Electrical Sensors

Employ electrical principles to detect phenomena.

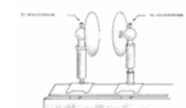
- May use changes in one or more of:
  - Electric charges, fields and potential
  - Capacitance
  - Magnetism and inductance

Some elementary electrical sensors

Thermocouple

Thermistor

Variable Capacitor



Review of Electrostatics

- In order to understand how we can best design electrical sensors, we need to understand the physics behind their operation.
- The essential physical property measured by electrical sensors is the electric field.

Electric Charges, Fields and Potential

Basics: Unlike sign charges attract, like sign charges repel

Coulombs' Law: a force acts between two point charges, according to:

$$\vec{F} = \frac{Q_1 Q_2 \hat{r}}{4\pi\varepsilon_0 r^2}$$

The electric field is the force per unit charge:

$$\vec{E} = \frac{\vec{F}}{Q_1}$$

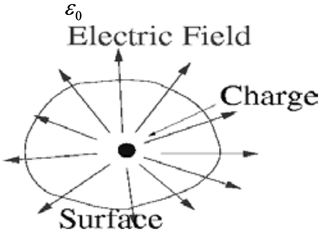
How do we calculate the electric field?

Electric Field and Gauss's Law

We calculate the electric field using Gauss's Law.

It states that: $\oint_S \vec{E}\cdot\vec{ds} = \frac{Q}{\varepsilon_0}$

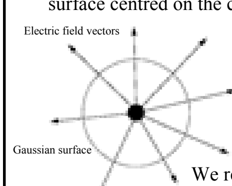
Seems very abstract, but is really useful



Point or Spherical charge

What is the field around a point charge (e.g. an electron)?

The electric field is everywhere perpendicular to a spherical surface centred on the charge.



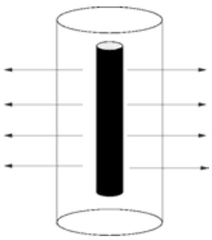
So $\oint_S \vec{E}\cdot\vec{ds} = E \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$

$$E = \frac{Q}{4\pi r^2}$$

We recover Coulombs Law!

The same is true for any distribution of charge which is spherically symmetric (e.g. a biased metal sphere).

Line of Charge



For a very long line of charge (eg a wire), the cylindrical surface has electric field perpendicular to a cylindrical surface.

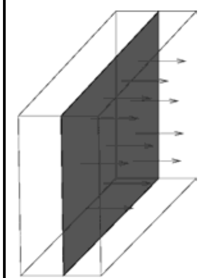
So $\oint_S \vec{E} \cdot \vec{ds} = E \times 2\pi r L = \frac{Q}{\varepsilon_0}$

$$\Rightarrow E = \frac{Q}{2\pi\varepsilon_0 r L} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Where λ = linear charge density

(Coulombs/meter)

Plane of Charge



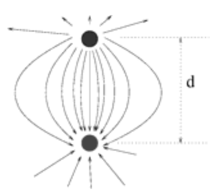
For a very large flat plane of charge the electric field is perpendicular to a box enclosing a segment of the sheet

So $\oint_S \vec{E} \cdot \vec{ds} = E \times A = \frac{Q}{\varepsilon_0}$

$$E = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Where σ = Charge/Unit area on the surface

Electric Dipole



- An electric dipole is two equal and opposite charges Q separated by a distance d.
- The electric field a long way from the pair is

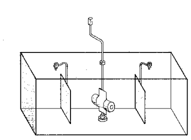
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qd}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

- $p = Q\,d$ is the Electric Dipole moment
- p is a measure of the strength of the field generated by the dipole.

Electrocardiogram

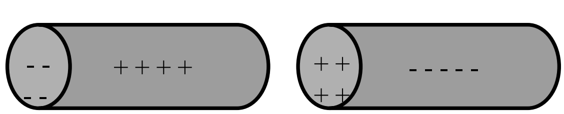
- Works by measuring changes in electric field as heart pumps
- Heart can be modeled as a rotating dipole

- Electrodes are placed at several positions on the body and the change in voltage measured with time



Electrocardiogram

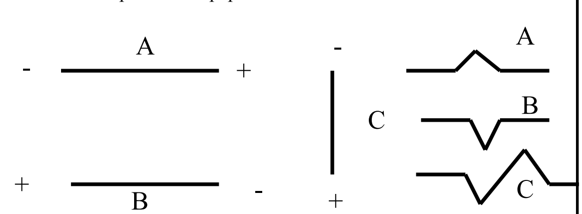
- Interior of Heart muscle cells negatively charged at rest
- Called "polarisation"
- K^+ ions leak out, leaving interior –ve
- Depolarisation occurs just proir to contraction:
  - Na^+ ions enter cells
  - Occurs in waves across the heart
  - Re-polarisation restores –ve charge in interior



Polarisation Depolarisation

Electrocardiogram

- Leads are arranged in pairs
- Monitor average current flow at specific time in a portion of the heart
- 1 mV signal produces 10 mm deflection of recording pen
- 1 mm per second paper feed rate



Electric Potential

The ECG measures differences in the electric potential V: $\vec{E} = -\vec{\nabla} V$

The Electric Potential is the *Potential* ability to do work.

