Astrophysical inference and transient gravitational wave astronomy

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OzGrav

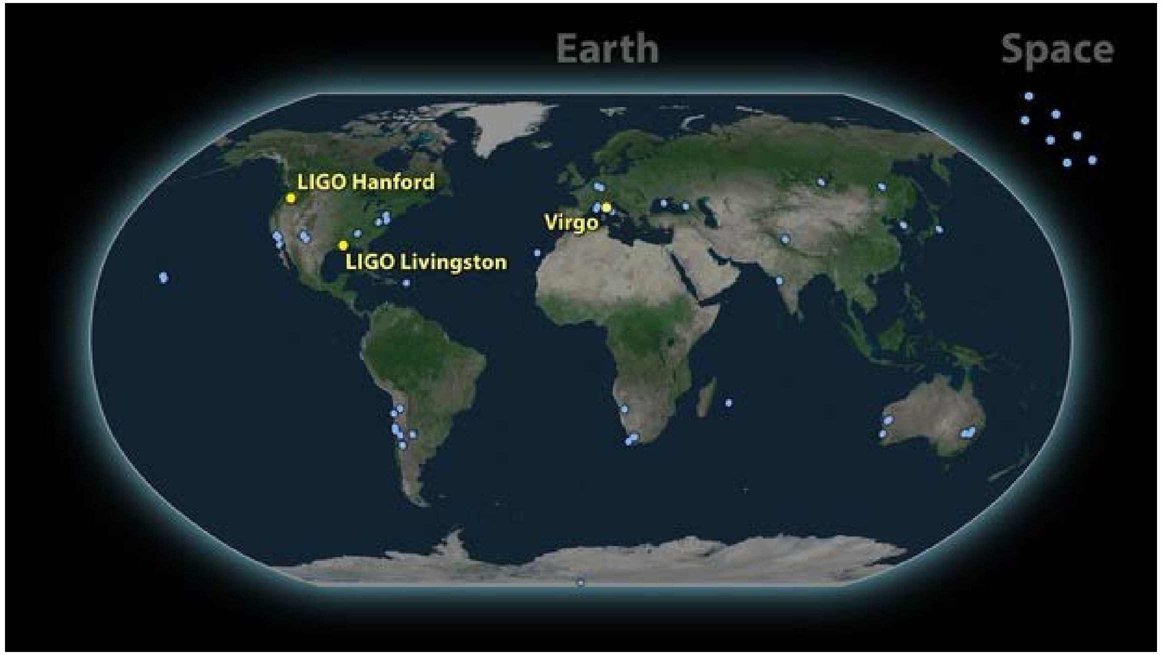
Talk overview

1. Transient gravitational wave astronomy
2. The tools of inference
3. Better understanding individual events
4. Better understanding a population of events
5. Inference as a detection tool
6. Introducing Bilby: the “Bayesian Inference Library”

Transient gravitational wave astronomy

Gravitational wave astronomy

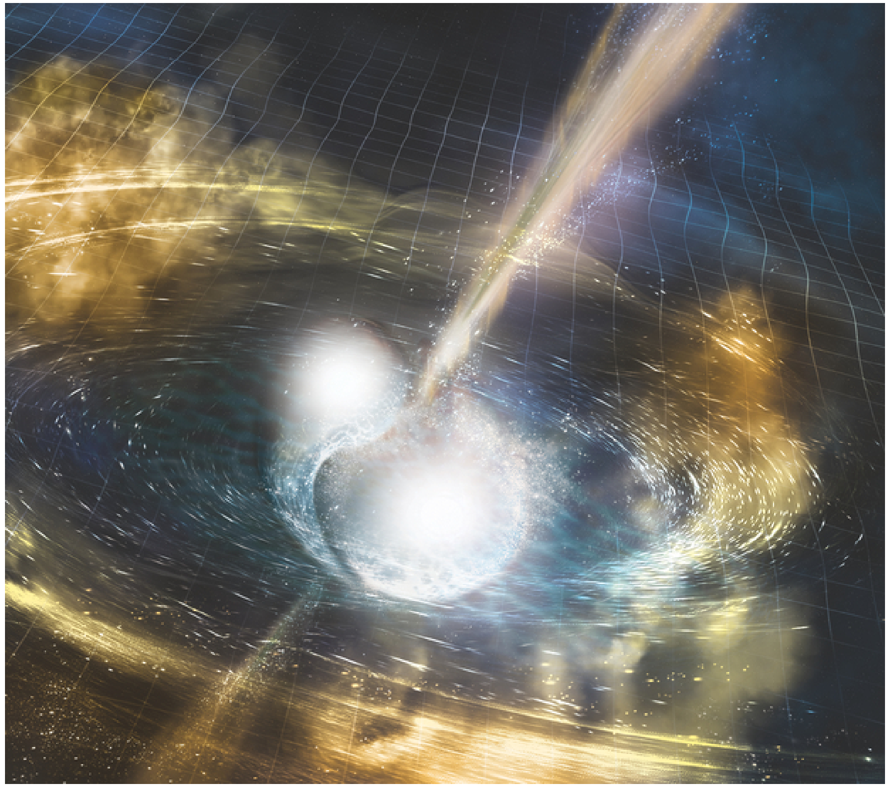
- Gravitational wave astronomy has already enjoyed much success
- Three ground-based interferometers
- Working in tandem with multi-messenger probes:
  - A multitude of telescopes spanning the electromagnetic spectrum
  - Neutrino detectors



[LIGO-Virgo]

Transient gravitational waves

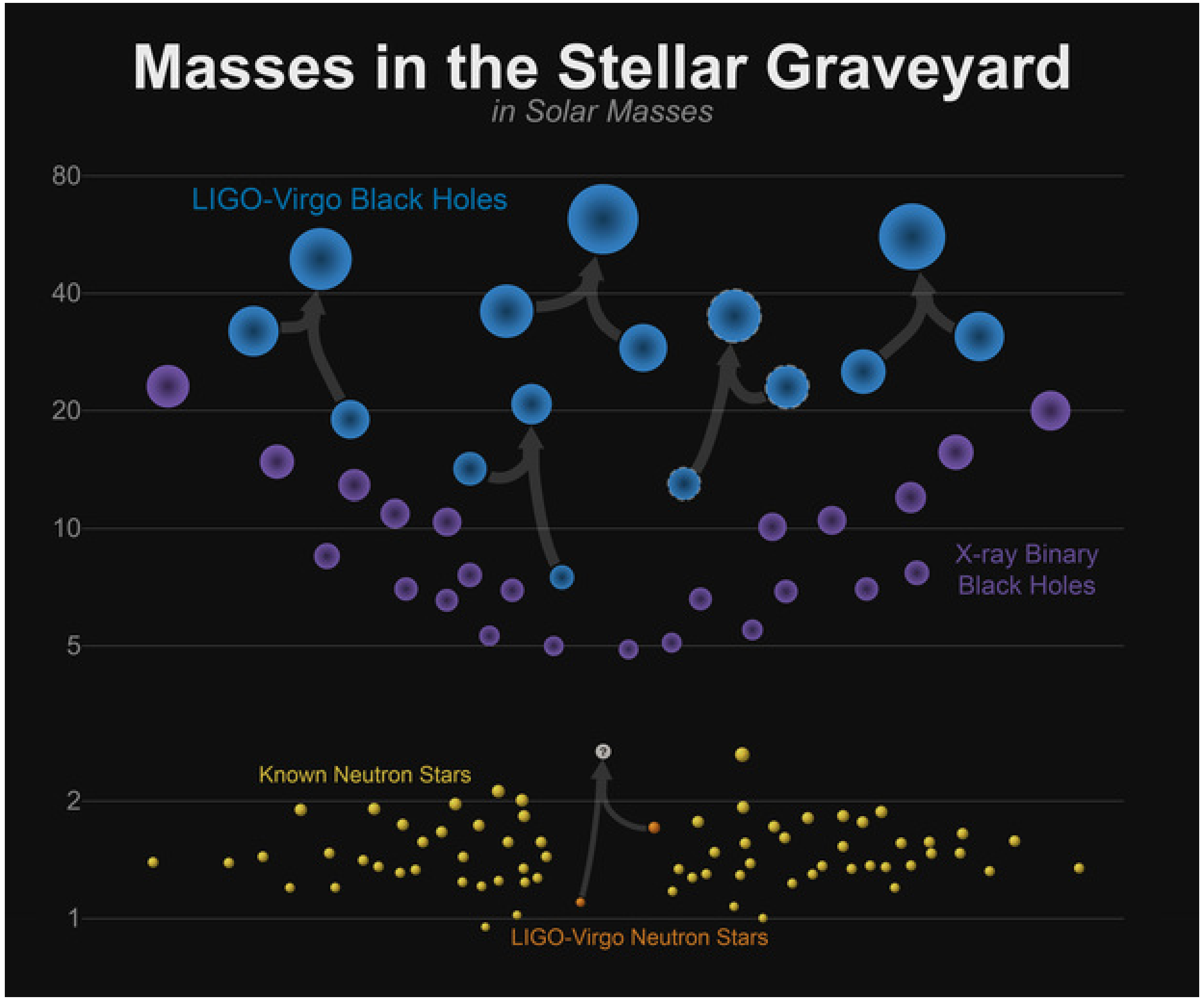
- Caused by catastrophic events
- Primarily considering CBC or *Compact Binary Coalescence*
- Typically "observable" for a fractions of a second up to a few tens of seconds
- Extra galactic in origin



