The Exterior Spacetime of Relativistic Stars in Quadratic Gravity

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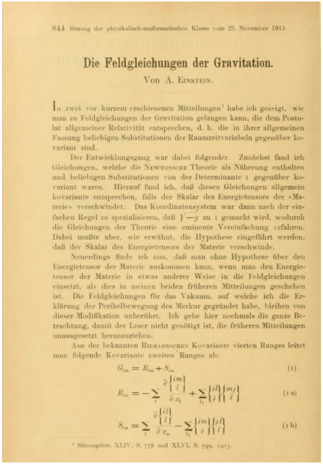
Outlook

- Motivation
- Why Quadratic Gravity
- Neutron Stars
- Finding the Metric

Brief History of GR


844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

Aus der bekannten Riemannschen Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \qquad (1)$$

$$R_{im} = -\sum_{l} \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{l\rho} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix} \qquad (1a)$$

$$S_{im} = \sum_{l} \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{l\rho} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix} \qquad (1b)$$

[1] Sitzungsber. XLIV, S. 778 und XLVI, S. 799. 1915.

Figure: Original 1915 Paper

1905 - Special Relativity

- Equivalence of observation
- Speed of light is constant
- Time dilation/length contraction

1915 - General Relativity

- Curvature of space-time is related to energy present
- Curvature representative of gravity

Passed Tests

What's wrong with GR?

- Perihelion Precession of Mercury

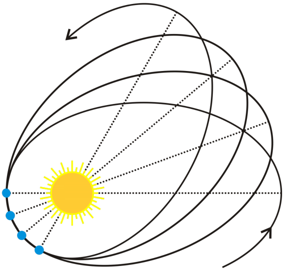


Figure: Mercury Orbit

Passed Tests

What's wrong with GR?

- Perihelion Precession of Mercury
- Light Bending

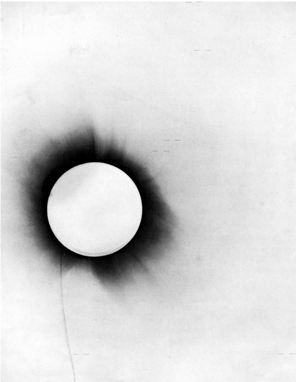


Figure: A. Eddington

Passed Tests

What's wrong with GR?

- Perihelion Precession of Mercury
- Light Bending
- Gravitational Redshift

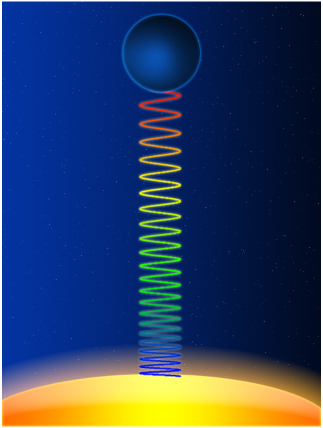


Figure: Gravitational Redshift (wikipedia)

Passed Tests

What's wrong with GR?

- Perihelion Precession of Mercury
- Light Bending
- Gravitational Redshift
- Shapiro Delay

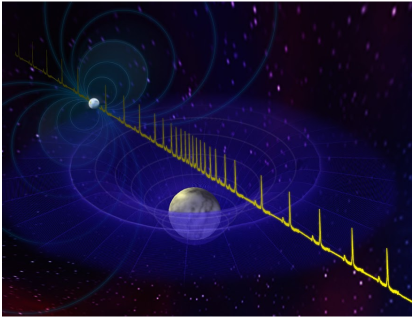


Figure: Shapiro Delay (Brian Koberlein)

Passed Tests

What's wrong with GR?

- Perihelion Precession of Mercury
- Light Bending
- Gravitational Redshift
- Shapiro Delay
- Frame Dragging

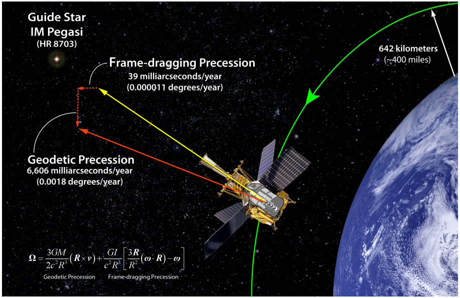


Figure: Gravity Probe B

Passed Tests

What's wrong with GR?

