GRAVITATIONAL WAVE ASTRONOMY

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1. **GW: physics & astronomy**

2. Current- & next-gen detectors & searches

3. Burst sources: CBC, SN → GR, cosmology

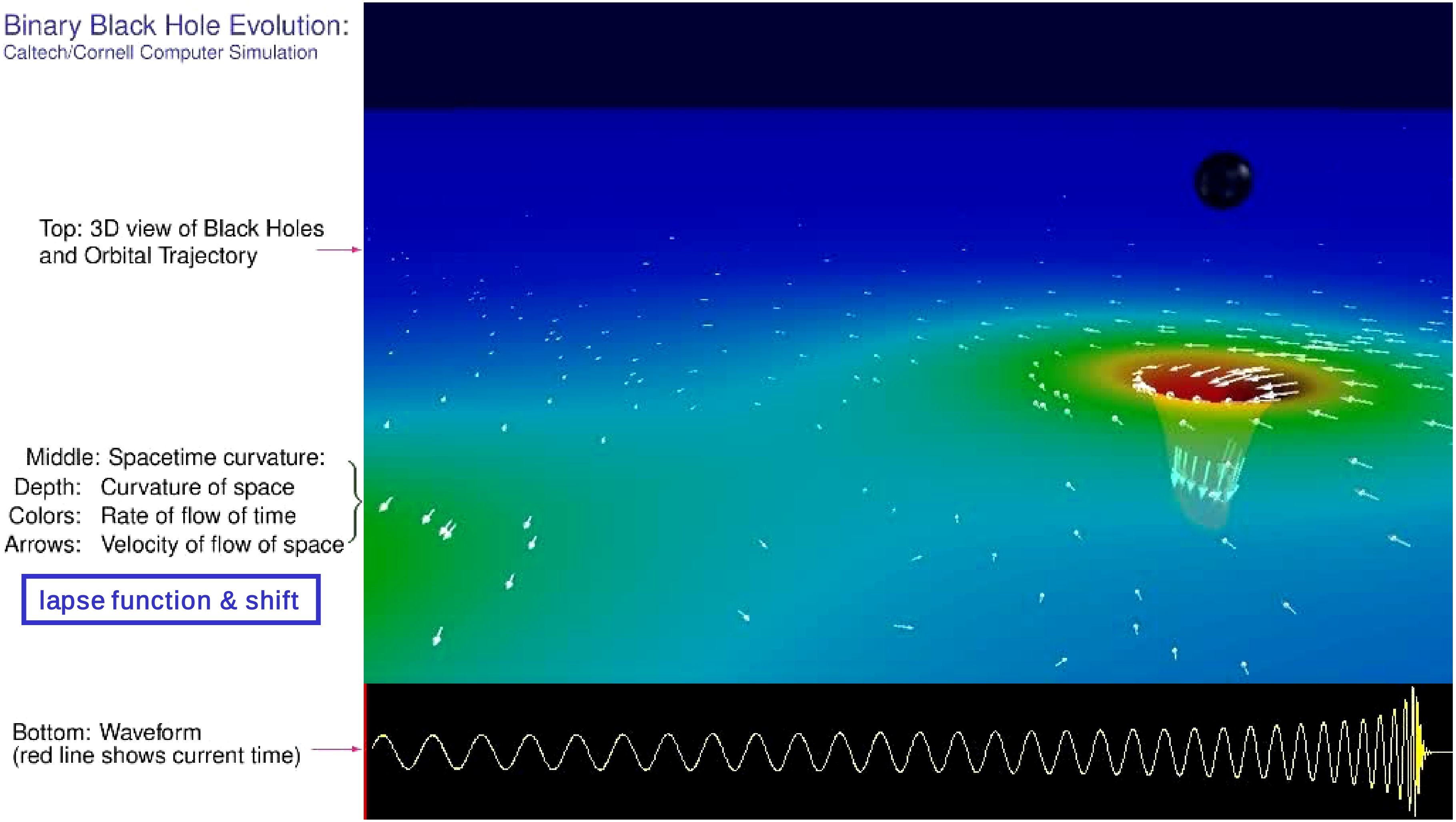
4. Periodic sources: NS → subatomic physics

LECTURE ONE

- GW in pictures and sounds
- Compendium of **science questions**

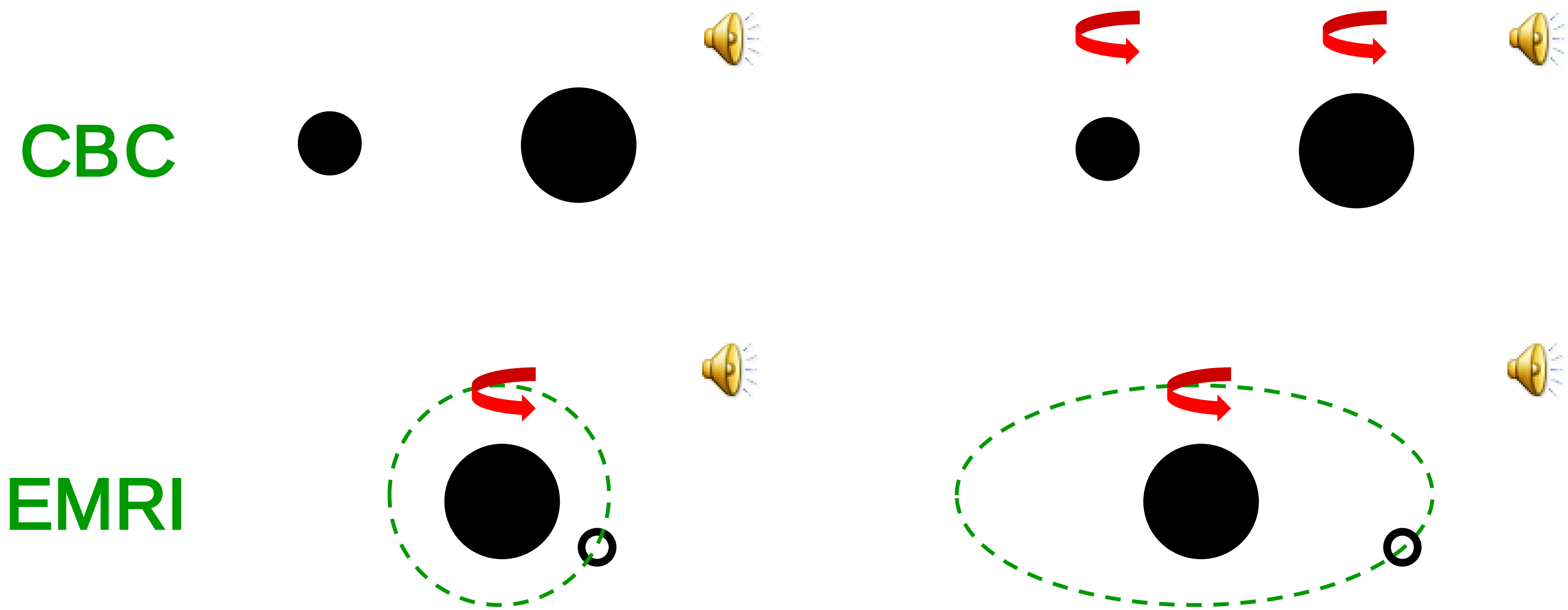
- GW physics: polarization, multipoles
- Compact binary coalescence

- The future is here… e.g. **GW cosmology**



Test strong-field GR via ring-down normal modes (Kerr?)

