Q1. The STM is reliant on the principle of Quantum mechanical tunnelling through a potential barrier. Classically transmission through such a barrier would not be possible.

- In an STM if 2 metals are brought together (within a few nm) electrons can tunnel through the 'insulating' gap between them. This tunneling of electrons can be detected as a current flowing between the tip and surface.

- The AFM is based on the attractive force that exists between atoms when their separation is reduced to 10's of Å. This force is known as Van der Vaals force, and is strongly dependant upon the interatomic spacing.

Q 2. (a)

$$E = \tfrac{1}{2} m v^2 = \frac{p^2}{2m} \quad \& \quad p = h/\lambda$$

$$\Rightarrow E = \frac{h^2}{2m\lambda} \quad \text{OR} \quad \lambda = \frac{h}{\sqrt{2mE}} = \frac{(6.6\times10^{-34})}{\left[2(9.1\times10^{-31})(50\times10^{3})(1.6\times10^{-19})\right]^{\frac{1}{2}}}$$

$$\Rightarrow \lambda = 5.47\times10^{-12}\,\text{pm}$$

(b) Interatomic spacing typically 3Å ($> \lambda$)

Q3

$$T(E) \propto e^{-2\alpha x} \qquad [x = \text{width of barrier}] \quad \alpha = \frac{\sqrt{2m(U-E)}}{\hbar}$$

$$\Rightarrow E_0 = 1\,\text{eV}, \quad E_1 = 1.5\,\text{eV}$$

$\Rightarrow$ let k be a constant of proportionality $\Rightarrow T(E) = ke^{-2\alpha x}$

for $E_0 \Rightarrow T(E_0) = ke^{-2\alpha_0 x}$ & $E_1 \Rightarrow T(E_1) = ke^{-2\alpha_1 x}$

& since $\dfrac{T(E_1)}{T(E_0)} = 5 \Rightarrow \dfrac{e^{-2\alpha_1 x}}{e^{-2\alpha_0 x}} = 5$

$$\Rightarrow \log_e\left(e^{-2\alpha_1 x}\right) = \log_e(5) + \log_e\left(e^{-2\alpha_0 x}\right)$$

$$-2\alpha_1 x = \log_e(5) - 2\alpha_0 x$$

$$\Rightarrow 2x(\alpha_0 - \alpha_1) = \log_e(5) \quad \text{OR} \quad x = \frac{\log_e(5)}{2(\alpha_0 - \alpha_1)}$$

$$\alpha_0 = \frac{\sqrt{2m(5-1\,\text{eV})}}{\hbar} = 1.027\times10^{10}\,\text{m}^{-1}$$

and $$\alpha_1 = \frac{\sqrt{2m(5\,\text{eV} - 1.5\,\text{eV})}}{\hbar} = 9.61\times10^{9}\,\text{m}^{-1}$$

$$\Rightarrow \alpha_0 - \alpha_1 = 0.659\times10^{9}\,\text{m}^{-1}$$

$$x = \frac{\log_e(5)}{2(61.48\times10^{6})} = 1.22\times10^{-9}\,\text{m}$$

Q4. 2 Modes → Contact & non-Contact.

- contact mode

designed to interact with short range interatomic forces.

- Predominant method → Constant force imaging

The force between the cantilever and surface is kept constant (using feedback circuit). Variations in the topography are detected as the cantilever adjusts ~~at~~ its vertical position in order to maintain the constant force. → Prone to surface damage

→ Non-contact mode

In this mode the tip is maintained at a larger separation (~10 → 100nm) and thus probes long range interatomic forces.

- This mode differs from contact mode, instead of measuring quasi-static deflections changes in the resonant response of the cantilever

- Changes in the force gradient between the cantilever & surface causes change in resonant freq. of cantilever.

- Less prone to surface damage → suitable for bio-work

Q5.

$\Rightarrow$ Energy of Thermal vibration is $\frac{1}{2}k_B T$

$\Rightarrow$ Amplitude of vibration can be determined from

$$E = \frac{1}{2}kA^2$$

if $\frac{1}{2}k_B T = \frac{1}{2}kA^2 \Rightarrow \quad A^2 = \frac{k_B T}{k} \quad$ or $\quad A = \sqrt{\frac{k_B T}{k}}$

(a) $T = 400k$

$$\Rightarrow A = \sqrt{\frac{(1.38 \times 10^{-23})(400)}{1.0}} = 0.743 \text{ Å}$$

(b) $T = 77k$

$$\Rightarrow A = \sqrt{\frac{(1.38 \times 10^{-23})(77)}{1.0}} = 0.326 \text{ Å}.$$

Magnetism & Superconductivity

Q6. (a)

$$B_c(4.2k) = B_c(0k)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$= (0.0829)\left(1 - \left(\frac{4.2}{4.47}\right)^2\right)$$

$$= 9.7 \times 10^{-3}\, T.$$

(b) $B = \frac{\mu_0 I}{2\pi r} \Rightarrow I_c = \frac{2\pi B_c r}{\mu_0} = \frac{2\pi(9.7 \times 10^{-3}\, T)(0.3 \times 10^{-3})}{4\pi \times 10^{-7}}$

$$\Rightarrow I_c = 14.568 \text{ A}.$$

Q7 (a) $B_{c_2}(4.2k) = B_{c_2}(0k)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$

$$= (15)\left(1 - \left(\frac{4.2}{9.3}\right)^2\right) = 11.94\, T.$$

(b) $I_c = \frac{2\pi B_{c_2} r}{\mu_0} = \frac{2\pi(11.94)(0.3 \times 10^{-3})}{4\pi \times 10^{-7}} = 17911 \text{ A}.$

→ Hence Type II more suited to generating high B-fields eg. → MRI, B-fields of 1.5T required.

→ Type I more suited to use in SQUID devices.

- Because squids used to detect very small changes in ~~B-field~~ magnetic flux. These changes in magnetic flux would not be large enough to destroy super conducting state.
- Also, since Type I tend to be single elements materials ⇒ easier to deposit using lithographic techniques than Type II (which are usually alloys of 2 different elements)

Q8. $E = hf$

$E = 2.73\,meV = 4.368 \times 10^{-22}\,J$

$\Rightarrow 1/f = h/E = \dfrac{(6.6 \times 10^{-34})}{(4.368 \times 10^{-22})} \Rightarrow f = 6.62 \times 10^{11}\,Hz$

$E_g = 3.53\, k_B T_c$

$\Rightarrow T_c = \dfrac{4.368 \times 10^{-22}\,J}{(3.53)(1.38 \times 10^{-23})} = 8.97\,K.$