Precession measurability
in black hole binary coalescences

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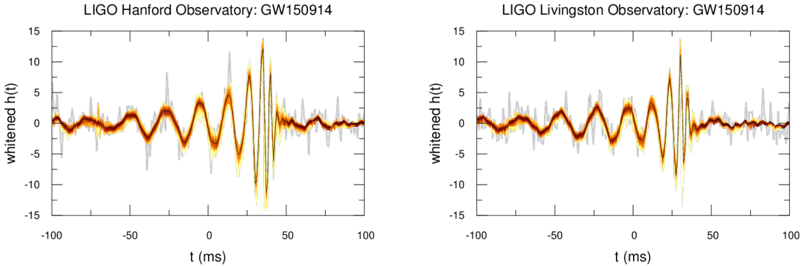
2018-06-13
University of Melbourne Astrophysics Colloquium

The Dawn of Gravitational-wave Astronomy

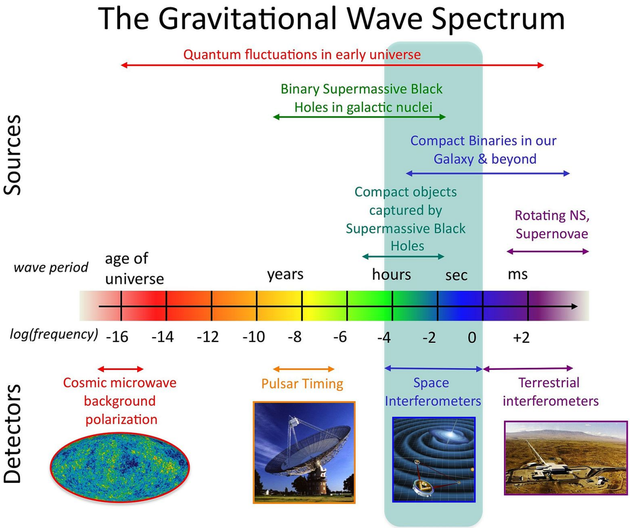


GW150914 at Hanford & Livingston Observatories
(plot credit: N. Cornish, J. Kanner, T. Littenberg, M. Millhouse;
LVC, *Phys Rev Lett* 116 (2016) 061102)

The Dawn of Gravitational-Wave Astronomy

Panorama at LIGO Hanford Observatory (credit: G.D. Meadors)

Opening the spectrum



Space for new observatories (credit: NASA Goddard Space Flight Center)

Introduction to Gravitational Waves

General relativity (GR): extremize *curvature* R,
when *cosmological constant* Λ, *matter* $\mathcal{L}_M$, *metric* g:

$$0 = \delta \int \left(\frac{1}{8\pi}(R - 2\Lambda) + \mathcal{L}_M \right) \sqrt{-|g|} d^4 x,$$

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GR's contribution: Einstein-Hilbert action S,

$$S \propto \int R \sqrt{-|g|} d^4x,$$

GR says, 'minimize/maximize Ricci curvature R' [1] ,
(as much as matter allows)

[1] Maybe someday this will turn out to be $f(R)$?

Introduction to Gravitational Waves

General relativity (GR): extremize *curvature* R,
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---

gives the Einstein field equations[2] for stress-energy tensor T:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(R + 2\Lambda) = 8\pi T_{\mu\nu},$$

[2] where R and $R_{\mu\nu}$ depend on $g_{\mu\nu}$

Introduction to Gravitational Waves

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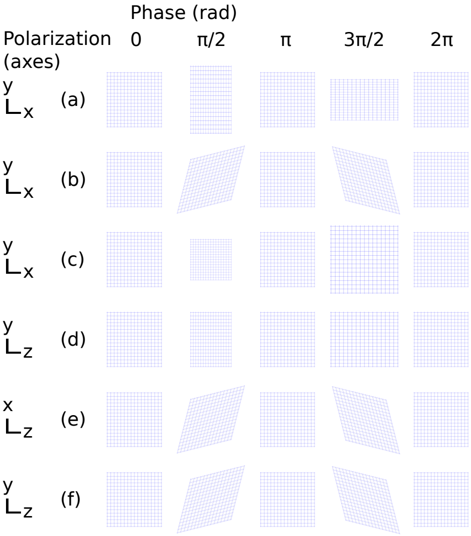
$$0 = \delta \int \left(\frac{1}{8\pi}(R - 2\Lambda) + \mathcal{L}_M \right) \sqrt{-|g|} d^4x,$$