EINSTEIN’S MUSIC



Courtesy Scott Hughes @ gmunu.mit.edu

$h_+ \rightarrow$ chirp; spin-orbit precession; GW circularizes

By “listening” closely to various GW tones,
we can probe fundamental physics
under conditions **inaccessible on Earth**

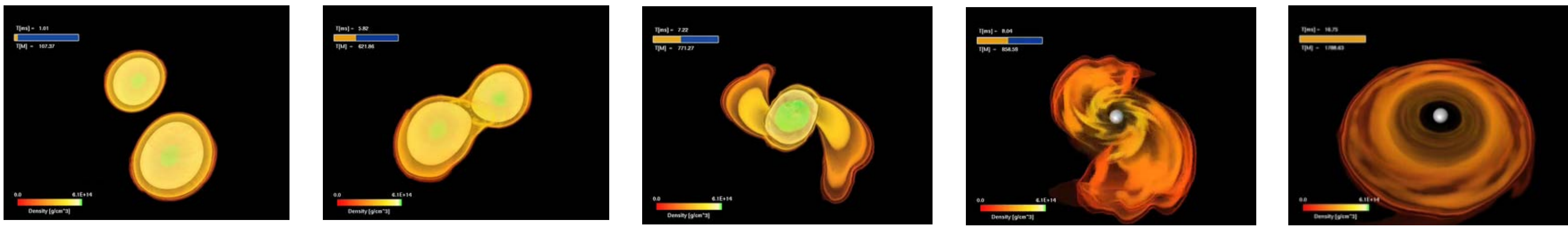
BIG QUESTIONS

- Strong-field tests of General Relativity
  - → do Nature’s black holes match the rotating Kerr solution?
  - → alternative gravities? (speed, mass, polarization of graviton)
  - → frame dragging, spin precession, ... (from merger waveforms)
  - → black hole oscillations and existence of naked singularities
- Tests of Quantum Chromodynamics
  - → nuclear equation of state (n radius, strange quark matter)
  - → many-body phenomena (QGP, superconductivity; cf. LHC)
- Birth of the Universe
  - → fluctuations at Planck scale before surface of last scattering
  - → precision cosmology (e.g. H_0) with EM counterparts
  - → phase transitions? (topological defects like strings)

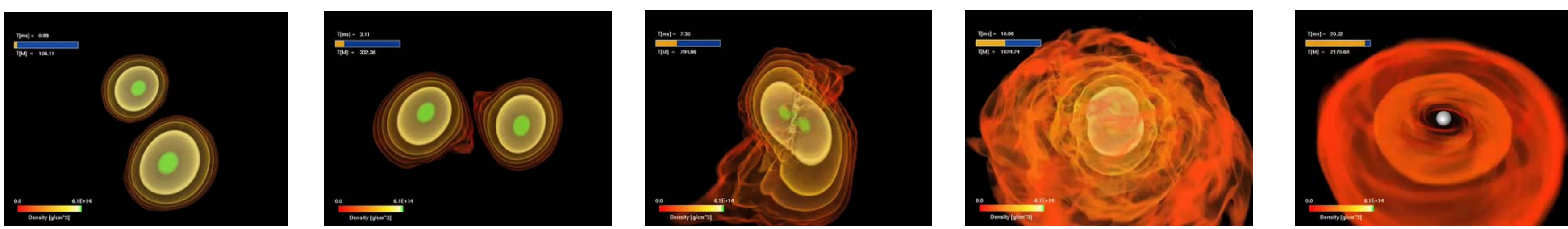
- Multi-messenger astronomy = EM + GW + ν
  - $\rightarrow$ hydro, particle, and magnetic physics in core-collapse SN
  - $\rightarrow$ what are γ-ray bursts? (brightest things in the sky...)
  - $\rightarrow$ extreme magnetism (10^{15} G), e.g. explosive annihilation
  - $\rightarrow$ how do black holes form, and how many are there?
  - $\rightarrow$ unexpected new sources!
- Macroscopic quantum metrology
  - $\rightarrow$ squeezed vacuum, quantum non-demolition measurements
  - $\rightarrow$ stability and control of opto-mechanical systems
  - $\rightarrow$ ultra-stable oscillators and lasers, ultra-smooth mirrors, AO
- Massively distributed computing

- **Cool pure-GW discoveries already!**
  - $\rightarrow$ NS deformation and internal magnetisation
  - $\rightarrow$ Kill off some superstring models

GW → NUCLEAR PHYSICS

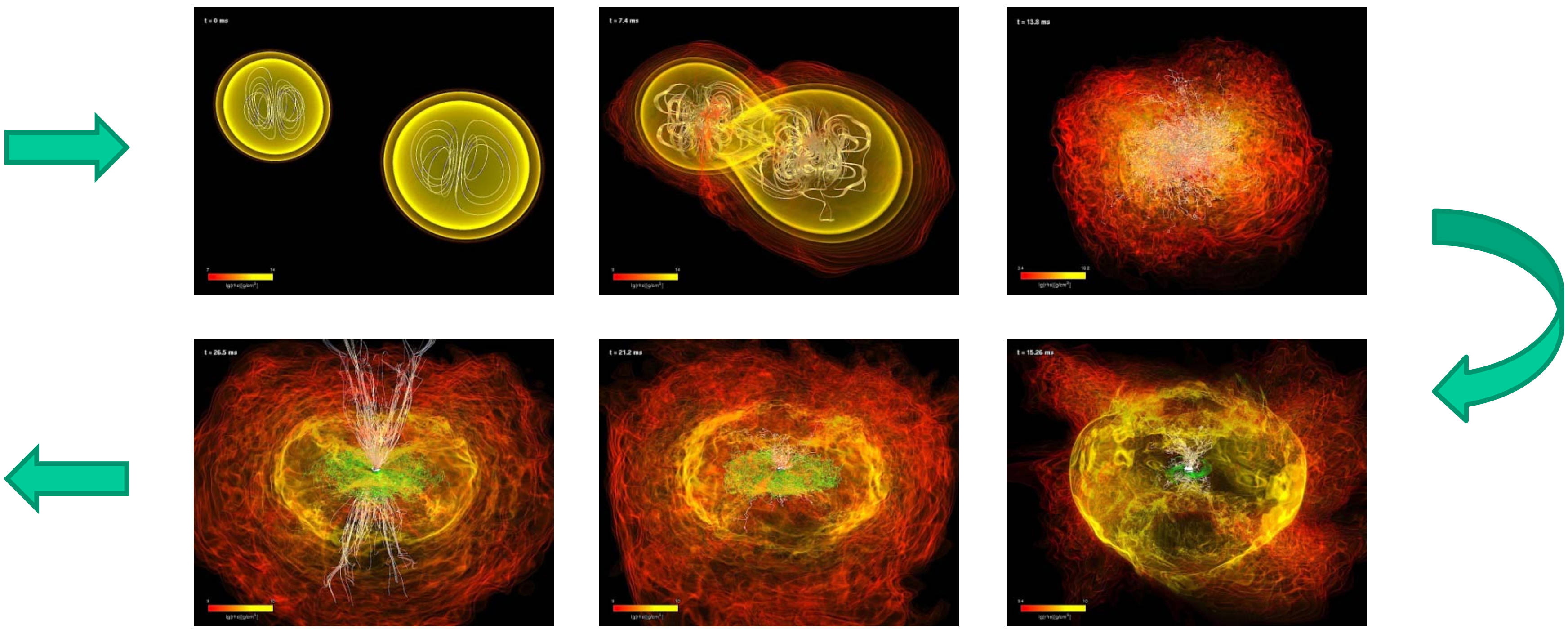


Polytropic equation of state (density contours)