NSF/LIGO/Sonoma State University/A. Simonnet

The view so far

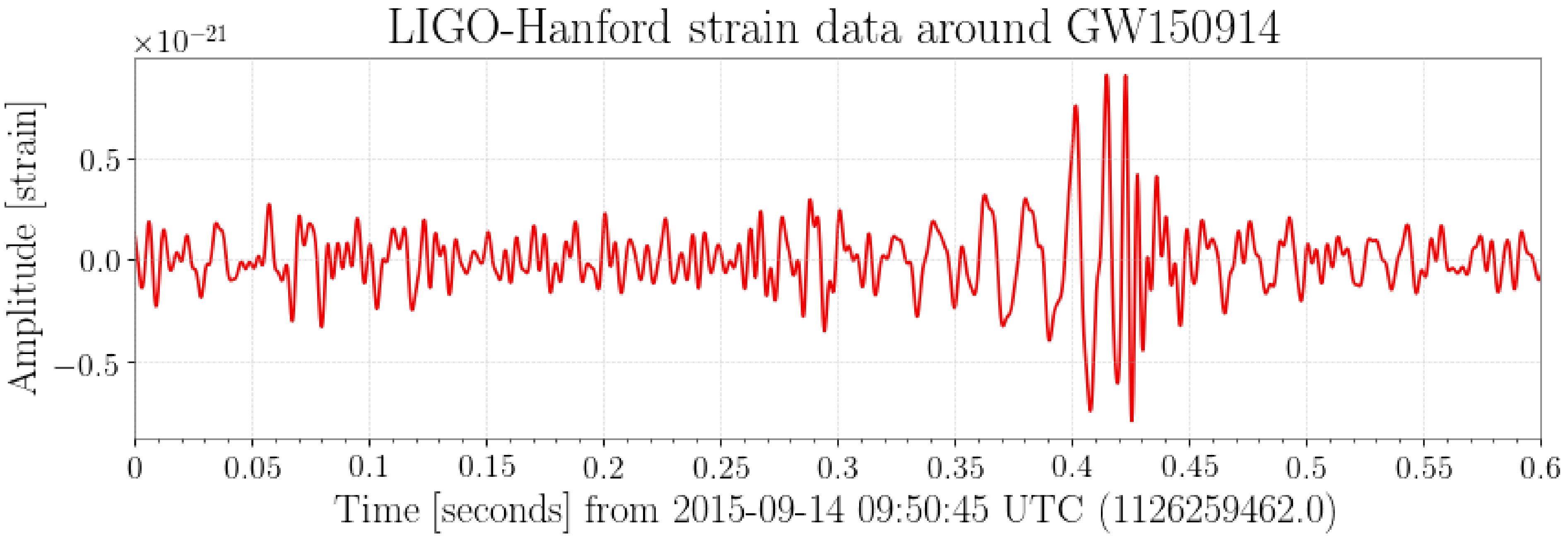
- After 2 observing runs
- 5 binary black hole mergers
- 1 binary neutron star merger:
  - Joint detection with Fermi-GBM/Integral
  - Subsequent detections across the spectrum
- 1 “LVT” (LIGO/Virgo trigger)



Credit: [LIGO-Virgo/Frank Elavsky/Northwestern University]

The tools of inference

Getting physics out of the interferometer



Source: [gwpy documentation]

Getting physics out of the interferometer

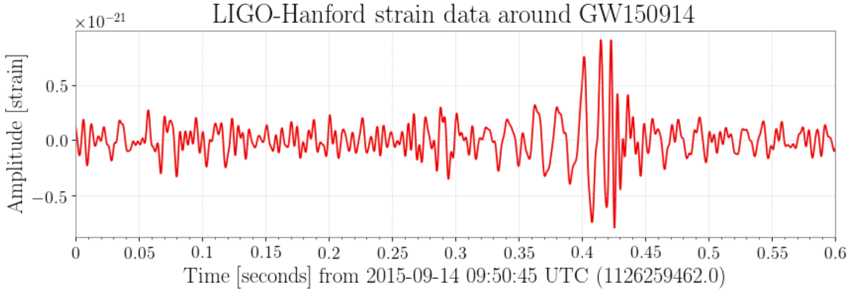


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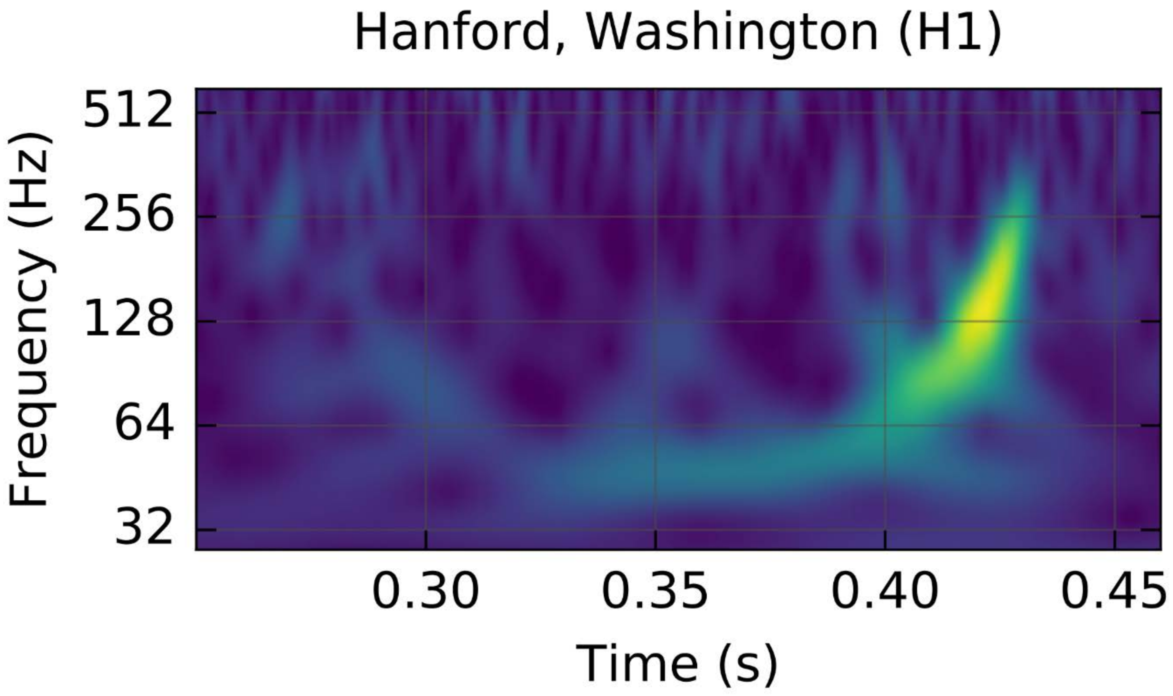
TABLE I. Source parameters for GW150914. We report median values with 90% credible intervals that include statistical errors, and systematic errors from averaging the results of different waveform models. Masses are given in the source frame; to convert to the detector frame multiply by $(1 + z)$ [90]. The source redshift assumes standard cosmology [91].

Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

Abbott et al (2016), PRL 116

From basic physics



Abbott et al (2016), PRL 116

- Abott et al (2016) Ann. Phys. 529
- Use time-frequency plot and peak amplitude
- Can obtain estimates such as

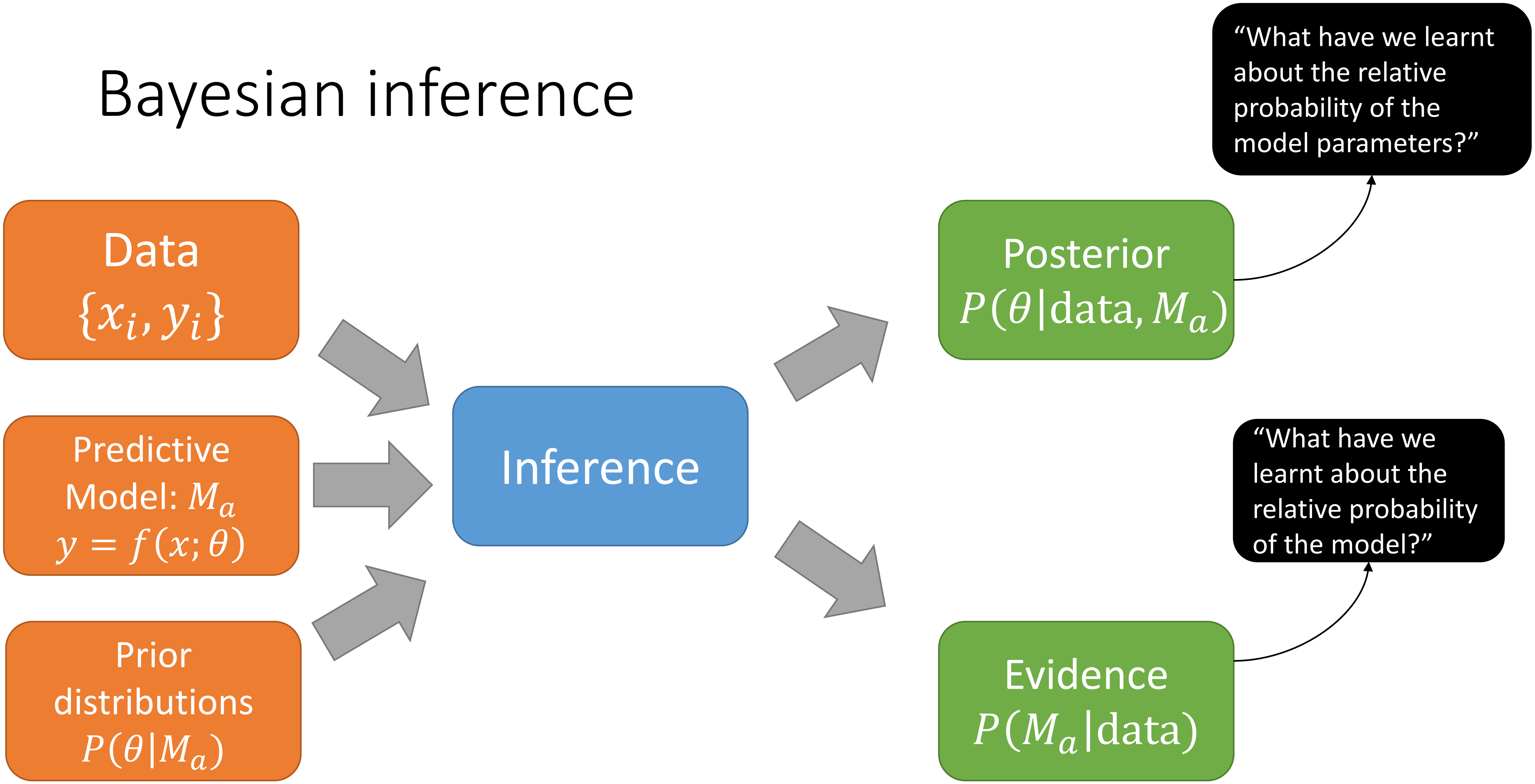
$$d_L \sim 300 \text{ Mpc}$$

- Not particularly accurate and difficult to estimate uncertainties

Bayesian inference

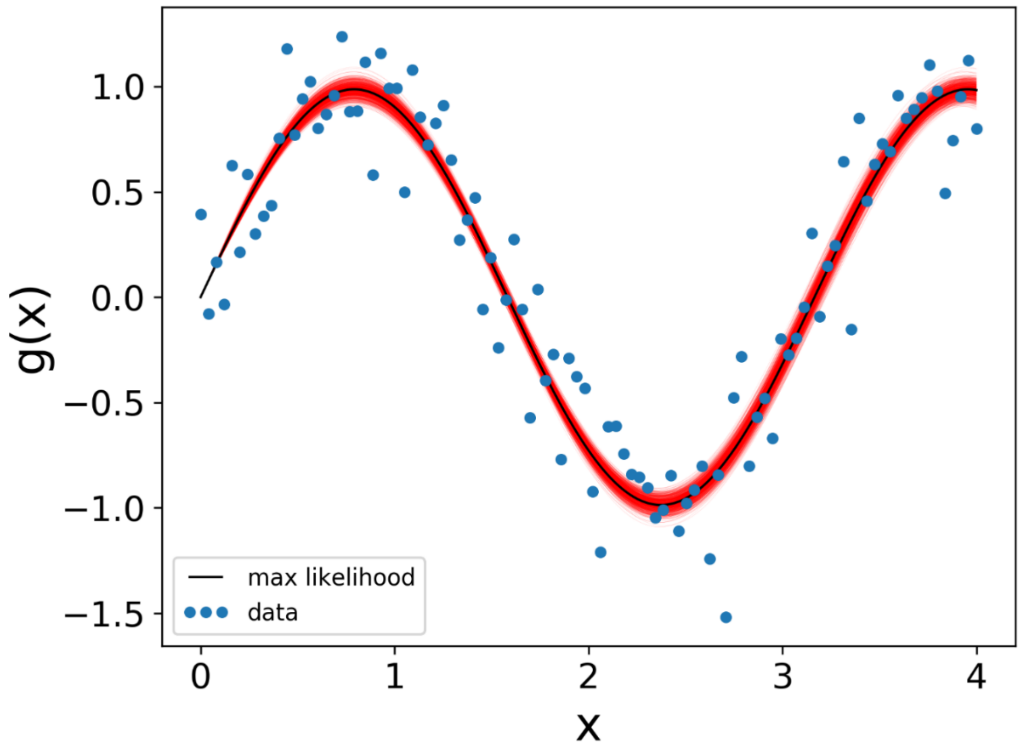
- Since we have uncertainties, we need to use probabilistic language
- Bayes rule: $P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$
- At its core, this is a statement of conditional probabilities
- But it provides tool set to:
    - Understand how well a given model fits the data
    - Compare between models

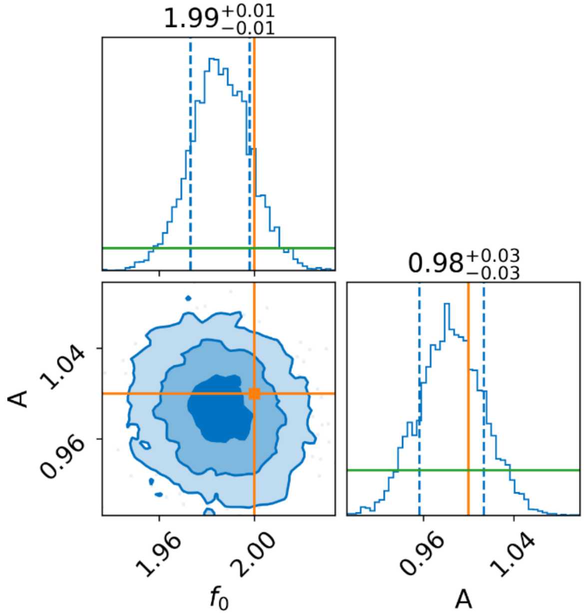
Bayesian inference



Simple example

- Model: $g(x; A, f_0) = A\sin(f\ x)$
- Data: simulate with some fixed values
- Priors:
  - $A \sim \mathrm{Uniform}(A_{\mathrm{min}}, A_{\mathrm{max}})$
  - $f_0 \sim \mathrm{Uniform}(f_{0\,\mathrm{min}}, f_{0\,\mathrm{max}})$
- Output:
  - Posteriors (corner plots)
  - Posterior predictive plots
  - Evidence (compare to other models)