$\rightarrow$ **wave equation** in transverse-traceless gauge if $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$, for flat space η and a small wave h:

$$(-\partial_t^2 + \partial_z^2) h_{\mu\nu} = 16\pi T_{\mu\nu}.$$

All conceivable wave polarizations



GR allows (a) and (b)

Introduction to Gravitational Waves

Wave equation is sourced by T:

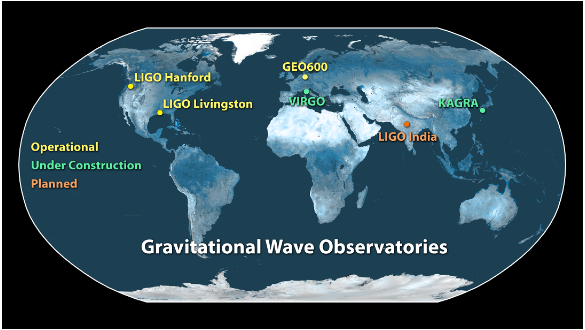
- Conservation of mass-energy $\rightarrow$ no monopole radiation
- Conservation of momentum $\rightarrow$ no dipole (unlike light)
- Quadrupoles (& higher) needed: **massive** astrophysical bodies

Direction wave-vector k_μ,
2 polarizations (h_+ & $h_\times$) of strain h:

$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -h_+ & h_\times & 0 \\ 0 & h_\times & h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Re\left(e^{\mathrm{i}(k_\mu x^\mu + \phi_0)}\right).$$

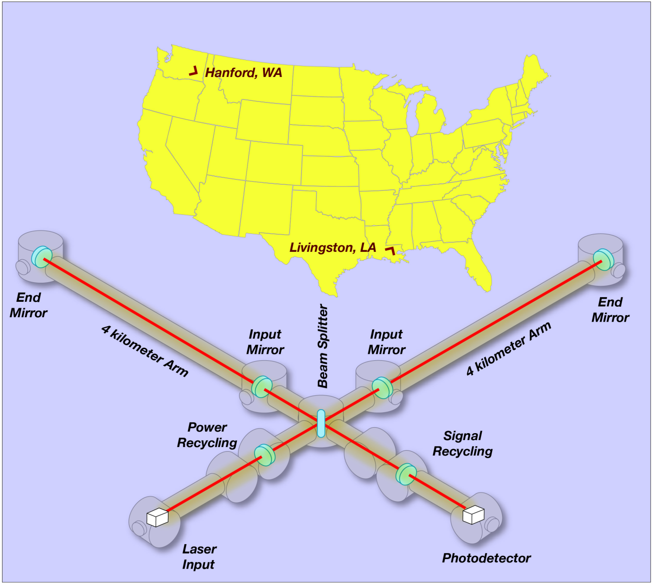
Space 'stretches' length L by ΔL in one direction, then another:
$\Delta L = hL \rightarrow$ *measure* ΔL

Gravitational-wave Observatories



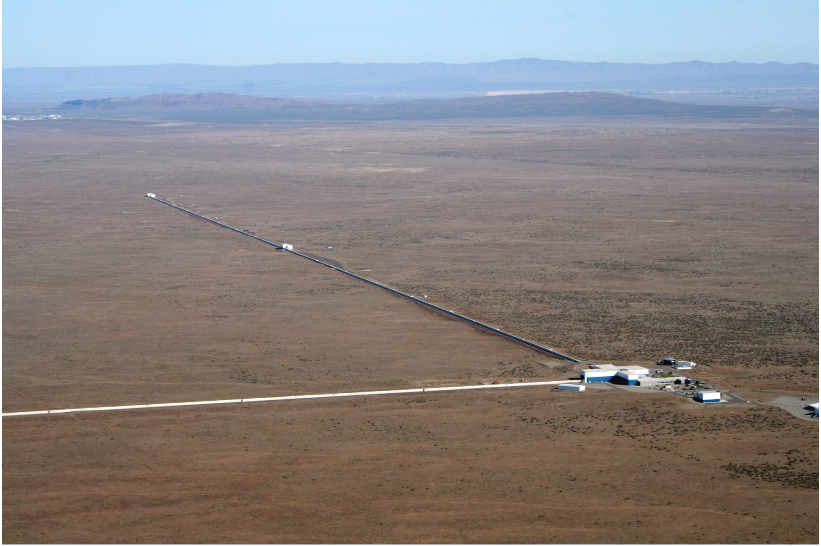
Global map out-of-date: Virgo now fully-*operational*,
LIGO India *under construction* (image credit: LIGO EPO)