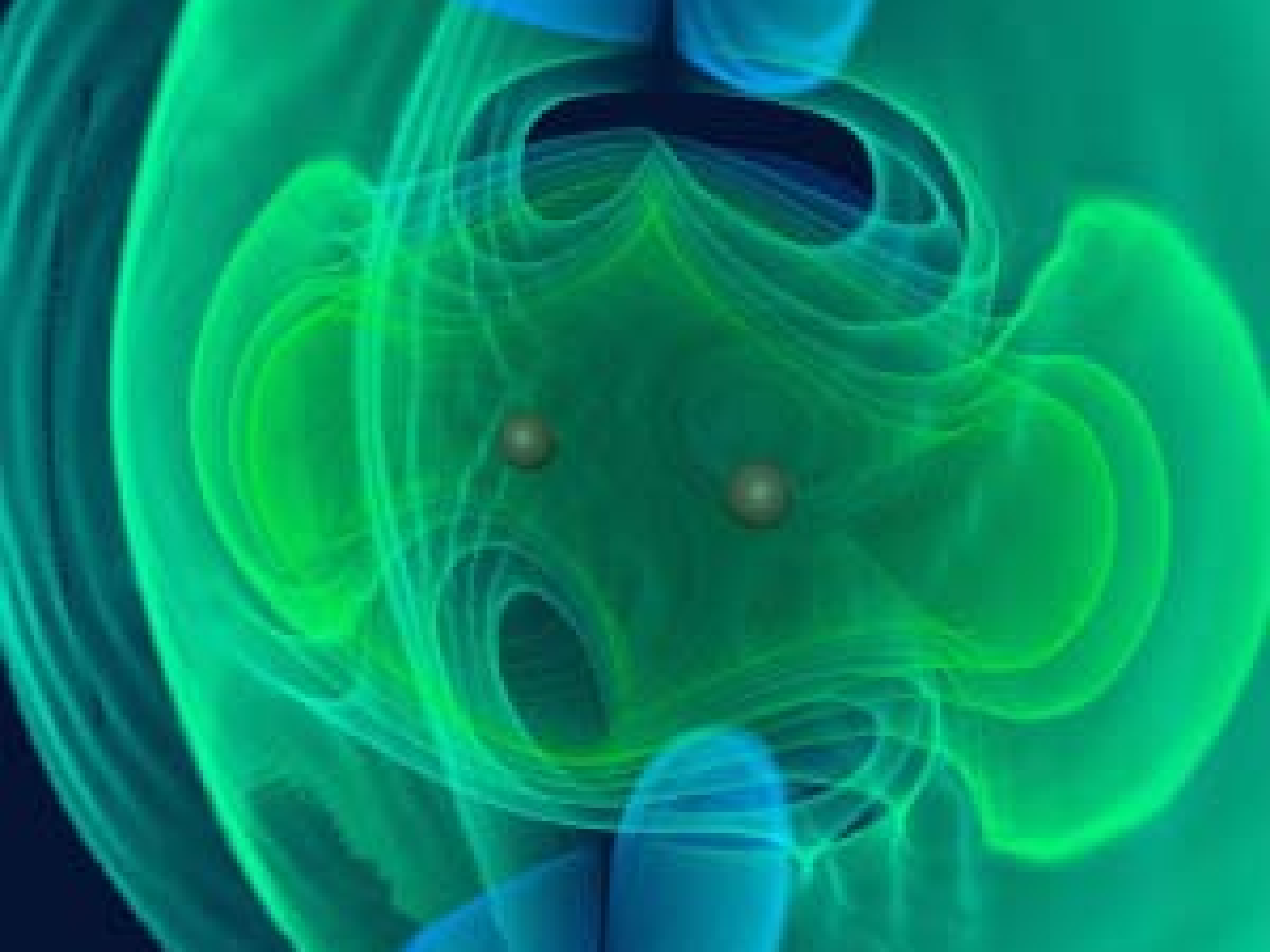


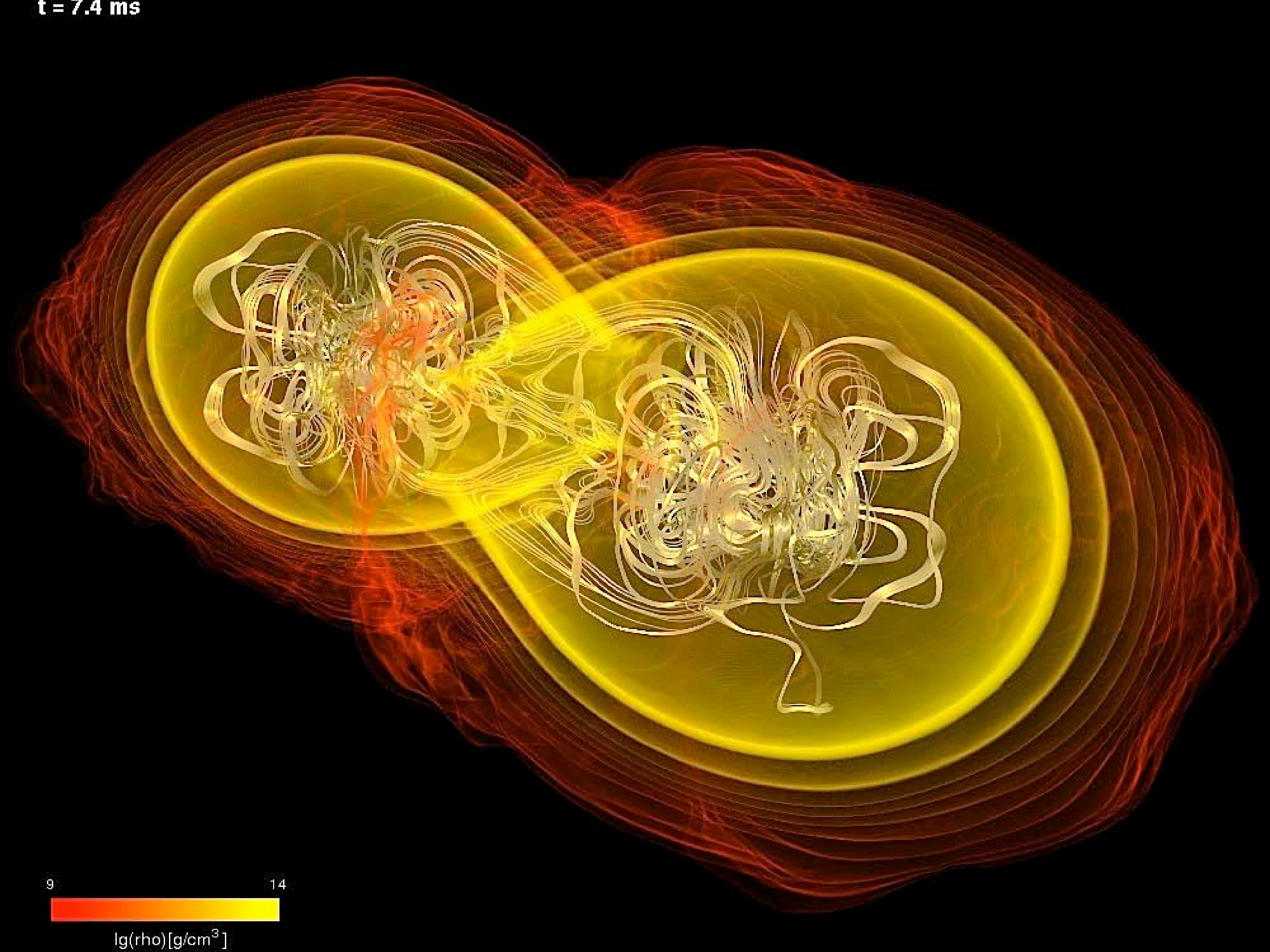
“Superfluid” equation of state

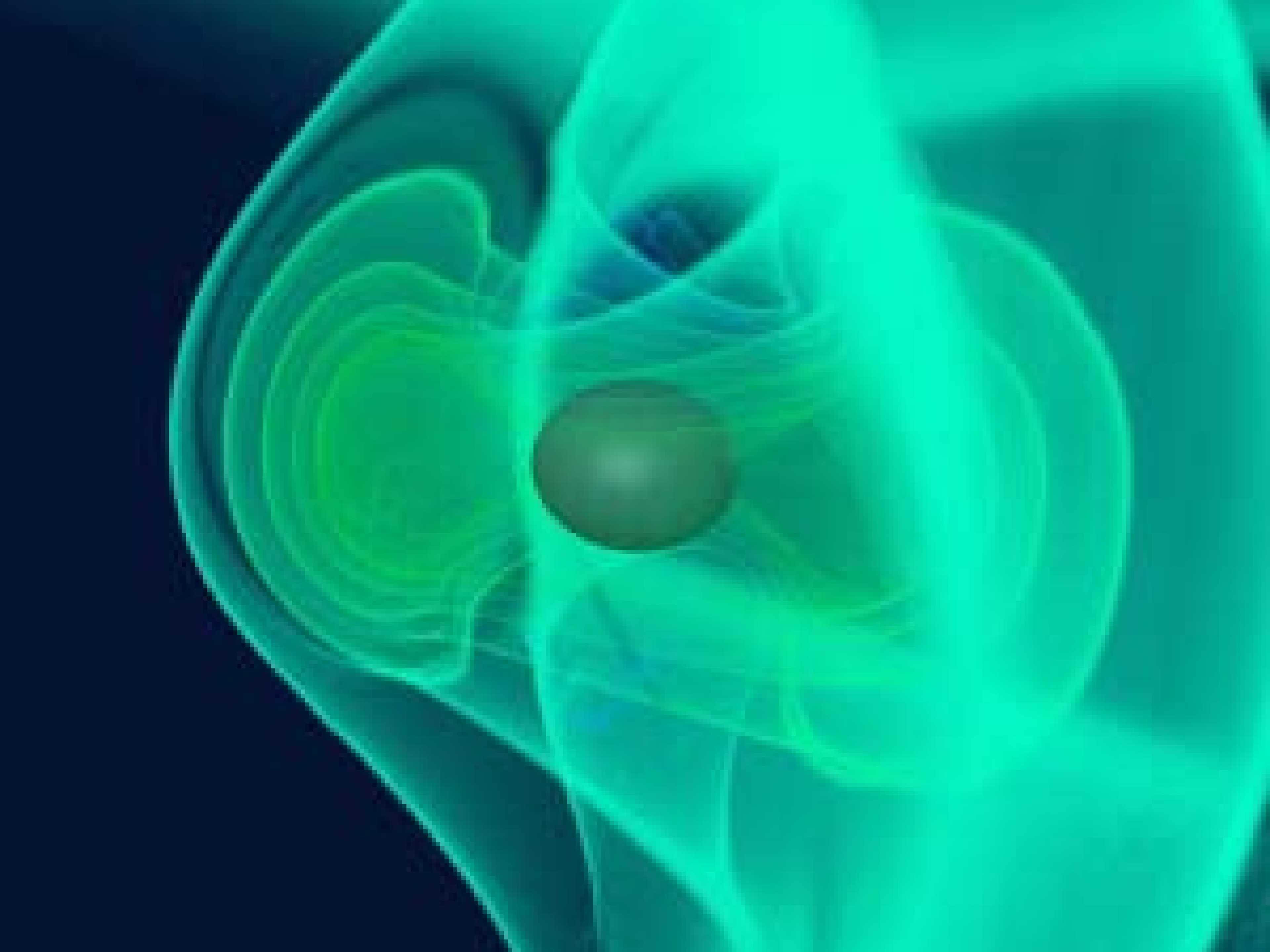
GW → ASTROPHYSICS

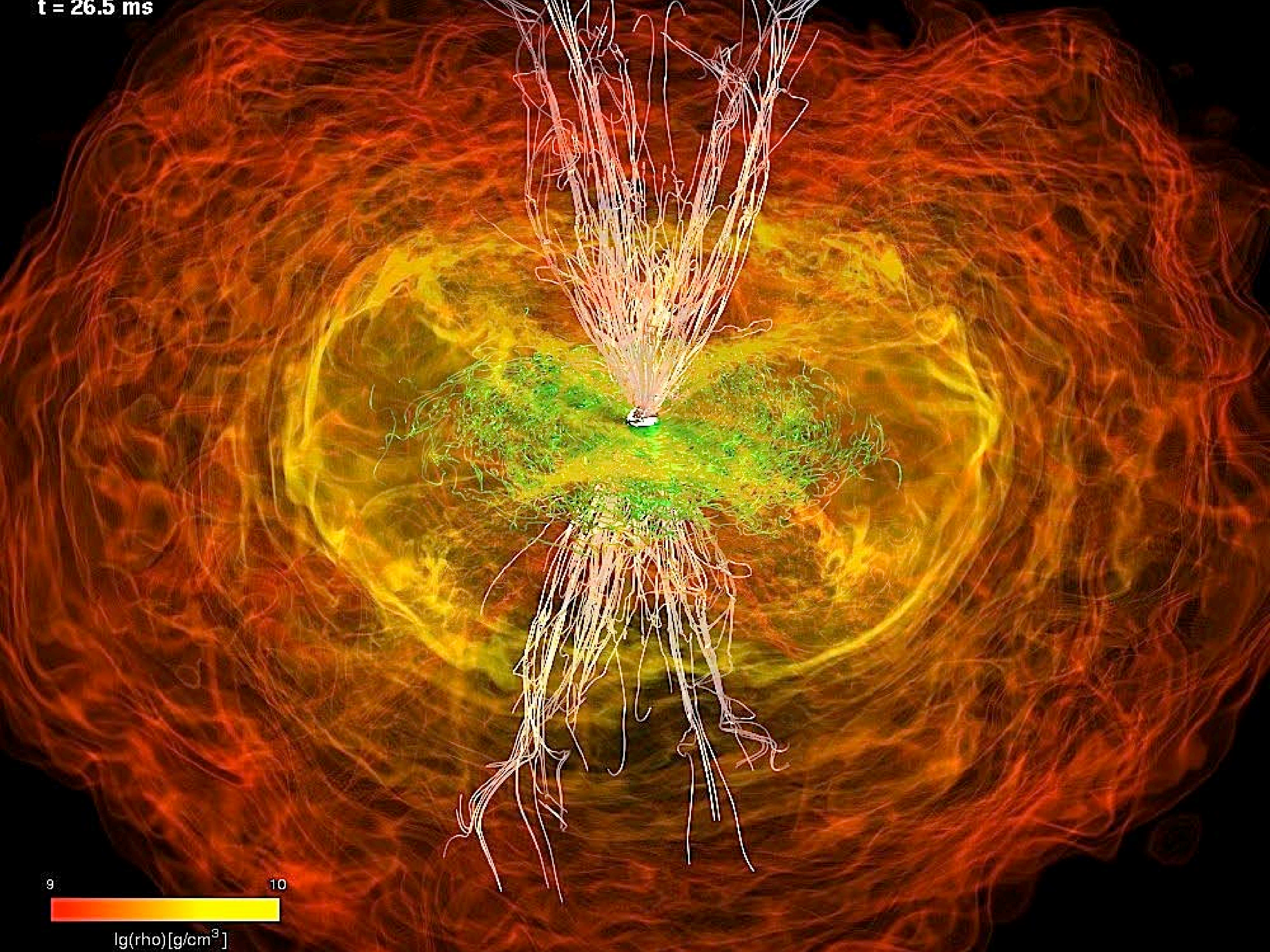


How to cook up a magnetized, relativistic jet
in a “short” (CBC) gamma-ray burst… (numrel.aei.mpg.de)




t = 7.4 ms
9
14
lg(rho)[g/cm^3]





t = 26.5 ms
9
10
lg(rho)[g/cm^3]

