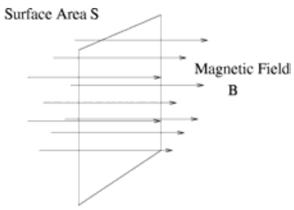
Magnetism

The density of a magnetic field (number of magnetic lines passing through a given surface) is the magnetic flux:

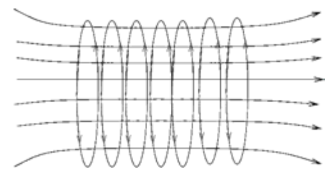


$$\Phi_B = \int \vec{B} \cdot d\vec{S}$$

Units of flux are Webers.
Tesla/m^2

Sources of Magnetism

Solenoid: Produces lines of flux as shown (in blue).



Note that the magnetic field lines are continuous with no source or sink

Inside the solenoid the magnetic flux density is:

$$B = n\mu I$$

Where n = number of turns of wire.
μ= permeability of the core material.
I = current through the core.

Active solenoids have many uses in sensor technologies.

There are permanent magnets (ferromagnets) too; these are very useful for small compact sensors.

Solenoids make inductive sensors which can be used to detect motion, displacement, position, and magnetic quantities.

Permanent Magnets

There are four main ways to characterise permanent magnets:

Residual inductance (B) in Gauss – how strong the magnet is

Coercive force (H) in Oersteds -Resistance to demagnetization

Maximum Energy Product (MEP), (B x H) in gauss-oersteds times 10^6. The overall figure of merit for a magnet

Temperature coefficient %/°C, how much the magnetic field decreaes with temperature.

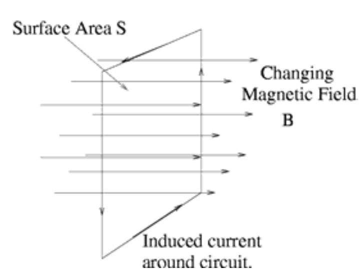
Some common permanent magnets.

Table 3-3 Properties of magnetic materials
(adapted from [2])

Material	MEP (G·Oe)·10^6	Residual induction (G)·10^3	Coercive force (Oe)·10^3	Temperature coefficient %/°C	Cost
R.E.-Cobalt	16	8.1	7.9	-0.05	highest
Alnico 1,2,3,4	1.3-1.7	5.5-7.5	0.42-0.72	-0.02 to -0.03	medium
Alnico 5,6, 7	4.0-7.5	10.5-13.5	0.64-0.78	-0.02 to -0.03	medium/high
Alnico 8	5.0-6.0	7-9.2	1.5-1.9	-0.01 to 0.01	medium/high
Alnico 9	10	10.5	1.6	-0.02	high
Ceramic 1	1.0	2.2	1.8	-0.2	low
Ceramic 2,3,4,6	1.8-2.6	2.9-3.3	2.3-2.8	-0.2	low/medium
Ceramic 5,7,8	2.8-3.5	3.5-3.8	2.5-3.3	-0.2	medium
Cunife	1.4	5.5	0.53	-	medium
Fe-Cr	5.25	13.5	0.6	-	medium/high
Plastic	0.2-1.2	1.4	0.45-1.4	-0.2	lowest
Rubber	0.35-1.1	1.3-2.3	1-1.8	-0.2	lowest

Magnetic Induction

Time varying fluxes induce electromotive force (emf, ie a voltage difference) in the circuit enclosing the flux

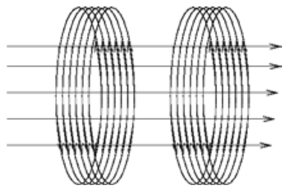


$$emf = -\frac{d\Phi_B}{dt}$$

Sign of the emf is such as to make a current flow whose magnetic field would oppose the change in the flux.

Making an inductor

Add a second solenoid to intercept the flux from the first



Assuming the same cross section area and no flux *leakage*, a voltage is induced in the second coil

$$V = -N\frac{d\Phi_B}{dt}$$

N= number of turns in the solenoid coil

Assuming B is constant over area A gives a more useful relation :

$$V = -N\frac{d(BA)}{dt}$$

This second coil is called the *pickup circuit.* We get a signal in this circuit if the magnitude of the magnetic field (B) changes or if the area of the circuit (A) changes.

We get an induced voltage if we:

- Move the source of the magnetic field (magnet, coil etc.)
- Vary the current in the coil or wire which produces the magnetic field
- Change the orientation of the magnetic field in the source
- Change the geometry of the pickup circuit, (eg. stretching or squeezing)

Example: Motion Sensor.

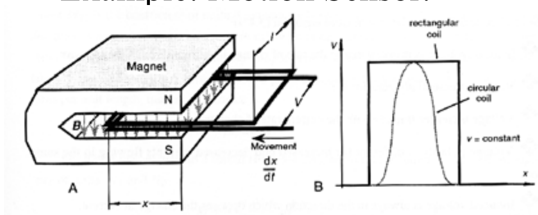


Fig. 3-4.1 Magnetic motion detector (A) and its transfer function (B).