Gravitational-wave Observatories



LIGO location and configuration (credit: S. Larson, Northwestern U)

Gravitational-wave Observatories



Overhead, toward X-arm (credit: C. Gray, LIGO Hanford)

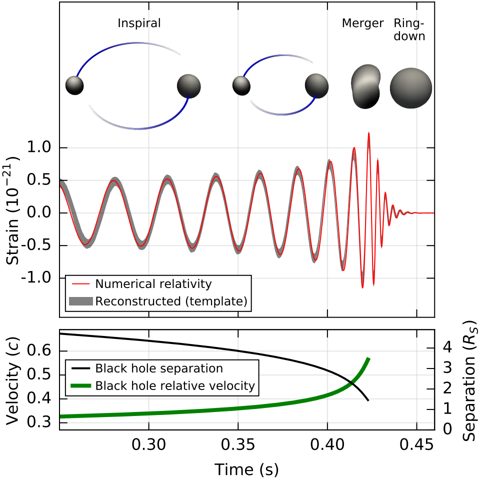
Gravitational-wave Data Analysis

	transient events	long-lasting
predicted form	CBCs[3]	CWs[4]
unknown form	bursts	stochastic

CBCs	'Inspirals' of merging neutron stars & black holes
CWs	'Pulsars' with mountains on neutron (quark?) crust
Bursts	from supernovae, hypernovae (GRBs)...
Stochastic	background of the Big Bang, white dwarf stars...

[3] Compact Binary Coalescences
[4] Continuous Waves

GW150914: an archetypical compact binary coalescence



Numerical relativity (NR) & template ('Observation of gravitational waves from a binary black-hole merger', LVC, *Phys Rev Lett* 116 (2016) 061102)

What does the template reveal?

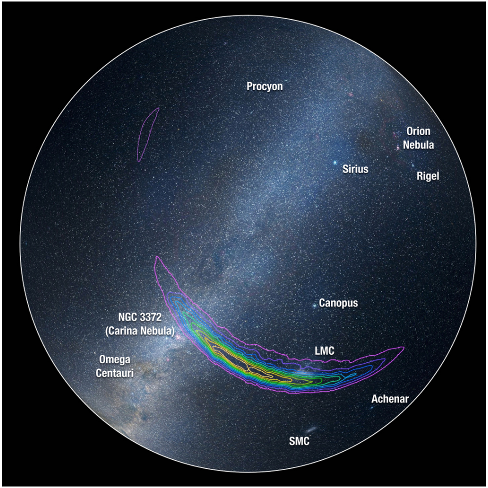
Inference

What does the template reveal?

Inference

Inference: learning about the model from the data
(By estimating parameters)

Inference, concrete example: GW150914 localization



Astronomical landmarks at time of event (probability deciles)
(credit: R. Williams, Caltech; T. Boch, CDS Strasbourg;
S. Larson, Northwestern U)

Inference, abstract example: GW150914 and stellar winds

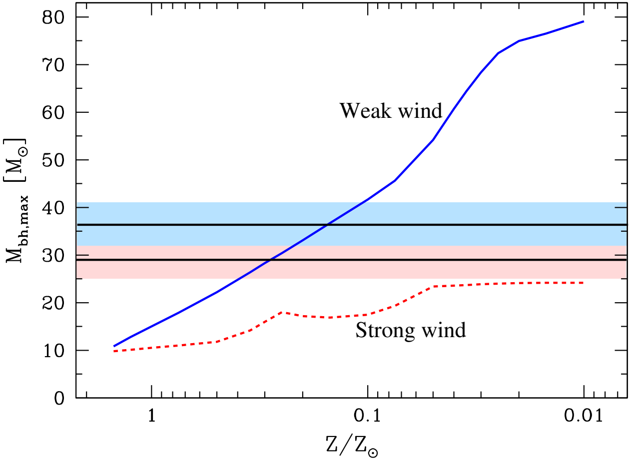


Figure 1, 'Astrophysical implications of the binary black-hole merger GW150914' (LVC, *ApJL* 818 (2016) L22), after Belcynzski et al 2010. Black-hole progenitor masses favor weak metallicity-wind models

