Sheet 3.

Q1 Don't Do.

Q2. $\rho = \rho_0(1 + \alpha(T - T_0))$

$T_0 = 293\,K$, $\rho_0 = 1.69\ \mu\Omega.cm$.

$\alpha = 3.9 \times 10^{-3} K^{-1}$

(a). $\rho(14K) = 1.69[1 + 3.9\times10^{-3}(14 - 293)]$

$= 1.69[1 + -1.0881]$

$= -0.704\ \mu\Omega.cm$ Non-sense!
(outside calibrated range!).

$\rho(900K) = 1.69[1 + 3.9\times10^{-3}(900 - 293)]$

$= 1.69[1 + 2.367]$

$= 5.69\ \mu\Omega.cm$.

$R(14K) = \frac{\rho L}{A} = \frac{-0.704 \times 3}{\pi \times (0.005)^2} = -26890\ \mu\Omega$.

$R(900K) = \frac{5.69 \times 3}{\pi \times (0.005)^2} = 217342\ \mu\Omega$

$= 0.217\ \Omega$.

(b) $R = R_0(1 + 39.08\times10^{-4}T - 5.8\times10^{-7}T^2$

difference $= R_0 \times 5.8\times10^{-7}(150)^2$

$= 1.305\ \Omega$.

$R(150) = 100(1 + 39.08\times10^{-4}\times150) + 1.305)$

$= 159.925$.

$1°\ \Delta T$ gives $0.3908\ \Omega$.

so $1.305\ \Omega \Rightarrow \Delta T = 3.34\ °C$.

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Q3. Under heating every dimension increases.

$$\frac{\Delta L}{L} = \alpha \Delta T$$

so % change in $L = 1.7 \times 10^{-5} \times 100 = 1.7 \times 10^{-3}$

Let A = cross section area $\propto L^2$

$A = cL^2$

suppose there is a small change in L_1, ΔL, then $L_2 = L_1 + \Delta L$

$$A_2 = cL_2^2$$

O.K. $$\frac{A_2}{A_1} = \frac{cL_2^2}{cL_1^2} = \frac{L_2^2}{L_1^2} = \frac{(L_1+\Delta L)^2}{L_1^2} = \frac{L_1^2 + 2\Delta L L_1 + \Delta L^2}{L_1^2}$$

$$\simeq 1 + \frac{2\Delta L}{L_1}$$

Then $A_2 = A_1 + \Delta A$, so $$\frac{A_1 + \Delta A_1}{A_1} = 1 + \frac{2\Delta L}{L_1}$$

$$\Rightarrow 1 + \frac{\Delta A_1}{A_1} = 1 + \frac{2\Delta L}{L_1}$$

ie $$\frac{\Delta A_1}{A_1} = \frac{2\Delta L}{L}$$

so a $1°$ increase in T gives a $2 \times 1.7 \times 10^{-5} \times 100$

$= 3.4 \times 10^{-3}$ % change in A.

$R = \rho \frac{L}{A}$; assuming ρ = constant for T constant,

$$R = \frac{\rho L}{cL^2} = \frac{\rho}{cL}$$

so $R_2 = \frac{\rho}{cL_2}$ corresponds to a change $L_2 = L_1 + \Delta L$

$$\frac{R_2}{R_1} = \frac{L_1}{L_2} = \frac{L_1}{L_1 + \Delta L}$$

$$\simeq 1 - \frac{\Delta L}{L_1}$$

$$\Rightarrow \frac{R_1 + \Delta R}{R_1} = 1 + \frac{\Delta R}{R} \simeq 1 - \frac{\Delta L}{L}$$

So $\frac{\Delta R}{R} = -\frac{\Delta L}{L}$ so $1°C$ change $\Rightarrow -1.7 \times 10^{-3}$ % decrease in R.

Sheet 3.

Q3 ⓐ If ρ is constant with T it is possible to measure temperature from linear expansion coefficients. Such effects lead to decreases in resistance with T.

Q4. Don't Do this.

Q5. From Sheet 2, Q2.

$Q = d_{11} F$ from Piezoelectric effect.

$$V = \frac{Q}{C} = \frac{d_{11} F}{C}\ ; \quad \text{Now } C = \frac{\varepsilon A}{d}$$

$$= \frac{d_{11} F d}{\varepsilon A}$$

d = thickness
A = area.

Then $\frac{\Delta V}{\Delta F} = \frac{d_{11} d}{\varepsilon A}$ = 52.1 Volts / Newton for $BaTiO_3$

= 2840 Volts / Newton for PVDF

since $V = 1600 \Rightarrow N = \frac{1600}{2840}$

= 0.563 Newtons.

For $BaTiO_3$ $V = 52.1 \times 0.563 = 29.3$ Volts.

Q6. $Q = P_Q A T$

$$V = \frac{Q}{C} = \frac{P_Q A T}{\frac{\varepsilon A}{d}} = \frac{P_Q T d}{\varepsilon}\ ; \quad \text{For } BaTiO_3 \ P_Q = 4 \times 10^{-4}$$

$$= \frac{4 \times 10^{-4} \times 50 \times 10^{-3}}{8.8 \times 10^{-12} \times 1700} = 1336 \text{ Volts.}$$

For PVDF $1336 = \frac{P_Q T d}{\varepsilon} \Rightarrow T = \frac{1336 \times 8.8 \times 10^{-12} \times 12}{0.4 \times 10^{-4} \times 10^{-3}} = 3.53\ ^\circ C.$

Q7 (a) Copper / Aluminium $\alpha_{diff} = 3.2\ \mu V K^{-1}$

(b) Iron / Nickel $(\alpha_{diff} = 13.4 - -20.4)\ \mu V K^{-1}$

$= 33.4\ \mu V K^{-1}$

(c) p-type Si / n-type Si $(\alpha_{diff} = (1000 - -1000)\ \mu V K^{-1}$

$= 2000\ \mu V K^{-1}$

for (a) $12\ mV = 3.2 \times 10^{-3}\ mV\ T$

$\Rightarrow T = 3750\ °C$

for (b) $12\ mV \Rightarrow T = 12/(33.4 \times 10^{-3}) = 359.3\ °C$

for (c) $12\ mV \Rightarrow T = 12/(2000 \times 10^{-3}) = 6.0\ °C$.

Clearly (c) has the highest sensitivity.