Alternatively: Work = Q × V

Where V = $V_1 - V_2$

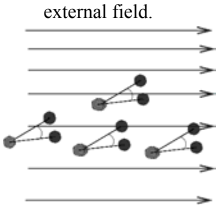
For uniform electric fields: $|E| = \frac{V}{d} \Rightarrow V = |E|\, d$

Electric fields on conductors.

- Conductors in static electric fields are at uniform electric potential.
- This includes wires, car bodies, etc.
- The electric field inside a solid conductor is zero.

Dielectric Materials

- Many molecules and crystals have a non-zero Electric dipole moment.
- When placed in an external electric field these align with external field.

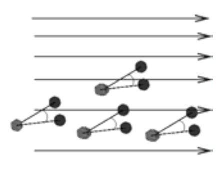


- The effect is to reduce the strength of the electric field within the material.
- To incorporate this, we define a new vector Field, the electric displacement, $\vec{D}$

Electric Displacement

$\vec{D}$ is independent of dielectric materials. Then the electric field $\vec{E}$ is related to $\vec{D}$ by: $\vec{E} = \frac{1}{\varepsilon_r \varepsilon_0} \vec{D} = \frac{1}{\varepsilon} \vec{D}$

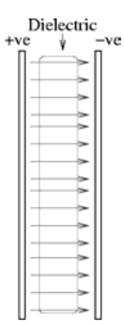
$\varepsilon_r, \varepsilon_0, \varepsilon$ Are the relative permittivity, the permittivity of free space and the absolute permittivity of the material.



As shown in the diagram, there is torque applied to each molecule. This results in energy being stored in the material, U. This energy is stored in every molecule of the dielectric:

$$U = \vec{p} \cdot \vec{E}$$

Capacitance.



Remember that the electric field near a plane of charge is:

$$E = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

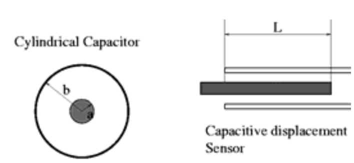
In the presence of a dielectric: $E = \frac{\sigma}{\varepsilon}$

$$E = \frac{V}{d} \Rightarrow V = \frac{d\sigma}{\varepsilon}$$

$$Since\ \ \sigma = \frac{Q}{A},\ V = \frac{dQ}{A\varepsilon}$$

So the Potential difference is proportional to the stored charge.

Cylindrical Capacitor



Can make a capacitor out of 2 cylindrical conductors

$$C = \frac{2\pi\varepsilon L}{\ln(\frac{b}{a})}$$

Sensing using capacitance.

So the charge Q = C·V

Where C = Capacitance, V = Potential difference.

For a parallel plate capacitor:

Easily Measured

Area of plate

$$C = \frac{A\varepsilon}{d}$$

Properties of Material

Distance between plates

We can sense change in A, ε, or d and measure the change in capacitance.

Measurement of Capacitance

Capacitors have a complex resistance

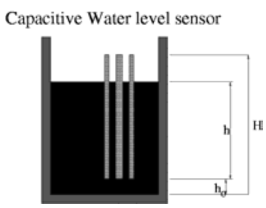
$$\frac{V(t)}{i(t)} = \frac{1}{j\omega C}$$

We measure capacitance by probing with an AC signal.

Directly measure current i(t) with known V(t) and frequency ω.

For extreme accuracy, we can measure resonant frequency with LC circuit.

Example: water level sensor



Measures the capacitance between insulated conductors in a water bath

Water has very different dielectric properties to air (a large κ)

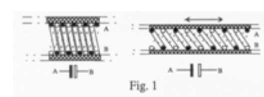
As the bath fills the effective permittivity seen increases, and the capacitance changes according to:

$$C_h = \frac{2\pi\varepsilon}{\ln(\frac{b}{a})}\left[H - h(1-\kappa)\right]$$

Example: The rubbery Ruler

Invented by Physicists here to measure fruit growth.

http://www.ph.unimelb.edu.au/inventions/rubberyruler/



Spiral of conductor embedded in a flexible "*rubbery*" compound

As the sensor expands, the *distance* between the plates increases causing capacitance to decrease.

$$C = \frac{A\varepsilon}{d}$$