Intro to Bayesian inference

Bayes' theorem is natural for inference
What is the 'posterior' probability of A, given B? $P(A|B)$ is,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)},$$

Ask, what's probability of a parameter λ given GW strain $h(t)$?

$$P(\lambda|h(t)) = \frac{P(h(t)|\lambda)P(\lambda)}{P(h(t))}$$

Intro to Bayesian inference

In the equation,

$$P(\lambda|h(t)) = \frac{P(h(t)|\lambda)P(\lambda)}{P(h(t))}$$

- $P(h(t)|\lambda)$ is the *likelihood*:
  many people use likelihoods
  (can be numerically-hard, depends on noise distribution)
- $P(\lambda)$ is the prior:
  the philosophical difference!
- $P(h(t))$ is the probability of the data (a normalization):
  usually hard to estimate
  get around by comparing $\frac{P(\lambda_A|h(t))}{P(\lambda_B|h(t))}$

Intro to Bayesian inference

Example

λ could be a vector $\vec{\lambda}$
$\lambda_1 = t_c$, 'when did the black holes coalesce?' or,
$\lambda_2 = \delta$, 'at what declination did they come from?'

More advanced Bayesian inference: prior

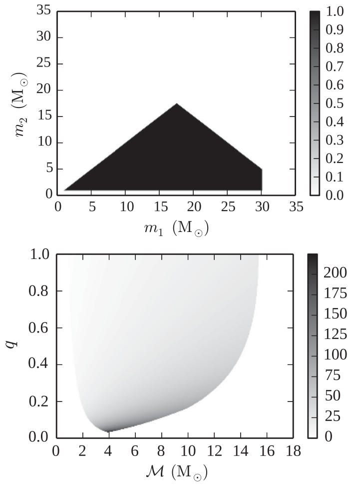


Figure 1, 'Parameter estimation for compact binaries...' (Veitch et al, *PRD* 91 (2015) 042003). Example prior on λ: black hole masses m_1, m_2 and mass ratio q & chirp mass $\mathcal{M}$

More advanced Bayesian inference: posterior

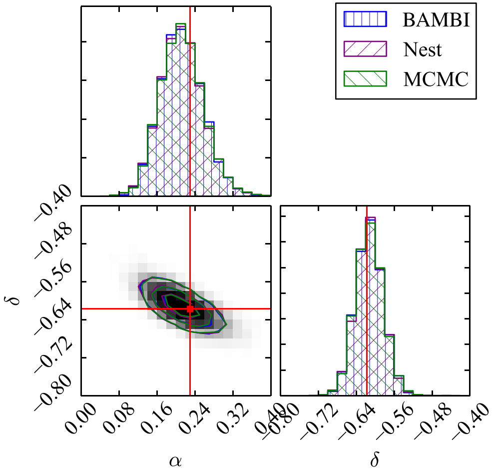


Figure 9, 'Parameter estimation for compact binaries...' (Veitch et al, *PRD* 91 (2015) 042003). Example posterior (three computational methods: BAMBI/Nest/MCMC) on right ascension α and declination δ.

More advanced Bayesian inference: hypotheses

A special 'parameter': a hypothesis H,

$$P(\vec{\lambda}|h(t), H) = \frac{P(h(t)|\vec{\lambda}, H)P(\vec{\lambda}|H)}{P(h(t)|H)}$$

The *Bayesian evidence* Z for H: integrate! (good sampling is hard)

$$Z = p(h(t)|H) = \int d\vec{\lambda} P(h(t)|\vec{\lambda}, H)p(\vec{\lambda}|H)$$

Between two hypothesis, *Bayes Factor* B_{ij} tells
how much data *supports* i over j:

$$B_{ij} = \frac{Z_i}{Z_j},$$

with final *Odds* O_{ij}
(ratio of posterior probabilities),

$$O_{ij} = \frac{P(H_i)}{P(H_j)} B_{ij}$$

Understanding merging binaries

Compact binary coalescence (CBC):
neutron stars
(GW170817)
black holes
(GW150914, LVT151012, GW151226, GW170104, GW170608, GW170814,. . .)
black hole coalescences: theoretically simple, numerically hairy
Model | Numerical Relativity (NR) $\propto$ General Relativity (GR):

$$[h(t)]_{\text{measured}} = \text{calibration}(\text{photodiode}(\text{interferometer}(t))),$$
$$[h(t)]_{\text{modelled}} = \text{approximant}(\text{NR}(\text{GR}(t)))$$