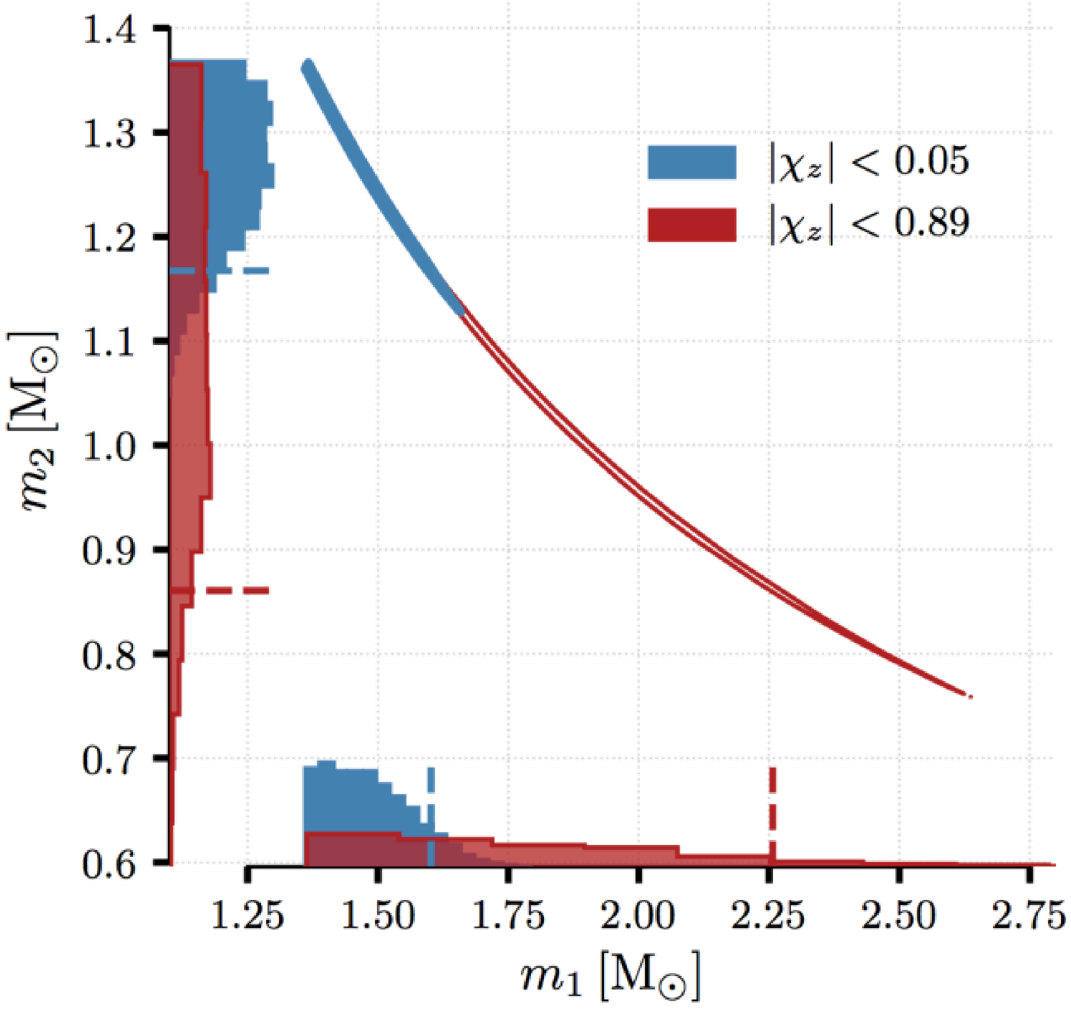
Inference for gravitational waves

- Fundamentally no different, but…
- Model of gravitational waveform
  - Typically has $\sim$15 parameters
  - Phenomological waveform tuned to numerical relativity
  - Slow to evaluate (typically $\sim$0.01 seconds/evaluation)
- Must also model the response of the detectors
  - E.g., frequency-dependent sensitivity
  - For long-lived signals this includes Doppler modulations

Better understanding individual events

Inference for individual events

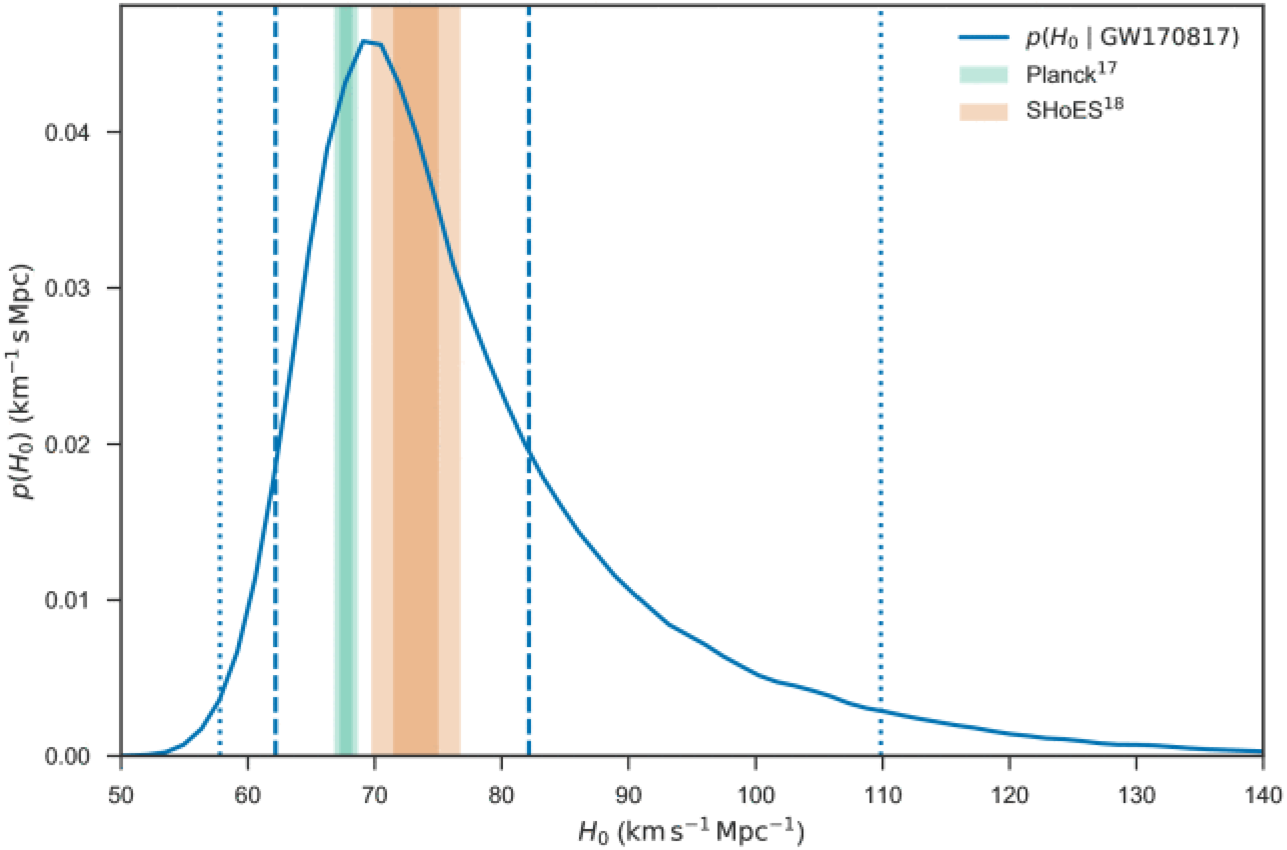
- GW170817 (the binary neutron star system)
- Masses clearly support that this is a pair of binary neutron stars
- Some wiggle room for a neutron star + black hole (but ruled out by other observations)



Abbott et al (2017) PRL 119

Inference for individual events

- GW170817 (the binary neutron star system)
- EM follow-up enabled a host-galaxy identification
- Giving an estimate of the distance
- Provides independent constraint on the Hubble constant



[Abbott et al (2017) PRL 119]()