WHAT ARE GW?

A bit of theory…

RIPPLES IN SPACETIME

- Harmonic vibration of the metric $g_{\alpha\beta}$ and hence curvature $R_{\alpha\beta\gamma\delta}[g_{\alpha\beta}]$ in vacuum

$$\boxed{R^{\mu}{}_{\alpha\mu\beta} - \tfrac{1}{2} R^{\mu\lambda}{}_{\mu\lambda} g_{\alpha\beta} = 0 = (8\pi G / c^4) T_{\alpha\beta}}$$

- Propagates at c

- Interval between two events = flat space + gravitational wave

$$ds^2 = c^2 dt^2 - d\mathbf{x}^2 + h_{\alpha\beta} dx^{\alpha} dx^{\beta}$$

TWIN POLARIZATIONS

- Bianchi $\rightarrow$ six polarizations (e.g. breathing)
- GR $\rightarrow$ $\boldsymbol{R^{\mu}{}_{\alpha\mu\beta}=0}$ $\rightarrow$ transverse, traceless gauge

$$h_{ij}(z,t) = h^{+}(t\text{-}z/c)\boldsymbol{e^{+}{}_{ij}} + h^{\times}(t\text{-}z/c)\boldsymbol{e}\ \boldsymbol{{}_{ij}}$$