$\Longrightarrow$ What is the strain $h(t)$?[5]

[5] *implicit: what is* $h(t, \lambda)$

Parameters λ of a CBC determine $h(t)$

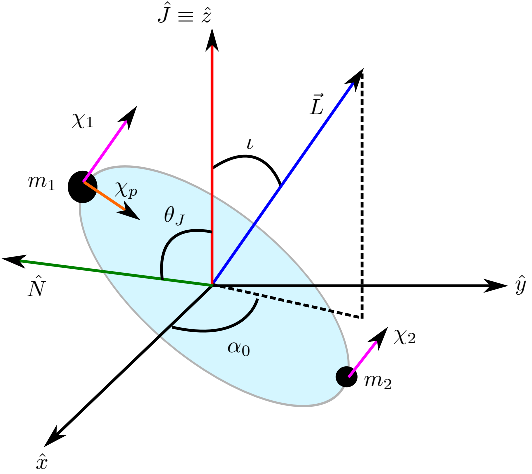


Figure 1, 'Fast and Accurate Inference...' (Smith et al, *PRD* 94 (2016) 044031). Illustration of CBC parameters in a *J*-aligned source frame, with precession.

Parameter estimation for compact binary coalescences

(for the simplest, non-eccentric binary black hole (BBH) case)
Strain $h(t)$ depends on 15 binary parameters λ

- $+2$: masses $\{m_1, m_2\}$,
- $+6$: 3-D spin-vectors $\{\mathbf{S}_1, \mathbf{S}_2\}$,
- $+3$: sky location of frame in BBH frame (r, ι, ψ),
- $+1$: coalescence time t_c,
- $+2$: sky location of BBH in detector frame (θ, φ),
- $+1$: polarization angle ψ_p
- $\sum = 15$ parameters to estimate

GR non-linear → *simulate* BBH with NR
Too high-dimensional to simulate all with NR → **approximants**

Numerical relativity's approximant waveforms

`LALInference` (Veitch *et al* 2015)
– Bayesian evidence for parameter estimation w/. . .
families of approximants to NR

`SEOBNR`
(Spinning Effective One Body-Numerical Relativity)
`IMRPhenom`
(Inspiral-Merger-Ringdown Phenomenological Model)
(and others)

– many motivations, known to differ: what is best?

Inferring evidence for approximants in data

Differences in NR (Williamson *et al* 2017; Pang *et al* 2018)

$\Longrightarrow$ *What about data?*

Two phases of questions [‖]

1. **What** is the typical difference?
2. **Where** is it biggest in parameter space?
3. **Which** fits better?
4. **How much data** is needed distinguish these approximants? *e.g.*, how many events?

---

5. Can data tune **better** approximant models?
6. “” tune NR?

[‖] CONCERNS: spin effects (not) included, higher-order modes, *etc*. . .

How model-comparison uses Bayes Factors

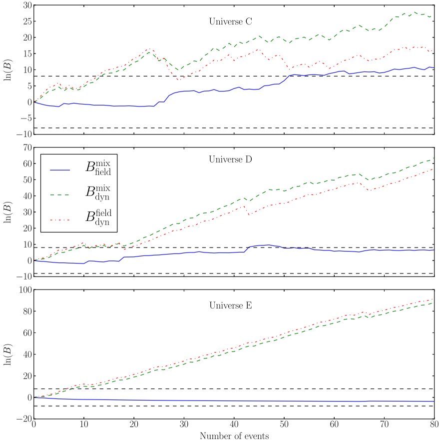


Figure 1, 'Determining the population properties...' (Tablot & Thrane, *PRD* 96 (2017) 023012). Using Bayes Factors B to distinguish population models: individual event evidence small, but cumulative grows, allows model comparison

Introduction to spin-precession

So far, talked about `SEOBNR` *vs* `IMRPhenom`
→ Hone in on the difference between `IMRPhenomD` & `IMRPhenomP`:

$$\text{Mass ratio (above unity)} : q = m_2/m_1, \tag{1}$$
$$\text{Total mass} : M = m_1 + m_2 \tag{2}$$