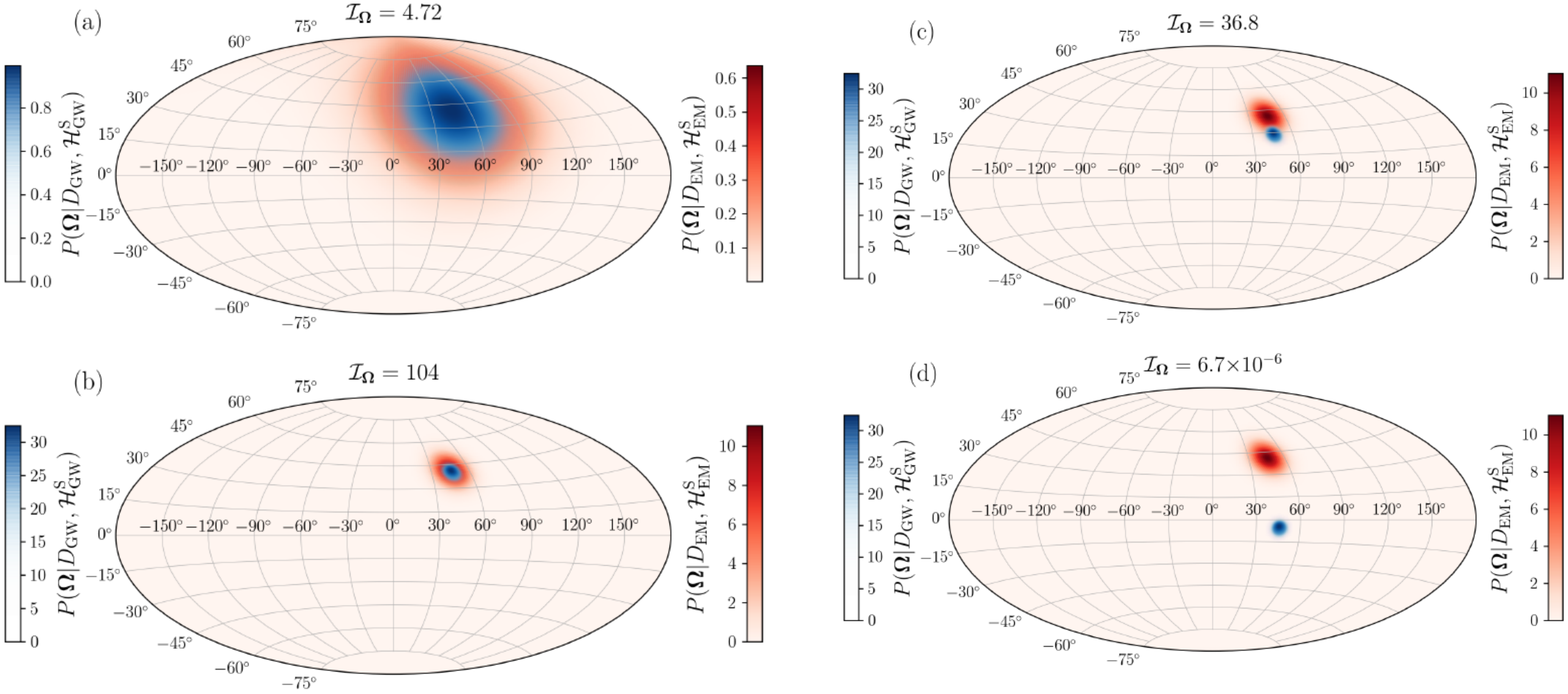
Joint inference between distinct data sets

- Given two data sets A and B both with "events" detected in them
- How do we decide if the events share a common origin?
- Generally, we look for common-parameters θ
- Ashton et al. (2018) APJ 860:

$$\frac{P(\mathrm{A}|\mathrm{Common})}{P(\mathrm{A}|\mathrm{Noise})} \times \frac{P(\mathrm{B}|\mathrm{Common})}{P(\mathrm{B}|\mathrm{Noise})} \times \int \frac{P(\theta|A)P(\theta|B)}{P(\theta)} d\theta = \frac{P(\mathrm{A,B}|\mathrm{Common})}{P(\mathrm{A,\,B}|\mathrm{Noise})}$$

Evidence for event in d_A × Evidence for event in d_B × Posterior overlap I_θ = Evidence for a common event

Example: sky-overlap



- Application to GW170817 and GRB 170817A favours a common signal by a factor of $\sim 10^6$
- Agreement with the original analysis of [Abbott et al (2017) APJL 848]

Better understanding a population of events

Hierarchical modelling

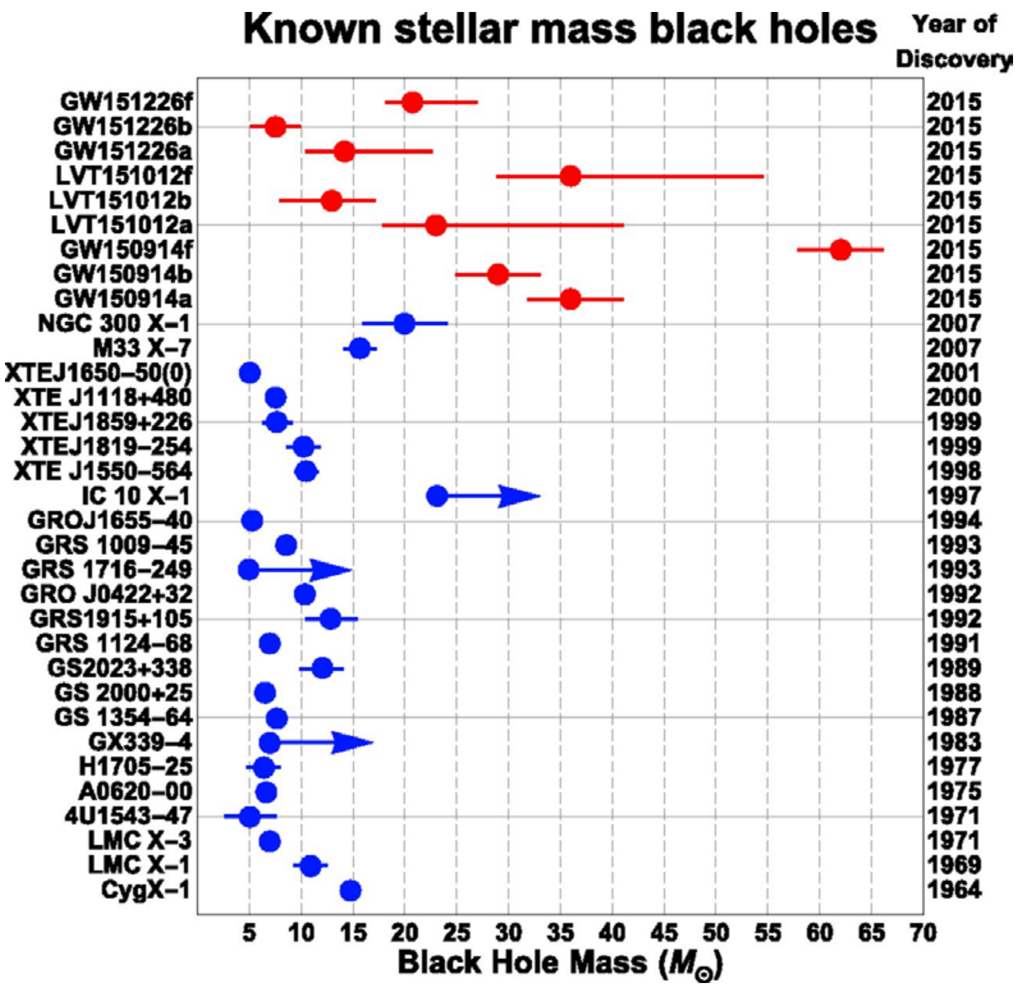
Hierarchical models

<u>Individual inference</u>

- Given $d_1, d_2, d_3 \ldots, d_N$ independent data sets
- Infer model parameters: θ for each data set :$P(\theta|d_i)$

<u>Hyperparameter inference</u>

- Infer hyperparameters Λ
- Hyperparameters model population-level effects
- Requires a predictive model



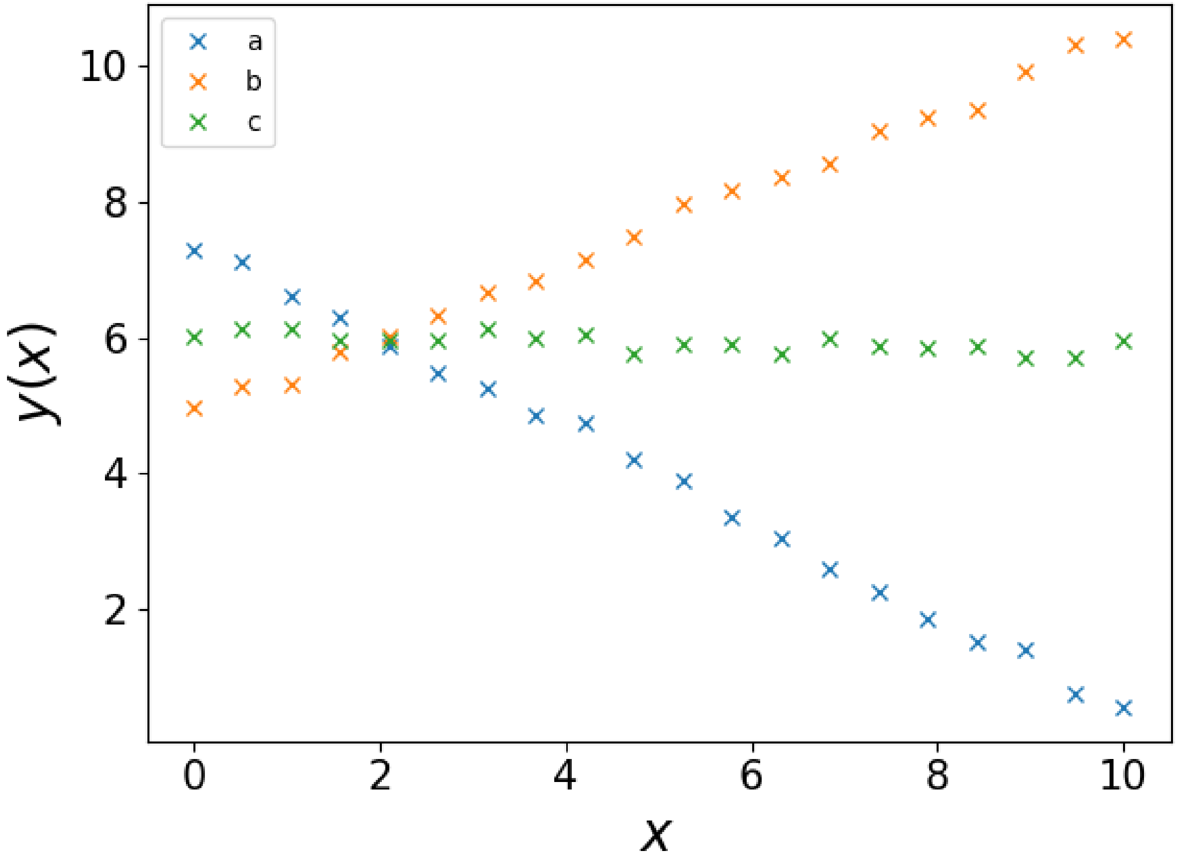
[Kalogera, Lazzarini (2017)](#)