Pickup coil with N turns, moves into the gap of a permanent magnet

Flux enclosed by the loop is: $\Phi_b = Blx$

The induced voltage is: $V = -\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BLx) = -nBl\frac{dx}{dt} = nBlv$

Self Induction.

The magnetic field generated by a coil also induces an emf in itself. This voltage is given by:

$$V = -\frac{d(n\Phi_B)}{dt}$$

The number in parenthesis is called the flux linkage, and is proportional to the current in the coil.

$$n\Phi_B = Li$$

The constant of proportionality is labeled the inductance, L.

$$V = \frac{d(n\Phi_B)}{dt} = -L\frac{di}{dt}$$

We can therefore define the inductance $L = -\frac{V}{\frac{di}{dt}}$

Induction notes.

The defining equation is: $L = -\frac{V}{\frac{di}{dt}}$

Induced voltage is proportional to current change

Voltage is zero for DC (inductors look like short circuit to DC)

Voltage increases linearly with rate of change of coil current

Voltage polarity different for increased and decreased current in same direction

Induced Voltage in direction which acts to oppose change in current

Calculating inductance

Inductance can be calculated from geometry

For a closely packed coil it is $L = \frac{n\Phi_B}{i}$

If n is the number of turns per unit length, the number of flux linkages in a length *l* is

$$N\Phi_B = (nl)\cdot(BA)$$

Plugging in the expression B for a solenoid gives:

$$L = \frac{N\Phi_B}{i} = \mu_0 n^2 lA$$

Note that *lA* is the volume of the solenoid, so keeping n constant and changing the geometry changes L

Inductors and complex resistance

In an electronic circuit, inductance can be represented as *complex resistance*, like capacitance.

$$\frac{V}{i} = j\omega L$$

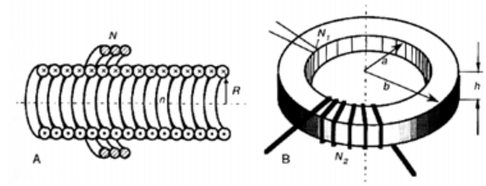
i(t) is a sinusoidal current having a frequency $\omega = 2\pi f$

Two coils brought near each other one coil induces an *emf* in the other

$$V_2 = -M_{21}\frac{di_1}{dt}$$

Where M_{21} is the coefficient of mutual inductance between the coils.

Mutual inductance.



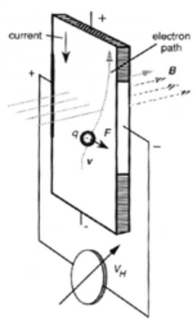
For a coil placed around a long cylinder:

$$M = \mu_0 \pi R^2 n N$$

For a coil placed around a torus, mutual inductance is

$$M = \frac{\mu_0 N_1 N_2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Hall Effect.



When an electron moves through a magnetic field it experiences a sideways force:

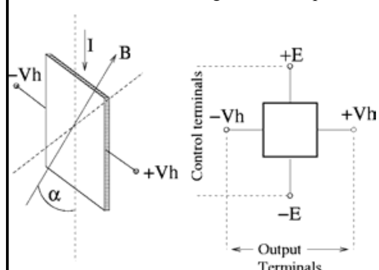
$$F = qvB$$

Q is the electron charge, *v* is the electron velocity, B the magnetic field

This gives rise to an potential difference across an appropriate sensor.

Qualitative Hall effect

The direction of the current and magnetic fields is vital in determining size of the potential difference.



The deflecting force shifts the electrons in the diagram to the right side.

This deflection produces the *transverse Hall potential* V_H

Quantitative hall effect

At fixed temperature, $V_H = h\,I\,B\sin(\alpha)$

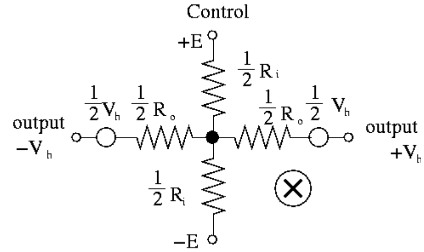
I is the current, B is the magnetic field, α is the angle between the magnetic field and the Hall plate, h is the *Hall coefficient.*

h depends on the properties of the material and is given by:

$$h = \frac{1}{Ncq}$$

- N is the number of free charges per unit volume
- c is the speed of light
- q is the charge on the carrier (+ve if holes).

Effective Circuit for Hall sensor



R_i is the *control resistance*

R_o is the *differential output resistance*

Control current flows through the control terminals

Output is measured across the *differential output terminals*

Hall effect sensors are almost always Semiconductor devices.

Parameters of a Typical sensor.

Control Current	3 mA	
Control Resistance, Ri	2.2 k Ohms	
Control Resistance, Ri vs Temperature	0.8%/C	
Differential Output Resistance, Ro	4.4 K Ohms	
Output offset Voltage	5.0 mV (at B=0 Gauss)	
Sensitivity	60 micro-Volts/Gauss	
Sensitivity vs Temperature	0.1%/C	
Overall Sensitivity	20 V/Ohm-kGauss	
Maximum B	Unlimited	

Note the significant temperature sensitivity.

Also note need to use a constant current source for control.

Piezoresistance of silicon should be remembered; makes semiconductor sensors very sensitive to shocks.