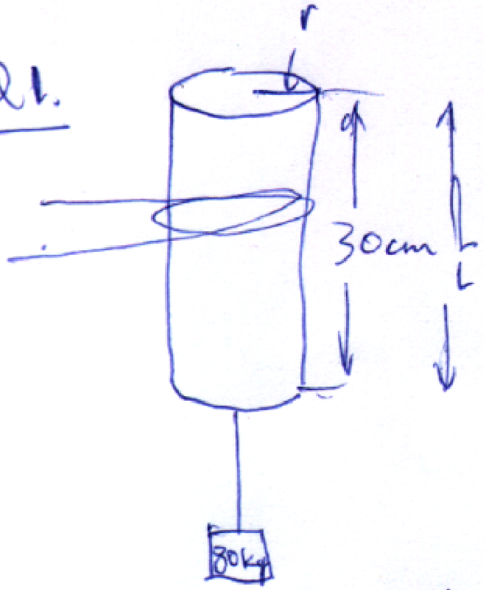
Sheet 2.

Q1.



Q1. Cylinder is ~~s~~ streched by 80 kg mass. Volume is constant but L increases

$$\frac{\Delta L}{L} = \cancel{\text{stress}}\ \text{strain} \qquad Y = \frac{\text{stress}}{\text{strain}} = 70 \times 10^{9}\ Nm^{-2}$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{\text{stress}}{Y}$$

$$\text{stress} = \text{Force}/\text{Area} = \frac{80 \times 9.8}{\pi r^2} = 2496815$$

$$\text{so} \quad \frac{\Delta L}{L} = \frac{2.49 \times 10^5}{70 \times 10^9} = 3.55 \times 10^{-6}$$

$$\text{Volume} = A_1 L_1 = A_2 L_2 \qquad L_2 = L_1 + \Delta L$$

$$\Rightarrow \frac{A_2}{A_1} = \frac{L_2}{L_1} = \frac{L_1 + \Delta L}{L_1} = 1 + \frac{\Delta L}{L}$$

$$\Rightarrow A_2 = A_1 + A_1 \frac{\Delta L}{L}$$

$$\Rightarrow \Delta A_2 = A_1 \frac{\Delta L}{L}$$

$$\pi r_1^2 = A_1, \quad \pi r_2^2 = A_2$$

$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{A_1}{A_2}$$

$$\Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{A_1}{A_2}} \qquad \text{Now, } r_2 = r_1 + \Delta r$$

$$\Rightarrow \frac{r_2}{r_1} = \sqrt{\frac{A_2}{A_1}} \Rightarrow \frac{r_1 + \Delta r}{r_1} = 1 + \frac{\Delta r}{r}$$

$$\text{so} \quad 1 + \frac{\Delta r}{r} = \sqrt{\frac{A_2}{A_1}} = \sqrt{1 + \frac{\Delta L}{L}} \simeq 1 + \frac{1}{2}\frac{\Delta L}{L}$$

Sheet 2 Q1 continued.

so $\frac{\Delta r}{r} \simeq \frac{1}{2}\frac{\Delta L}{L}$

$\frac{\Delta r}{r} = \cancel{3.55}\ 1.78 \times 10^{-6}$

O.K. circumference $= 2\pi r$, what length change of wire?

ΔC = change in circumference $= 1.78 \times 10^{-6} \times 2 \times 10^{-2} \times \pi$

$= 1.12 \times 10^{-7}$ m.

~~Change in Resistance ΔR =~~

Resistance $= \frac{\rho L}{A}$ so $\Delta R = \frac{1.69 \times 10^{-8} \times 1.12 \times 10^{-7}}{\pi \times (5 \times 10^{-4})^2}\ \Omega$

$= \frac{1.89 \times 10^{-15}}{7.85 \times 10^{-7}}\ \Omega$

$= 2.41 \times 10^{-9}\ \Omega$

Change in diameter of the Al cylinder $= 2\Delta r$

$= 2 \times 1.78 \times 10^{-6} \times 0.01$

$= 3.56 \times 10^{-8}$

Sensitivity $= \frac{\Delta R}{R}$; $R = \frac{\rho L}{A} = \frac{1.69 \times 10^{-8} \times 2 \times \pi \times 0.01}{7.85 \times 10^{-7}}$

$= 1.35 \times 10^{-3}\ \Omega$

so $\frac{\Delta R}{R} = 1.79 \times 10^{-6}$

(b) ΔR for cylinder $= \Delta R = \frac{\rho \Delta L}{A} = \frac{2.75 \times 10^{-8} \times 3.55 \times 10^{-6} \times 0.3}{\pi \times (0.01)^2}$

$= 9.322 \times 10^{-11}$

R for cylinder $= R = \frac{\rho L}{A} = \frac{2.75 \times 10^{-8} \times 0.3}{\pi \times (0.01)^2}$

$= 2.626 \times 10^{-5}$

so $\frac{\Delta R}{R} = 3.55 \times 10^{-6}$ is the sensitivity

Measuring the length has 2x the sensitivity however it is very difficult to measure such small absolute resistance. I'd use Cu.

Sheet 2

Q2. (a) Sensitivity = $\frac{\Delta V}{V}$ for $\frac{\Delta F}{F}$ is $\frac{\Delta V}{\Delta F}$

Induced charge $Q = d_{11}F$

Capacitance $C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} = \frac{d_{11}F}{C}$

Now $C = \frac{\varepsilon A}{d} = \frac{1\times10^{-4}\times\varepsilon_0\varepsilon_r}{1\times10^{-3}}$

so $V = \frac{d_{11}F \times10^{-3}}{1\times10^{-4}\,\varepsilon_0\varepsilon_r}$

so $\frac{V}{F} = \frac{d_{11}\times10^{-3}}{\varepsilon_0\varepsilon_r\,10^{-4}} = \frac{10\,d_{11}}{\varepsilon_0\varepsilon_r}$

Then $\frac{\Delta V}{\Delta F}$ For $BaTiO_3$ $= \frac{78\times10^{-9}\times10}{8.9\times10^{12}\times1700}$

$= 52.1$ Volts/newton.

$\frac{\Delta V}{\Delta F}$ for PVDF $= \frac{10\times-30\times10^{-9}}{8.8\times10^{12}\times12} = 2840$ Volts/newton

$\frac{\Delta V}{\Delta F}$ for Quartz $= \frac{10\times2.3\times10^{-9}}{8.8\times10^{12}\times4.5} = 580$ Volts/newton

(b)

	21 °C	-15 °C
$BaTiO_3$	52100 V	51841 V
PVDF	284000 V	282591 ✓
Quartz	58000 V	57712 V

$\Delta T = 36$ °C slope of ~~change~~ $-0.016\%/C$.
so fractional change $= 0.00016\times31$
$= 0.00496$