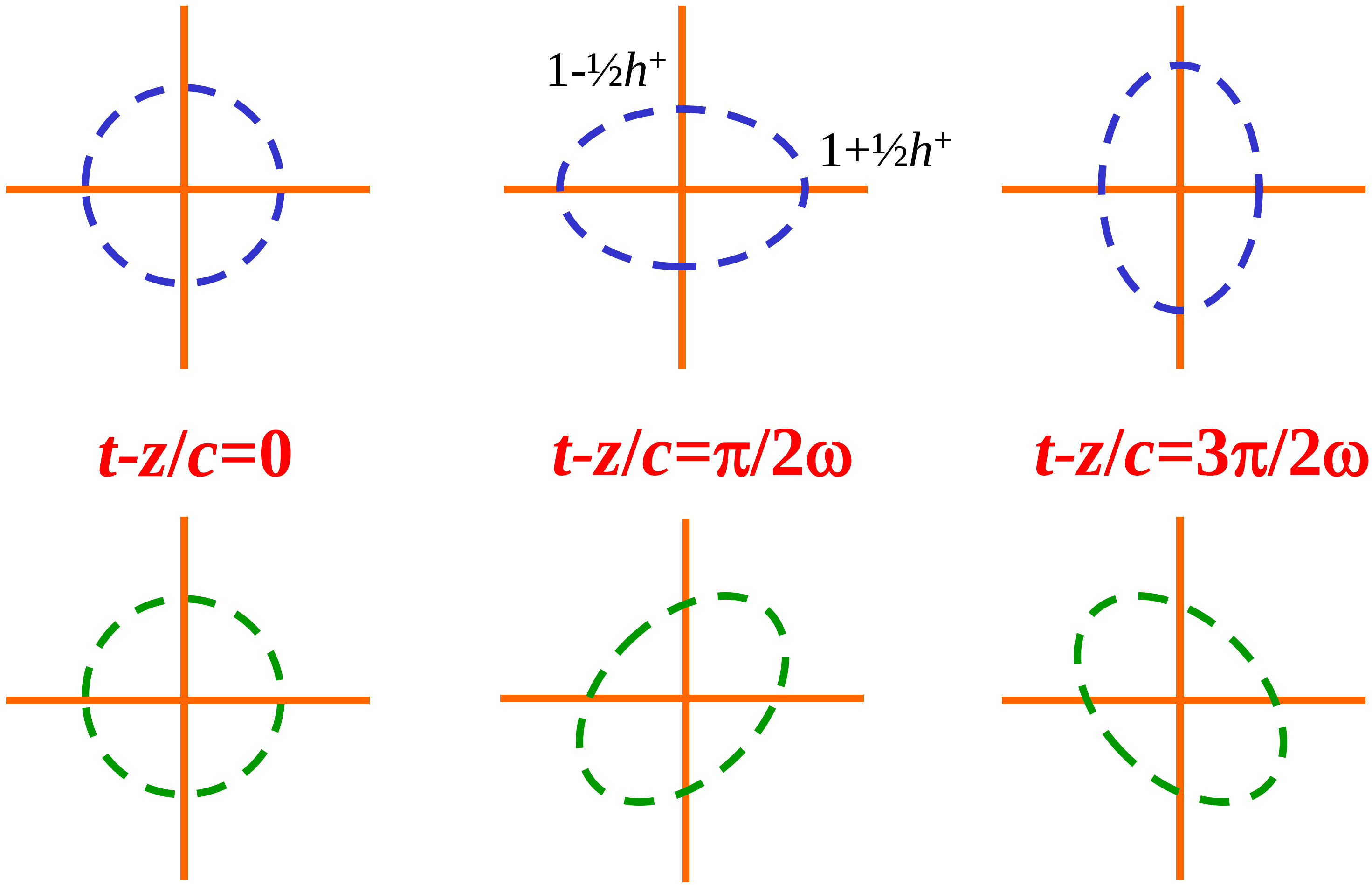
- TIDAL acceleration: no “00” component

Plus mode: $e^{+}{}_{xx} = 1 = -e^{+}{}_{yy}$

- $ds^2 = c^2dt^2 - (1-h^{+})dx^2 - (1+h^{+})dy^2 - dz^2$

Cross mode: $e^{+}{}_{xy} = 1 = e^{+}{}_{yx}$

- $ds^2 = c^2dt^2 - dx^2 - dy^2 - 2h^{\times}dx\,dy - dz^2$



Locus of events “equidistant” from (0,0)
(ring of test masses)

DIPOLE ANTENNA

- Mass dipole moment

$$q_i = \Sigma m x_i$$

- Power output $\propto (d^2 q_i / dt^2)^2$ **vanishes** (momentum conserved if zero external force)

$$d^2 q_i / dt^2 = \Sigma m \, d^2 x_i / dt^2 = 0$$

- Nonzero external force ("giant wrench") $\rightarrow$ equal but opposite dipole moment

GW SOURCE PHYSICS

- Mass quadrupole $\propto$ energy density $\propto T^{00}$

$$\boxed{h_{lm} = \frac{G}{c^4 d}\frac{\partial^2}{\partial t^2}\int d^3x Y_{lm}^{*} r^l \rho}$$

$$\boxed{\frac{GM}{Rc^2}\left(\frac{Rf}{c}\right)^2\left(\frac{d}{R}\right)^{-1}\varepsilon_\rho}$$

- Current quadrupole $\propto$ momentum flux $\propto T^{0i}$

$$\boxed{h_{lm} = \frac{G}{c^5 d}\frac{\partial^2}{\partial t^2}\int d^3x Y_{lm}^{*} r^l \mathbf{x}\cdot \mathrm{curl}(\rho \mathbf{v})}$$

$$\boxed{\frac{GM}{Rc^2}\left(\frac{Rf}{c}\right)^2\left(\frac{d}{R}\right)^{-1}\frac{\varepsilon_{\rho\mathbf{v}} V}{c}}$$

SOME USEFUL FORMULAS

- **Metric perturbation** (Thorne 80 RevMP)

$$h_{jk}^{TT} = \frac{G}{c^4 d} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \left[\partial_t^{(l)} I^{lm} T_{jk}^{E2,lm} + \partial_t^{(l)} S^{lm} T_{jk}^{B2,lm} \right]$$

MASS CURRENT

- **Antenna pattern**

$$T_{jk}^{E2,lm}(\theta,\varphi) \propto r^2 [\nabla\nabla Y^{lm}]_{jk}^{STT} \qquad T_{jk}^{B2,lm}(\theta,\varphi) \propto [(\mathbf{x}\times\nabla)(r\nabla Y^{lm})]_{jk}^{STT}$$

- **Total power**

$$\frac{dE}{dt} = \frac{G}{32\pi c^4} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \left[\left| \partial_t^{(l+1)} I^{lm} \right|^2 + \left| \partial_t^{(l+1)} S^{lm} \right|^2 \right]$$

COMPACT SOURCES

$h \approx (V/c)^2 (r/R_s)^{-1}$ $L \approx c^5 G^{-1} (V/c)^6 (R/R_s)^{-2}$

- Hulse-Taylor binary: M_{Sun}, 10^9 m, 10^5 ms^{-1}

  $h \approx \mathbf{10^{-24}}$ @ 10 kpc $\leftrightarrow$ $L \approx 10^{27}$ erg s^{-1} $\boxed{\text{kHz}}$

- NS mountain: εM_{Sun}, 10^4 m, 10^7 ms^{-1}

  $h \approx \mathbf{10^{-16}\varepsilon}$ @ 1 kpc $\leftrightarrow$ $L \approx 10^{49}\varepsilon^2$ erg s^{-1} $\boxed{\text{kHz}}$

- Merging galaxies: $10^6 M_{Sun}$, 1 pc, 10^6 ms^{-1}

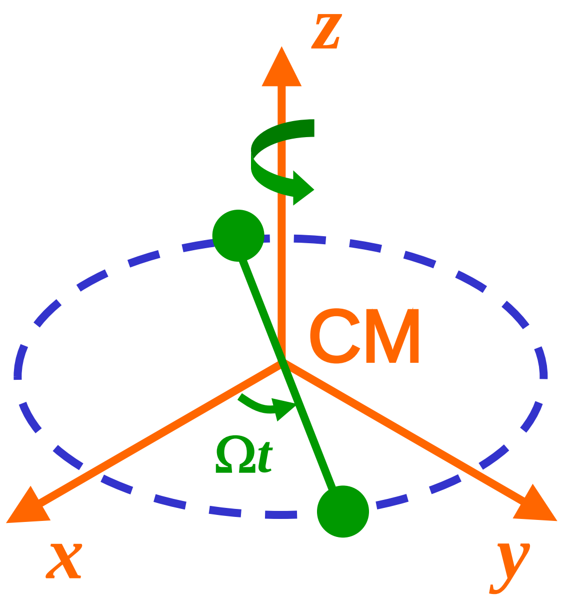
  $h \approx \mathbf{10^{-18}}$ @ 1 Mpc $\leftrightarrow$ $L \approx 10^{36}$ erg s^{-1} $\boxed{\text{mHz}}$

EX: COALESCING BINARY

- $I_{xx} = -I_{yy} = \frac{1}{2}\mu a^2\cos(2\Omega t)$
  $I_{xy} = I_{yx} = \frac{1}{2}\mu a^2\sin(2\Omega t)$

- Observer along z axis:
  $h^{+} = \cos(2\Omega t)$, $h^{\times} = \sin(2\Omega t)$

- Observer along y axis:
  $h^{+} = \frac{1}{2}\cos(2\Omega t)$, $h^{\times} = \mathbf{0}$