Hierarchical modelling: toy example

- Three data sets: a, b, and c
- For the ith individual data set:

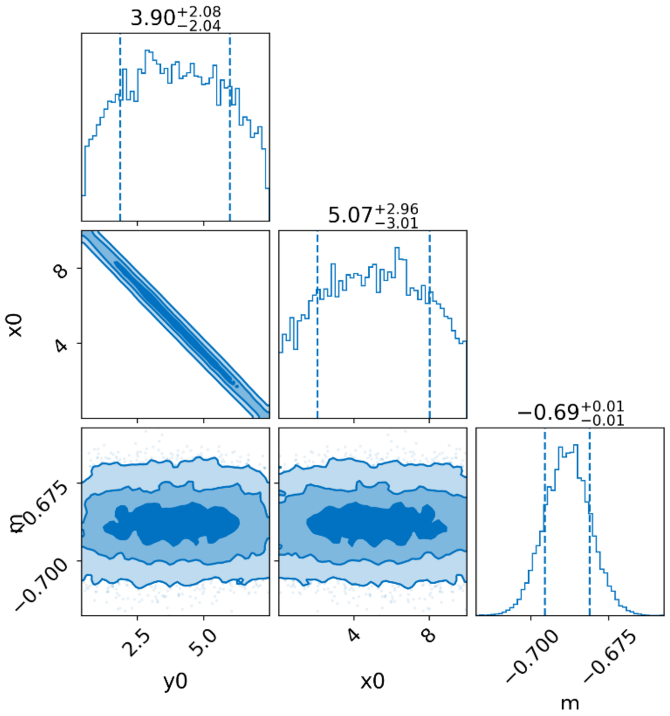
$$y(x) = y_0 + m_i\,(x - x_0)$$

- Where y_0, x_0 are population-parameters while m_i is not
- How can we infer x_0 and y_0 ?
- Note: x_0 and y_0 are degenerate

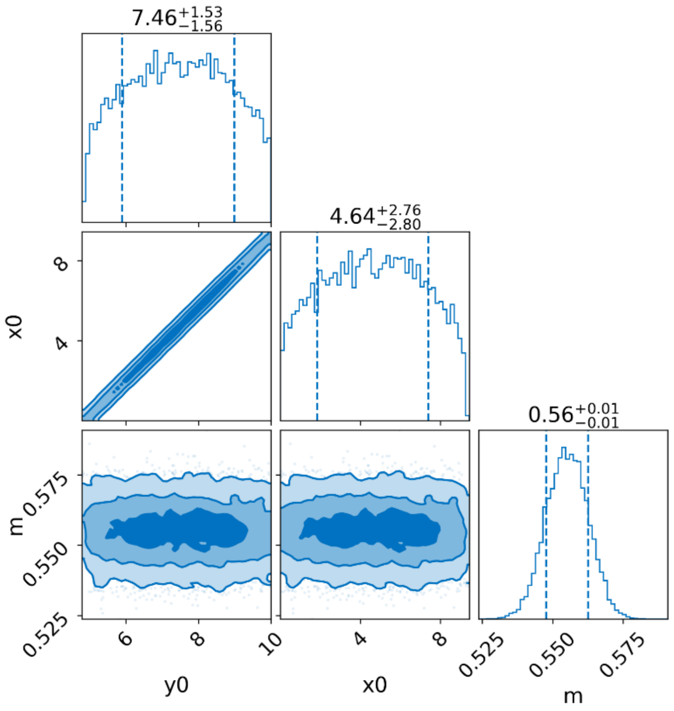


Individual posteriors

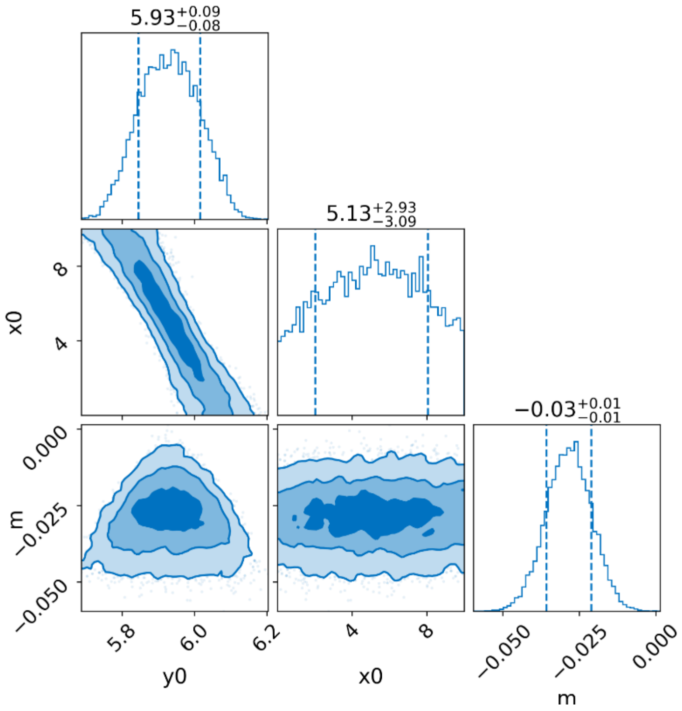
For an individual event, the gradient is well measured but x_0 and y_0 are not



