} \leq 1 \times 10^{-7} \left(\frac{[m_\nu]_{\alpha\beta}}{1 \text{ eV}} \right) \left(\log \frac{\Lambda}{M_Z} \right)^{-1} \left| \frac{m_\tau^2}{m_\alpha^2 - m_\beta^2} \right|$$

Mild logarithmic dependence on Λ

At $\Lambda \sim 1$ TeV both bounds are weaker than experimental limits

Do we understand the difference of Dirac vs Majorana bounds ?

Yes.

The difference can be traced to different nature of “Voloshin’s symmetry” protecting Dirac and Majorana mass terms, and the way this symmetry is broken by SM interactions.

Do we understand the difference of Dirac vs Majorana bounds ?

- Voloshin's mechanism: identify $SU(2)_V$ under which
  - magnetic moment transforms as a singlet (allowed)
  - mass transform as a triplet (forbidden)

$$\psi_V = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$ is a doublet under $SU(2)_V$ (Dirac)

Two flavors $$\psi_V = \begin{pmatrix} \nu_L^1 \\ \nu_L^2 \end{pmatrix}$$ Is a doublet under $SU(2)_V$ (Majorana)

- If underlying s.d. dynamics is invariant under $SU(2)_V$, then one can have large magnetic moments and small mass.

Do we understand the difference of Dirac vs Majorana bounds ?

- Voloshin's mechanism: identify $SU(2)_V$ under which
  - magnetic moment transforms as a singlet (allowed)
  - mass transform as a triplet (forbidden)

$$\psi_V = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$ is a doublet under $SU(2)_V$ (Dirac)

Two flavors: $$\psi_V = \begin{pmatrix} \nu_L^1 \\ \nu_L^2 \end{pmatrix}$$ Is a doublet under $SU(2)_V$ (Majorana)

- Naturalness bounds rely on the fact that:
  - $SU(2)_V$ (Dirac) is broken by SM gauge interactions
  - $SU(2)_V$ (Majorana) is broken by SM Yukawa interactions

Conclusions

- Naturalness bounds on μ_ν obtained by requiring that mass induced by rad. corr. does not exceed 1 eV:

Dirac

$$\frac{|\mu_\nu|}{\mu_B} \lesssim 3 \times 10^{-15} \times \left(\frac{\delta m_\nu}{1\ \mathrm{eV}}\right) \times \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2$$

Majorana

$$\frac{[\mu_\nu]_{\alpha\beta}}{\mu_B} \leq 4 \times 10^{-9} \left(\frac{[m_\nu]_{\alpha\beta}}{1\ \mathrm{eV}}\right) \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 \left|\frac{m_\tau^2}{m_\alpha^2 - m_\beta^2}\right|$$

- Dirac vs Majorana difference understood in terms of different Voloshin's symmetries

- Discovery of neutrino magnetic moment near present limits would strongly suggest that neutrinos are Majorana particles

Additional Material

Voloshin's symmetry for Dirac neutrinos

$$\psi = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$ transform as a doublet under $SU(2)_V$

Singlet →
$$\mathcal{L}_\mu \;\propto\; \mu_\nu \,\psi^T \left\{ [C\sigma_{\mu\nu}] \otimes \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\} \psi \cdot F^{\mu\nu}$$

↑

$SU(2)_V$ structure

↓

Triplet →
$$\mathcal{L}_m \;\propto\; m_\nu \,\psi^T \left\{ [C] \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\} \psi$$

- $SU(2)_V$ broken by gauge interactions $\Rightarrow$ $\delta m_\nu \;\propto\; \frac{\alpha}{\pi} \Lambda^2 \mu_\nu$

- Voloshin's symmetry for Majorana neutrinos (two flavors)

$$\psi = \begin{pmatrix} \nu_L^1 \\ \nu_L^2 \end{pmatrix}$$ transform as a doublet under $SU(2)_V$

Singlet →

$$\mathcal{L}_\mu \propto [\mu_\nu]_{12}\, \psi^T \left\{ [C\sigma_{\mu\nu}] \otimes \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\} \psi \cdot F^{\mu\nu}$$

$SU(2)_V$ structure

Triplet →

$$\mathcal{L}_m \propto \psi^T \left\{ [C] \otimes \begin{pmatrix} (m_\nu)_{11} & (m_\nu)_{12} \\ (m_\nu)_{12} & (m_\nu)_{22} \end{pmatrix} \right\} \psi$$

- $SU(2)_V$ broken by Yukawa interactions $\Rightarrow$

$$\delta[m_\nu]_{\alpha\beta} \propto \frac{\alpha}{\pi} \Lambda^2 [\mu_\nu]_{\alpha\beta} \left(Y_\alpha^2 - Y_\beta^2 \right)$$