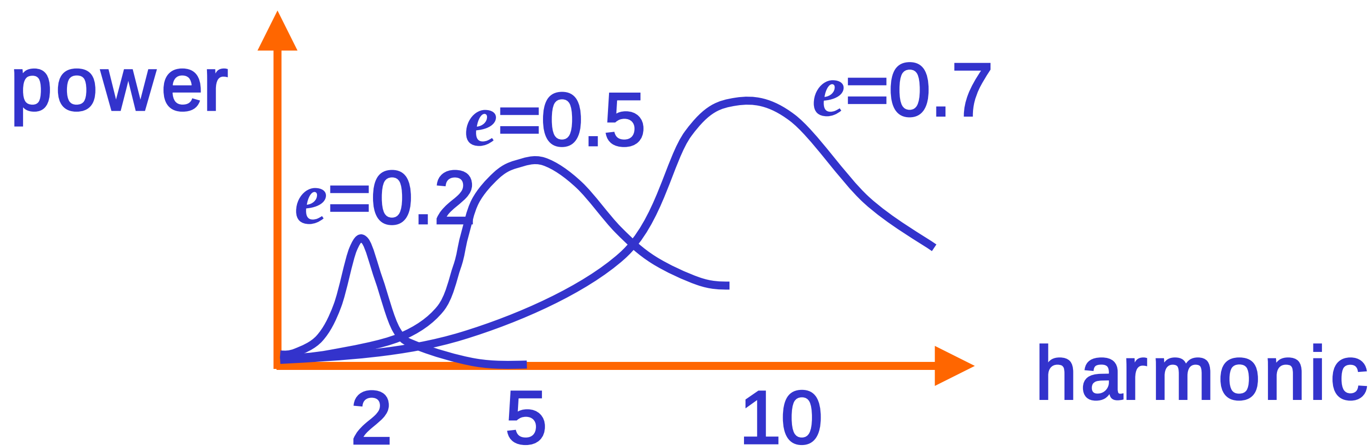
CIRCULAR ORBIT

- h_{ij} oscillates at $2\Omega_{\mathrm{orb}}$
- Orbit **shrinks**
- Coalesces after

$$47\ \mathrm{Gyr}\ (P_{\mathrm{orb}}/1\ \mathrm{day})^{8/3} (M_1/M_{\mathrm{Sun}})^{-5/3}\ q(1+q^{-1})^{1/3}$$

ELLIPTICAL ORBIT

- **All** harmonics of Ω_{orb}
- Orbit shrinks and **circularizes**
- GW enhanced ($\times 10$ for $e=0.6$) (Peters 73)



GW are neither scattered nor absorbed;
they map $T^{\mu\nu}$ transparently

(e.g. Thorne's "lattice" of NSs in space)

PRECISION GW COSMOLOGY

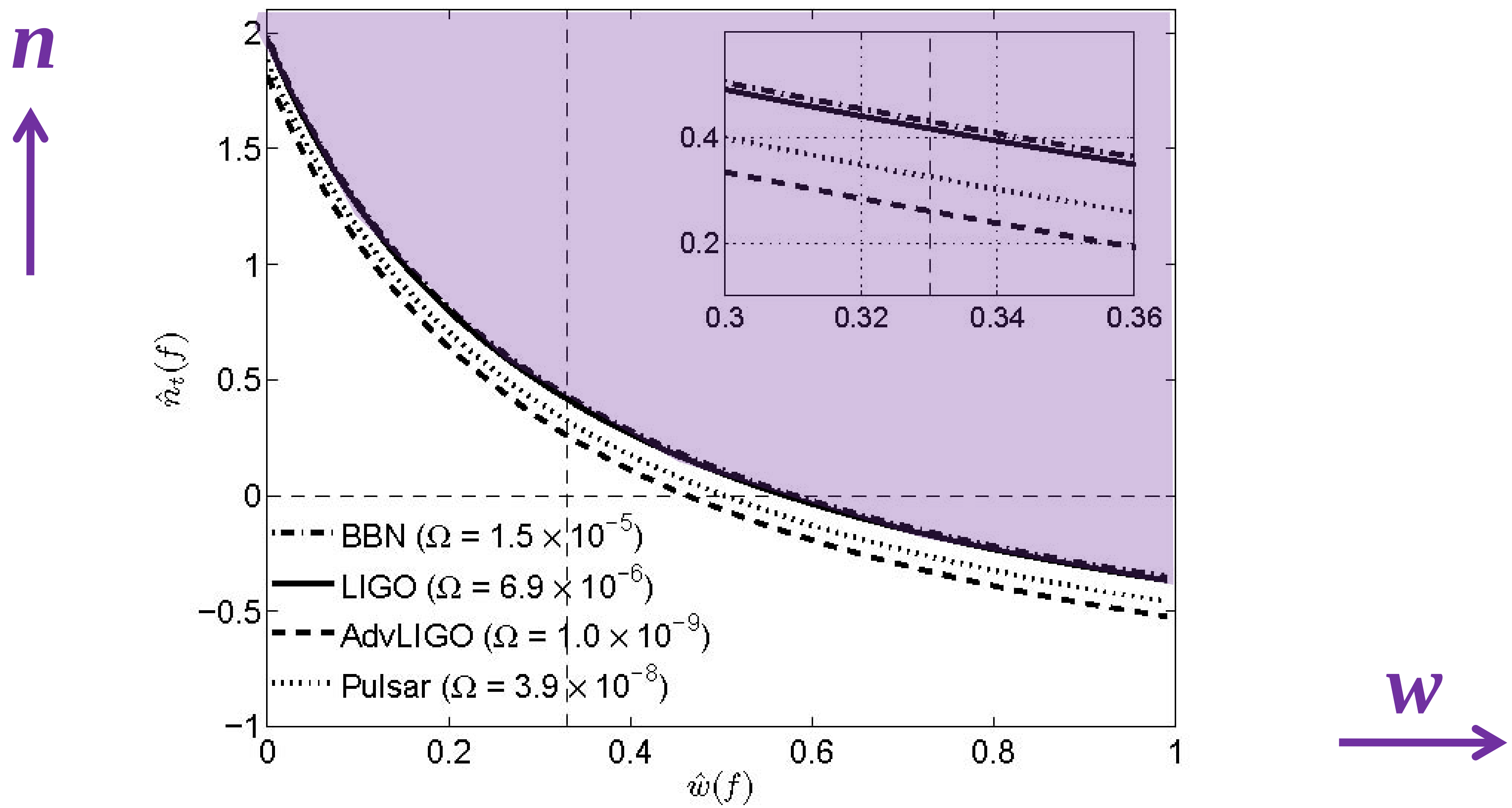
New results already!