- Perihelion Precession of Mercury
- Light Bending
- Gravitational Redshift
- Shapiro Delay
- Frame Dragging
- Geodetic Effect

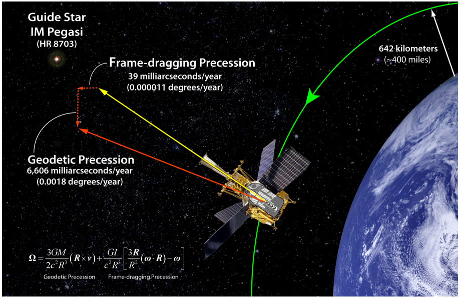


Figure: Gravity Probe B

Passed Tests

What's wrong with GR?

- Perihelion Precession of Mercury
- Light Bending
- Gravitational Redshift
- Shapiro Delay
- Frame Dragging
- Geodetic Effect
- Binary Pulsars

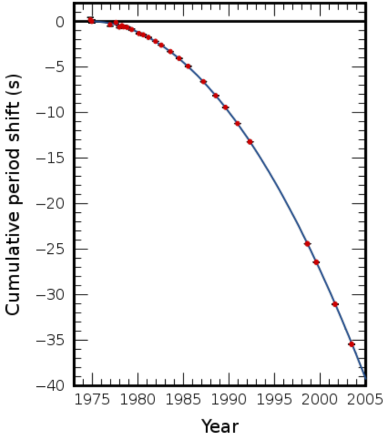


Figure: Hulse-Taylor Binary

Passed Tests

What's wrong with GR?

- Perihelion Precession of Mercury
- Light Bending
- Gravitational Redshift
- Shapiro Delay
- Frame Dragging
- Geodetic Effect
- Binary Pulsars
- Gravitational Waves

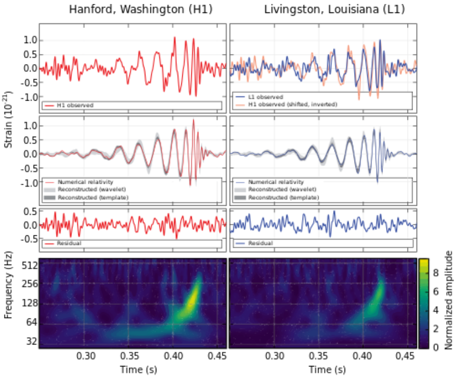
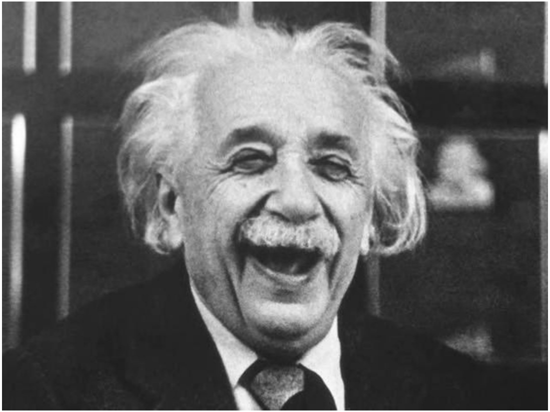


Figure: (LIGO) Gravitational Waves

When asked how he would have felt if his theory would fail

Then I would feel sorry for the dear Lord. The theory is correct anyway.

- Albert Einstein (1919)



Why Test GR?

- Quantum Mechanics
  - GR is a classical theory
  - Not quantized
  - Cannot reconcile with other forces
- Cosmology
  - Inflation
  - Dark Matter
  - Dark Energy
- More testing in strong-field
  - Solar System tests passed
  - Probe area near compact objects

Quadratic Gravity

- Motivation
  - Some quantum gravity theories (string theory, loop quantum gravity) induce higher order curvature terms naturally
  - GR may be corrected at low energy scales to gain higher order curvature terms
- Curvature squared terms
  - R^2
  - $R_{ab}R^{ab}$
  - $R_{abcd}R^{abcd}$
  - ${}^*R_{abcd}R^{abcd}$

Einstein-dilaton-Gauss-Bonnet* (EdGB)