Effective spin parameters

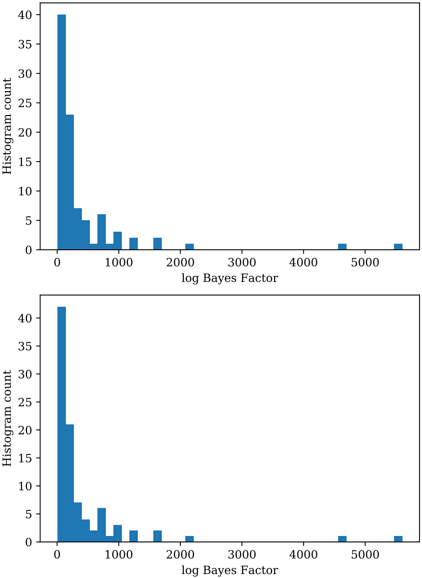
$$\chi_{\text{eff}} = (\mathbf{S}_1/m_1 + \mathbf{S}_2/m_2)\cdot\hat{\mathbf{L}}/M \tag{3}$$
$$= \frac{a_1\cos\theta_1 + qa_2\cos\theta_2}{1+q}$$
$$\chi_p = \max\left(a_1\sin\theta_1, \left(\frac{4q+3}{4+3q}\right) qa_2\sin\theta_2\right). \tag{4}$$

Can we measure precession?

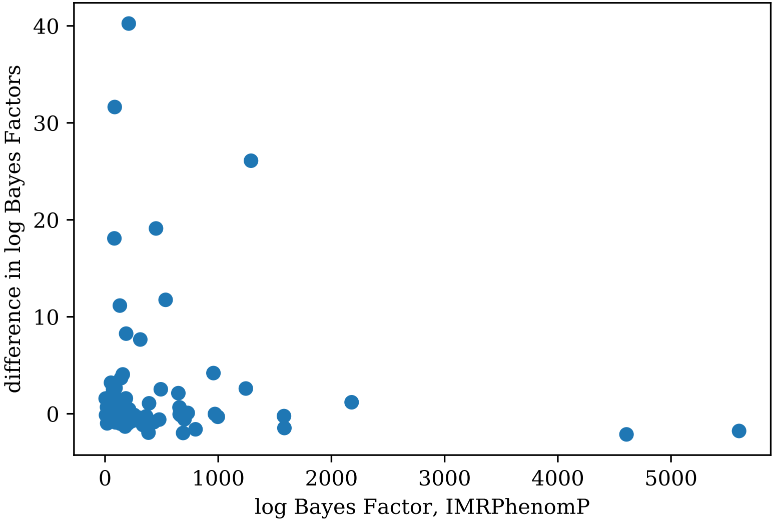
Simulations (no real data)

Ask the question:
What is the total Bayes Factor difference
between models *with* & *without* precession?

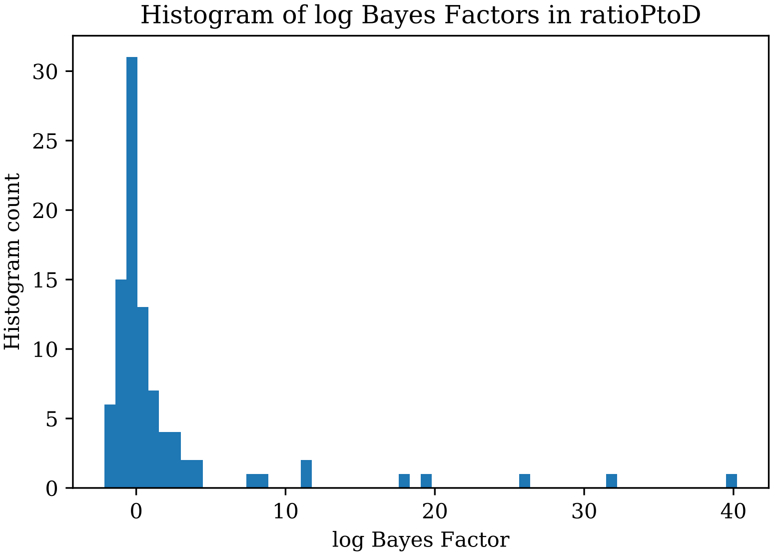
Histogram of log Bayes Factors D (top) & Pv2 (bottom)



log Bayes Factor (Pv2 - D) vs log Bayes Factor (P)