Posterior for data A



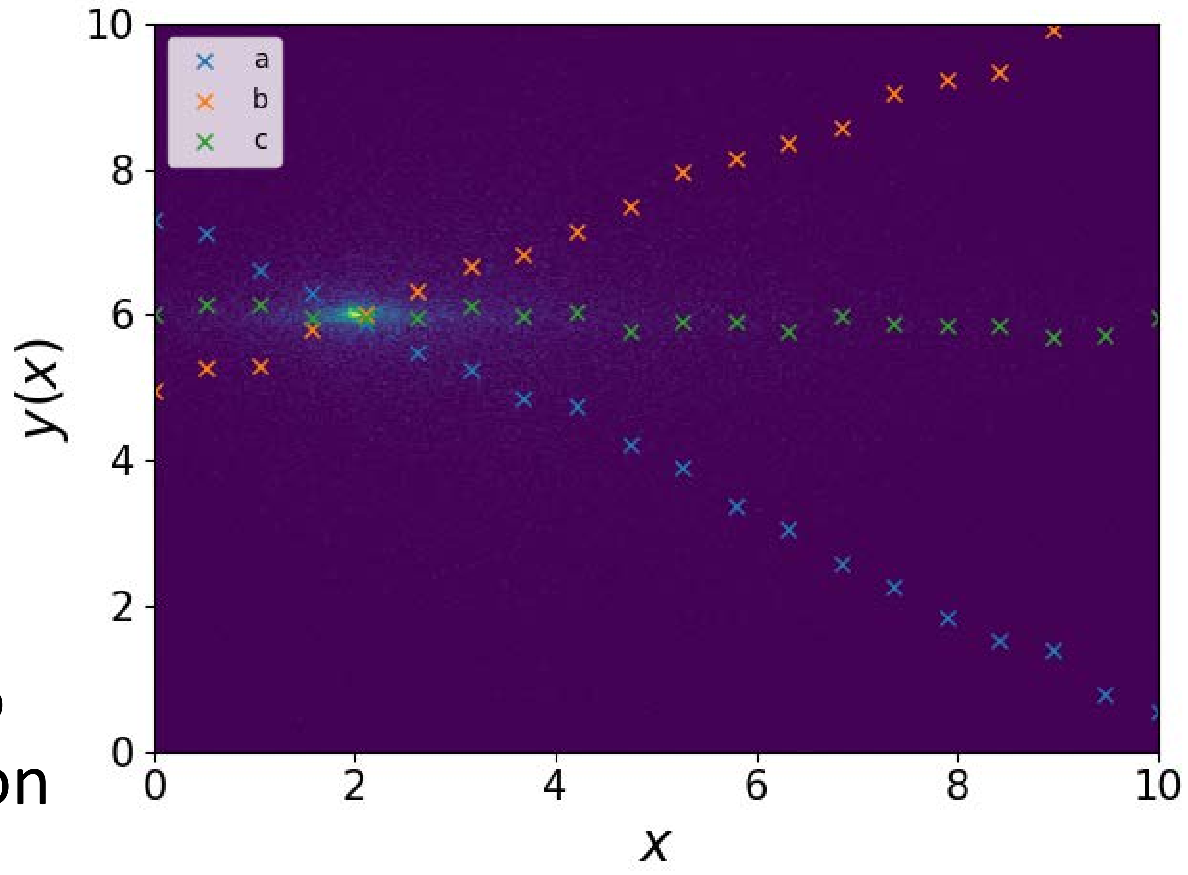
Posterior for data B



Posterior for data C

Hyperparameter inference

- We can model the population behaviour as follows:
- Let:
  - $x_0 \sim \mathrm{Normal}(\mu_{x_0}, \sigma_{x_0})$
  - $y_0 \sim \mathrm{Normal}(\mu_{y_0}, \sigma_{y_0})$
- Then infer the posterior on the hyperparameters: $\mu_{x_0}, \sigma_{x_0}, \mu_{y_0}, \sigma_{y_0}$
- Finally look at the predicted position for x_0, y_0



Applying hierarchical models to gravitational wave transients

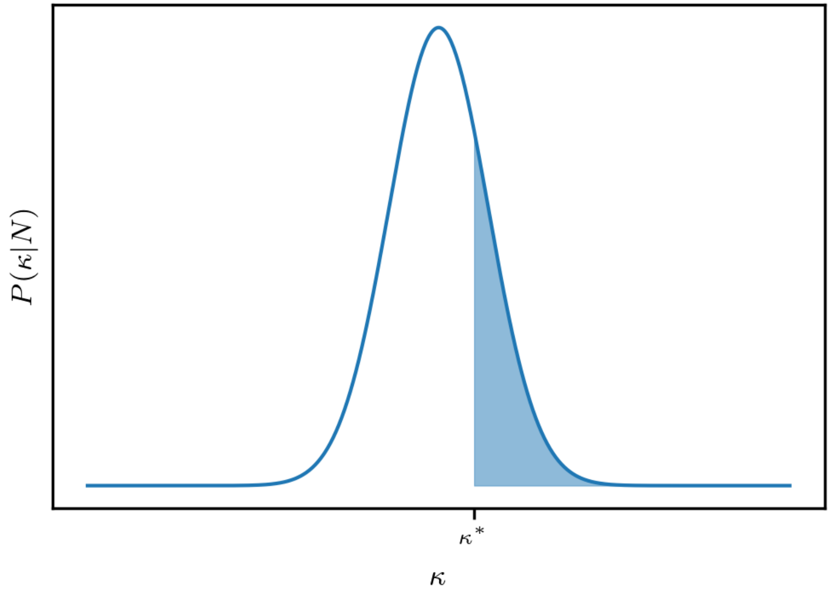
- Use the observed events to determine population properties
  - E.g., Talbot & Thrane (2017) PRD 96
  - Determine formation channels and identify sub-populations
- Constrain formation rates and test cosmology
- Search for an astrophysical background of events
  - Smith & Thrane (2018) PRX 8
  - Time to detect the background reduced from 40 months (using traditional cross-correlation) to 1 day of design-sensitivity data

Inference as a detection tool

Inference for detection

- Detection methods currently rely on frequentist method:
  - Define background distribution (time-slides)
  - Compute a p-value (probability of event or louder give background)
  - Small p-value ⇒ unlikely to be background
- This has some issues
  - Calculation of background is difficult/expensive
  - Saturation, e.g., Was et al (2009) arXiv:0906.2120
  - How to incorporate this in a Bayesian framework

$$p = \int_{\kappa *}^{\infty} P(\kappa |\, N)\, d\kappa$$



$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

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Time-slides

Hyperparameterised odds

- In a normal setting, we ask

"what is the evidence for a signal in the i$^{\text{th}}$ segment?"

- Amounts to calculating

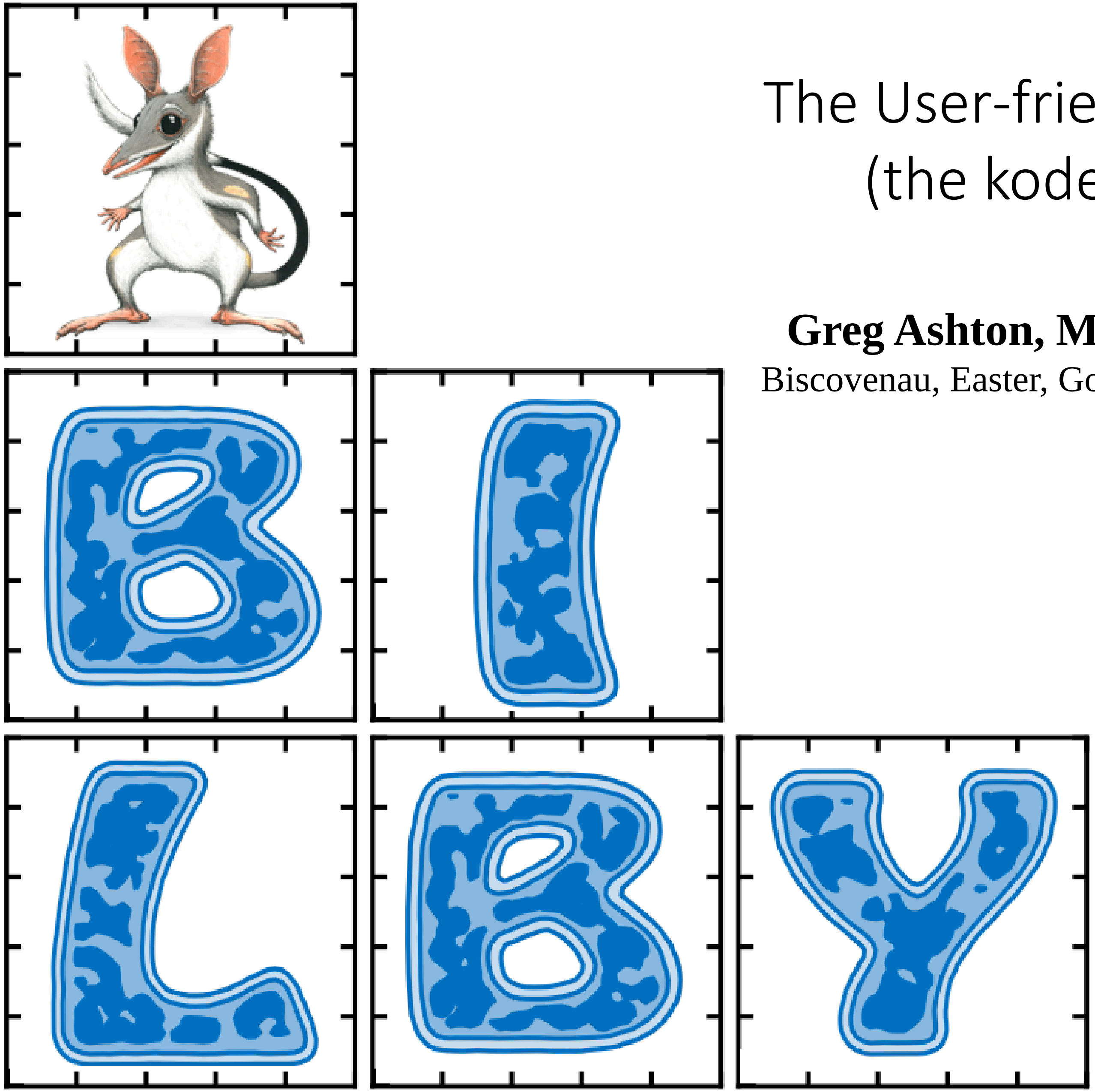
$$O_{S_i/N_i}(d_i) = \frac{P(S_i \mid d_i)}{P(N_i \mid d_i)} = \frac{P(d_i|S_i)P(S_i)}{P(d_i|N_i)P(N_i)}$$

- Instead, we want to ask

"what is the evidence for a signal in the i$^{\text{th}}$ segment, given **all** the data?"

$$O_{S_i/N_i}(\boldsymbol{d}) = \frac{P(S_i \mid \boldsymbol{d})}{P(N_i \mid \boldsymbol{d})} = \frac{\int d\Lambda\, P(d_i|S_i,\Lambda)P(S_i|\Lambda)\, P(\Lambda|\boldsymbol{d}_{i\neq k})}{\int d\Lambda\, P(d_i|N_i,\Lambda)P(N_i|\Lambda)\, P(\Lambda|\boldsymbol{d}_{i\neq k})}$$

Ashton, Smith & Thrane (in prep.)