- Action

$$S = \int \sqrt{-g} \left[\kappa R + \alpha \, \phi \, \mathcal{R}_{GB} - \frac{1}{2} \left(\nabla_a \phi \right) \left(\nabla^a \phi \right) - V(\phi) \right] + S_m$$

with

$$\kappa = (16 \pi G)^{-1}$$

$$\mathcal{R}_{GB} = R^2 - 4 \, R_{ab} R^{ab} + R_{abcd} R^{abcd}$$

- Small corrections to GR $\left(\alpha / L^2 \ll 1 \right)$
- Field Equations

$$\kappa \, G_{ab} + T^{GB}_{ab} = T^{m}_{ab} + T^{\phi}_{ab}$$
$$\Box \phi = -\alpha \mathcal{R}_{GB}$$

Introduction to Neutron Stars

- Formed from collapse of large star
- Mass $\sim$ 1.4 - 2 $M_{\odot}$
- Radius $\sim$ 10 km

Introduction to Neutron Stars

- Huge densities

$$\sim 10^{15}\ [\mathrm{g/cm^3}]$$

- Huge surface gravity

$$\sim 10^{12}\ [\mathrm{m/s^2}]$$

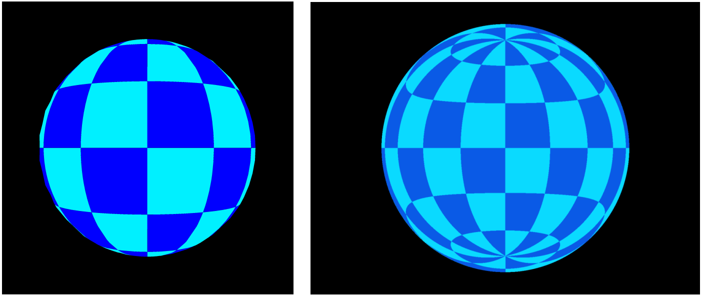


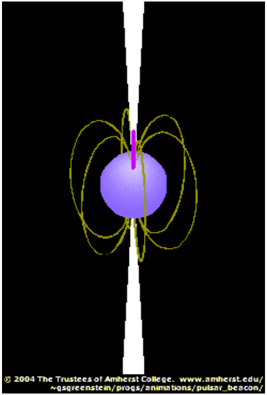
Figure: Corvin Zahn, Institut für Physik, Universität Hildesheim, Tempolimit Lichtgeschwindigkeit (M=1, R=4)

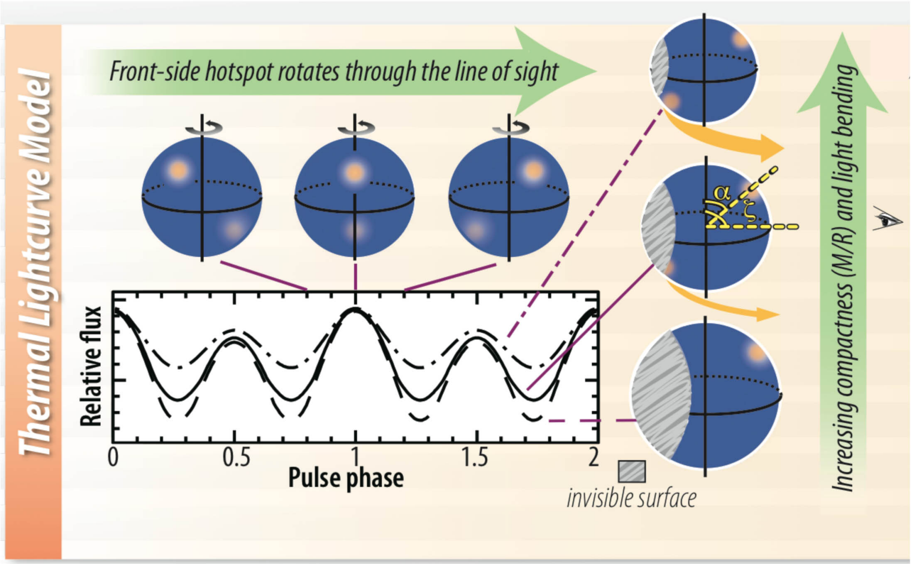
Introduction to Neutron Stars

- Huge magnetic fields

$$10^4 - 10^{11}\ [\mathrm{T}]$$

- Rotating NS → Pulsars





Thermal Lightcurve Model
Front-side hotspot rotates through the line of sight
Relative flux
0
0.5
1
1.5
2
Pulse phase
α
ζ
invisible surface
Increasing compactness (M/R) and light bending