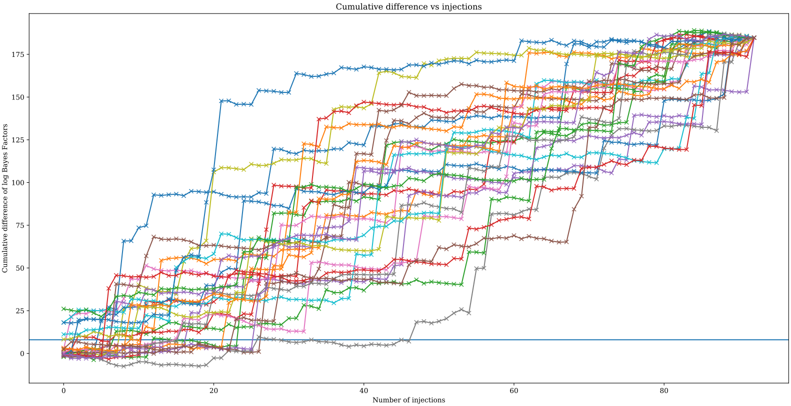
Histogram of log Bayes Factor (Pv2 - D)



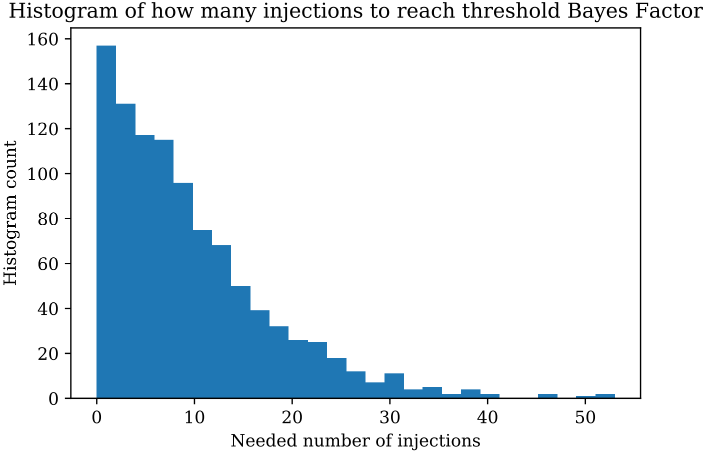
Measurability

Suppose detection = threshold
total log Bayes Factor difference between `D` and `Pv2` ≥ 8?

25 non-independent Shuffles to Reach $\log BF = 8$



Histogram: 10^3 non-independent Shuffles to Threshold



Pre-Conclusion

Plan:

1. As few as one event, but typically $\mathcal{O}(8)$ **assuming** $a_{\max} = 0.89$ where a is black hole spin
2. If $a_{\max}$ lower, probably *harder*
3. Hyper-parametrize as in model at RIT
4. Discern which events best indicate *precession*

Why care?

Precession $\implies$ GR test + astro (capture/common)

Conclusion

Gravitational-wave astronomy is beginning

Bayesian Inference tests hypotheses on this new data

Growing evidence w/ more events – and new types of observatories

Acknowledgments

Thanks again to my OzGrav colleagues, including H. Middleton for inviting this talk, as well as L. Sun and A. Melatos, and to my collaborators in the Monash Centre for Astrophysics (MoCA).

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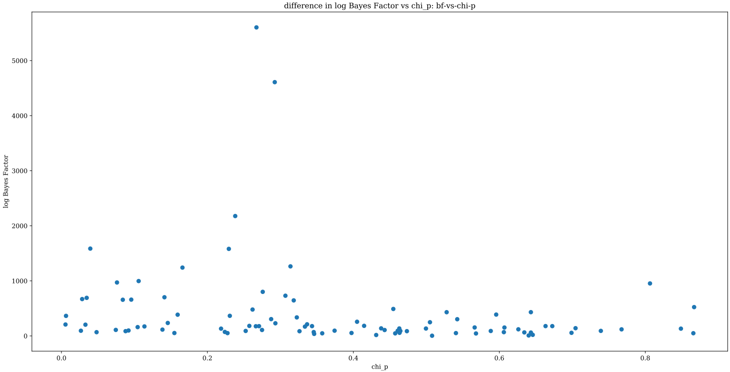
IMRPhenomD log Bayes Factor vs χ_p



IMRPhenom: Pv2 - D, log Bayes Factors, vs χ_p