The User-friendly Bayesian inference library (the kode formerly known as tupak)

Greg Ashton, Moritz Hübner, Paul Lasky, Colm Talbot
Biscovenau, Easter, Goncharov, Lower, Payne, Powell, Sarin, Smith, Thrane

- General code for inference
- Capable of handling arbitrary data/models
- Multiple samplers
- gw-specific and hyperparameter modules
- www.monash.docs.ligo.org/bilby/

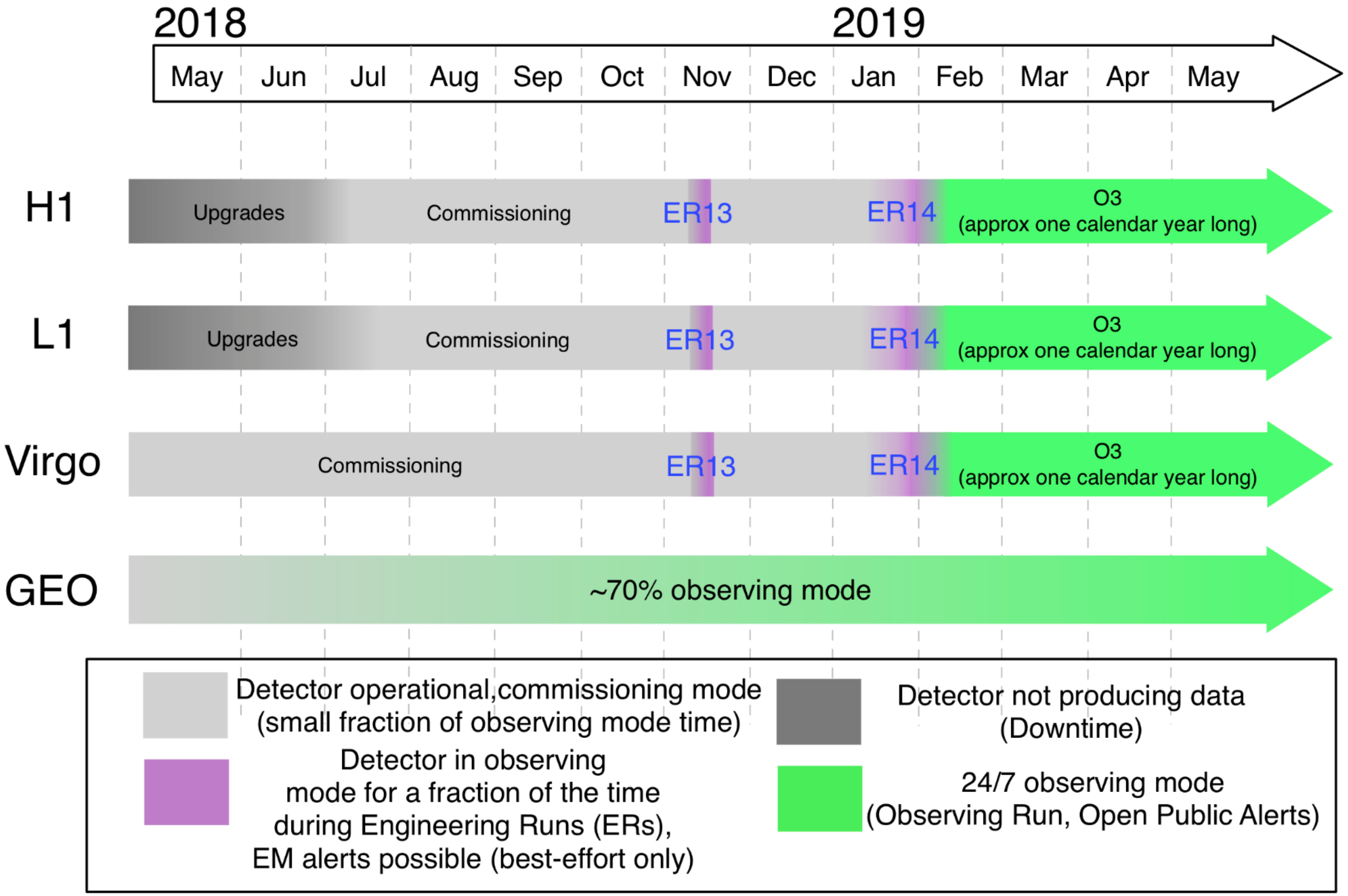
Overview

- Inference provides a universal tool for astrophysics
- Can help us understand individual events
- Hierarchical modelling can be used to
  - Understand populations
  - Detect backgrounds of events
  - Detect events
- Bilby: an OzGrav effort to build the next generation of inference tools

LIGO-VIRGO Joint Run Planning Committee

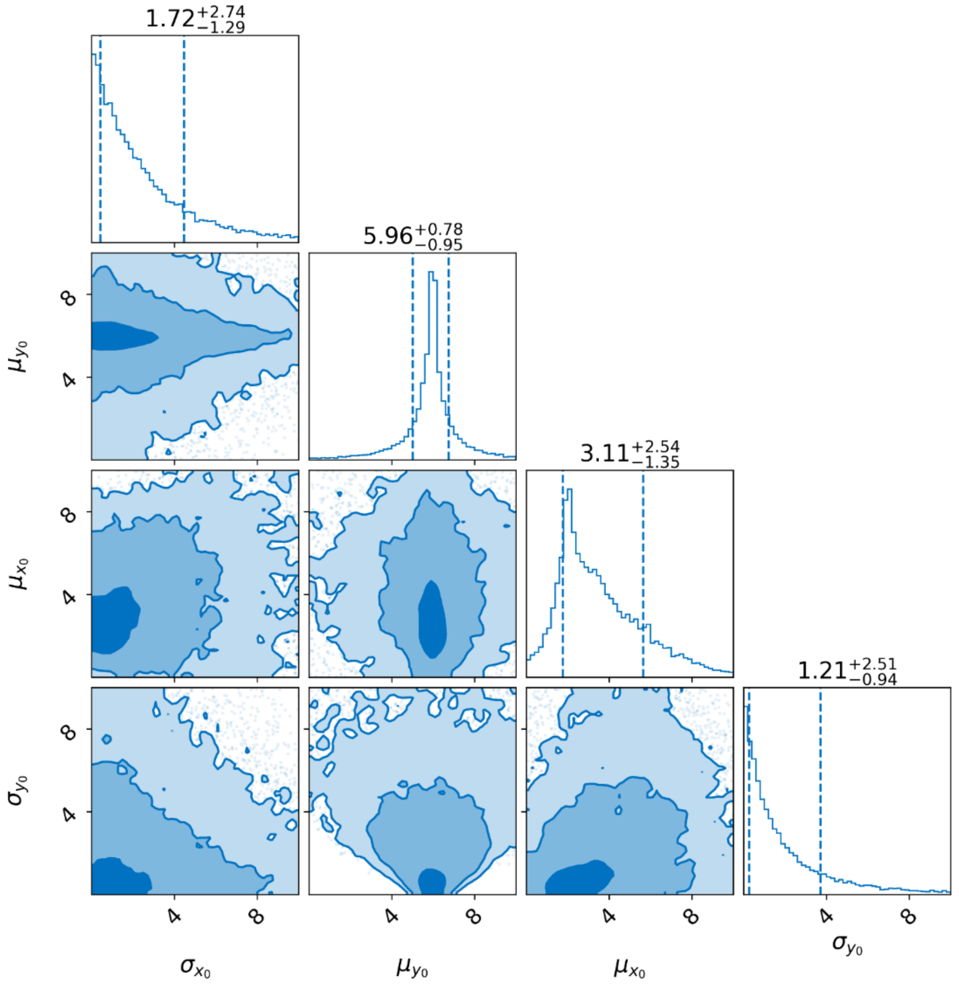
Working schedule for O3

(Public document G1801056-v2, based on G1800889-v5)



Credit: LIGO-Virgo

Posterior for hyperparameter example



Background and significance