Figure: NASA

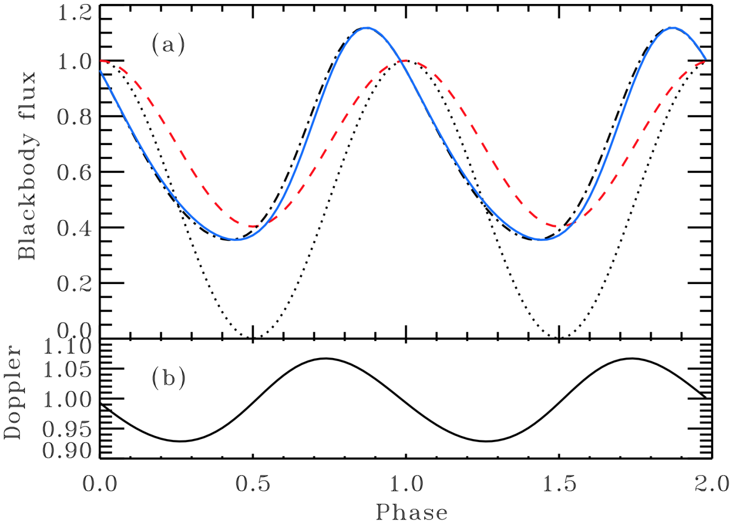


Figure: J. Poutanen - arxiv:0809.2400[astro-ph]

Scalar-Tensor Theory

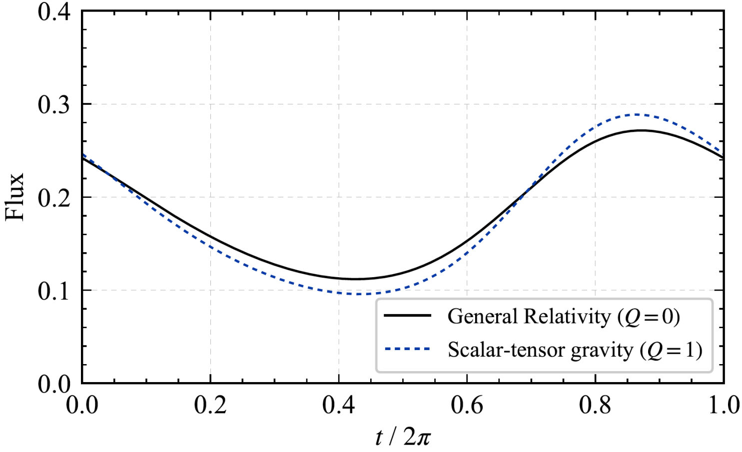
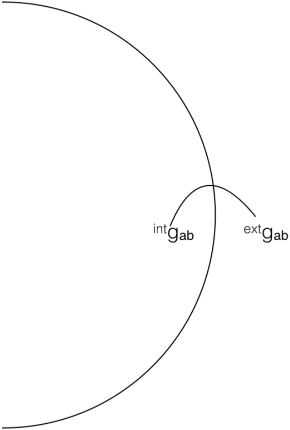


Figure: H.O. Silva and N. Yunes - arxiv:1808.04391[gr-qc]

What do we want



- Smooth
- Continuous
- Asymptotically flat
- Not singular

Ansatz

- Begin with the assumption

$$ds^2 = -e^{2\tau} dt^2 + e^{2\sigma} dr^2 + r^2 d\Omega^2$$

- Assume our expansions

$$\tau = \tau_0 + \alpha^2 \tau_2$$
$$\sigma = \sigma_0 + \alpha^2 \sigma_2$$
$$\phi = \phi_0 + \alpha \phi_1$$

- Solve order-by-order

Exterior $\mathcal{O}\left(\alpha^0\right)$

- Birkhoff's Theorem

$$e^{2\tau_0} = \left(1 - \frac{a}{r}\right)$$
$$e^{2\sigma_0} = \left(1 - \frac{a}{r}\right)^{-1}$$

- ...that was easy (too easy)