PRIMORDIAL BACKGROUND

- Vacuum fluctuations amplified during **inflation** *before the surface of last scattering*
- Stochastic → cross-correlate IFOs (42-169 Hz)

$$\Omega_{GW}(f) = \frac{d(\rho_{GW}/\rho_c)}{d(\ln f)} = \Omega_\alpha (f/100\,\mathrm{Hz})^\alpha \leq 6.9\times10^{-6}$$

- Beats limits on ρ_{GW} / ρ_c from (Abbott et al. 2009)
  → **light element** abundances: $\Omega_0 \leq 1.1 \quad 10^{-5}$
  → **CMB** spectrum: $\Omega_0 \leq 9.5 \quad 10^{-6}$



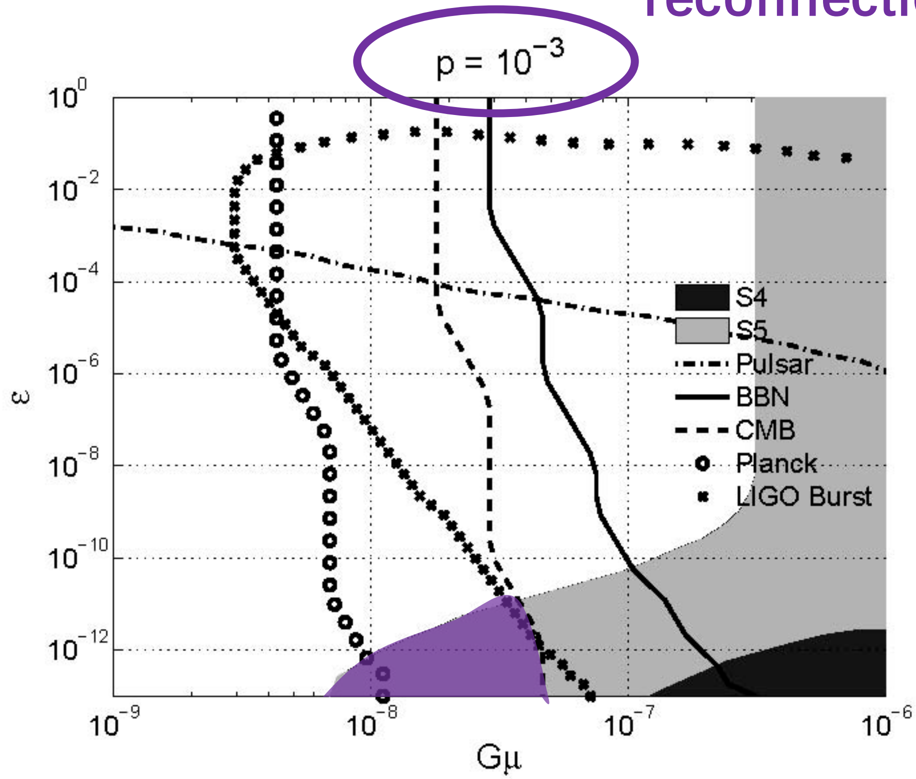
GW spectrum $\propto f^{\,\alpha}$ depends on $\alpha(n,w)$

$\rightarrow$ tilt in fluctuation spectrum $P(k) \propto k^{1+n}$

$\rightarrow$ dark energy equation of state $w = p/\rho$

reconnection probability

loop size



(Abbott et al. 09 Nat)

tension

Cosmic strings (reconnecting tangle)

→ radiate when cusps point at Earth

→ loop size ↔ GW radiation reaction

→ string tension ↔ vibration frequency

STANDARD SIRENS

- **Short GRB** with host galaxy **red shift** (Dalal et al. 06; Nissanke et al. 10) or without (Finn 96)
- In-spiral waveform known
- Absolute distance to surface of last scattering (angular scale of CMB peaks, matter density from peak heights)
- **Dark energy EOS:** w to 9% and H_0 to 2%
- Even at low z, **luminosity distance** $\propto \int dz\, H(z)^{-1}$ contains energy densities at all z

- GW data & Post-Newtonian waveform

  → red shifted chirp mass $(1+z)\,(m_1 m_2)^{3/5}(m_1+m_2)^{1/5}$

  → coalescence time & phase

  → luminosity distance

  → sky position & orbital axis

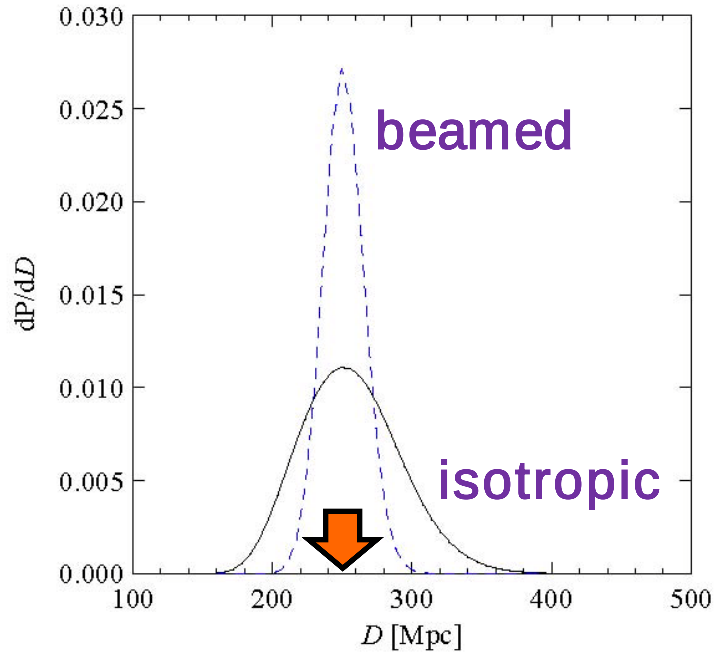
- Three "phase" parameters exquisite!
- Five "amplitude" parameters so-so…

- Distance and orbital inclination degenerate

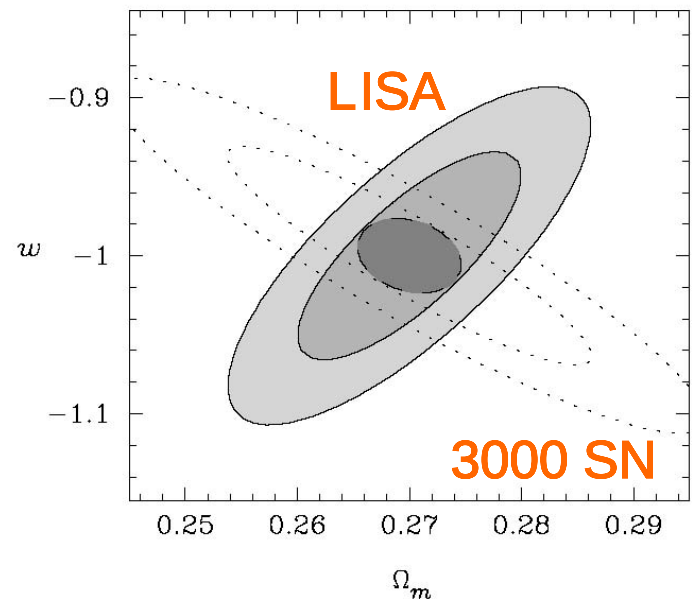
  → break with detector network (e.g. Australia)
- Are GRBs beamed? (Soderberg et al. 06)

  → line of sight ∥ orbital axis

- Monte-Carlo distances for random orientations
  (Dalal et al. 06)
- "No beaming" error
  $\approx 9\ (D/0.2\mathrm{Gpc})\ \%$
- CMB errors dominate



- LISA SMBH in-spirals
- Distance error $\approx 1\%$
- ☹ Lensing fluctuations
- EM counterparts?
  (Milosavljevic & Phinney 05)



SUMMARY

- GW in pictures and sounds
- Compendium of **science questions**

- GW physics: polarization, multipoles
- Compact binary coalescence

- The future is here… e.g. **GW cosmology**