Interior $\mathcal{O}\left(\alpha^0\right)$

- Assume perfect fluid

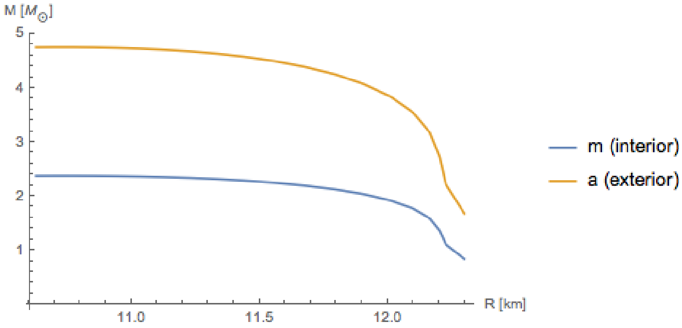
$$T_m^{ab} = (\rho + p)\, u^a u^b + p\, g^{ab}$$
$$u^a u_a = -1$$

- F.E. lead to Tolman-Oppenheimer-Volkoff equations

$$\partial_r m = 4\pi \rho r^2$$
$$\partial_r \tau_0 = \frac{4\pi p r^3 + m}{r\,(r - 2\,m)}$$
$$\partial_r p = -\frac{\left(4\pi p r^3 + m\right)(\rho + p)}{r\,(r - 2\,m)}$$

Interior $\mathcal{O}\left(\alpha^0\right)$

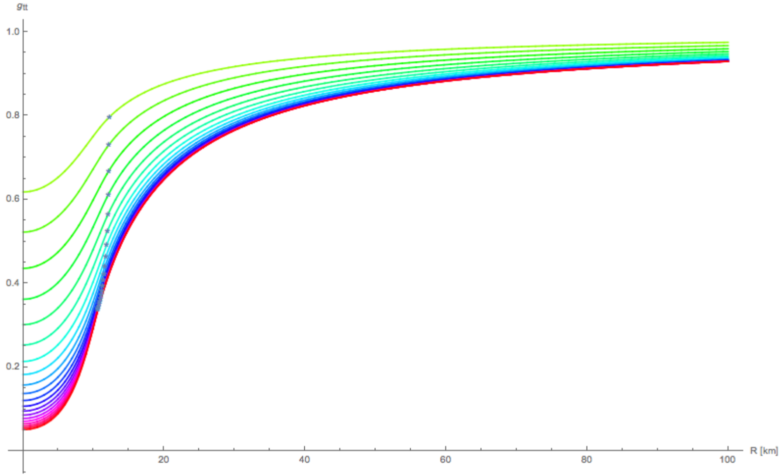
- Mass-Radius Curves



- Yes, $a = 2\,m$

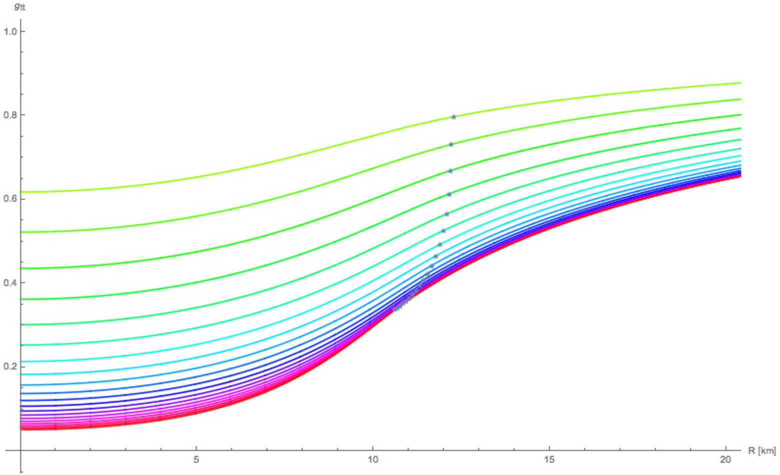
Interior $\mathcal{O}\left(\alpha^0\right)$

- g_{tt} Metric Solution



Interior $\mathcal{O}\left(\alpha^0\right)$

- g_{tt} Metric Solution



$\mathcal{O}\left(\alpha^2\right)$ Terms

Finally

- What?
  - Finding the metric outside of a neutron star in modified gravity.
  - Specifically, we are using EdGB, which can be shown as an extension of string theory.
- Why?
  - To develop a model which can be tested with observations of NS pulse profiles.
  - In an effort to place constraints on the theory.
- How?
  - By building the analytic metric using perturbation theory and solving the field equations order by order.

Thank You

Questions?