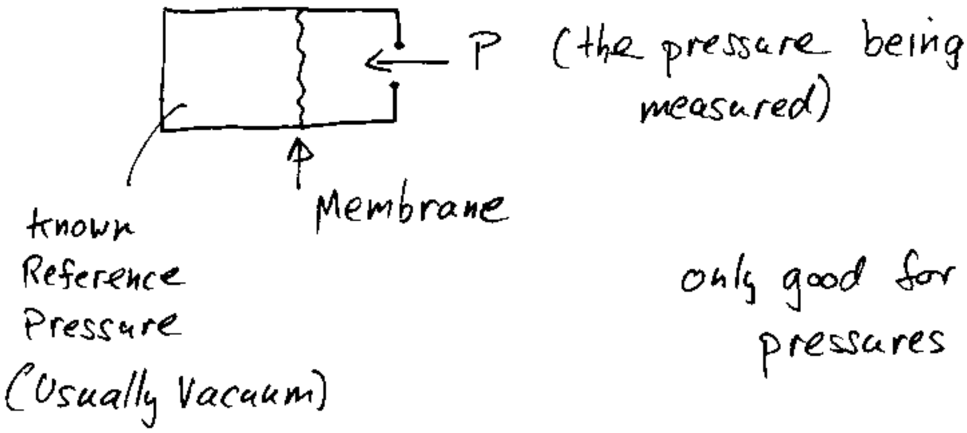
Solutions to Problem sheet 3: Mechanical Sensors

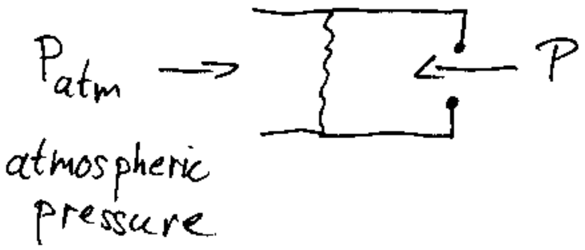
①

(a) (i) Absolute Sensor (of pressure)



only good for V. low pressures

(ii) Gage pressure sensor



good for pressures near atmospheric pressure

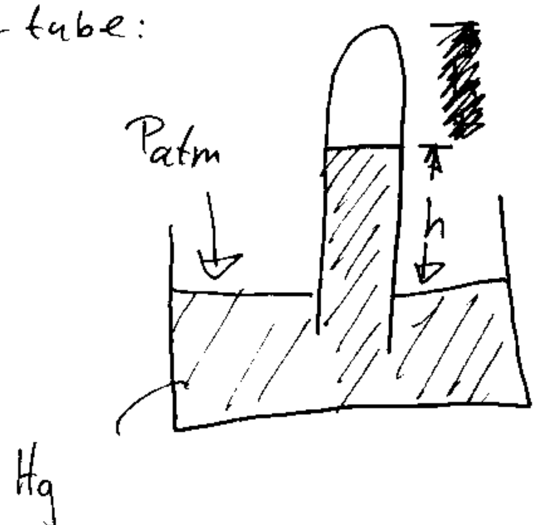
(iii) Differential pressure sensor

P_1 and P_2 are unknown pressures.

In all three cases, we measure the displacement of the membrane; by electrical or optical means, usually.

① (b) For a U-tube barometer, the ~~relationship~~ unknown pressure is directly related to the height of mercury measured.

This question is quite badly posed - In fact, it refers to a mercury barometer without a U-tube:



$$P_{atm} = \rho g h$$

9.8 m/s²

density of Hg

note that if $P_{atm} = 0$ all the Hg flows out.

This may run counter to our intuition about vacuum 'sucking'.

(ii)

$$h_{min} = \frac{P_{atm}}{\rho g}$$

$$= \frac{(99 \times 10^3 \text{ Pa})}{(1.36 \times 10^4 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}$$

$$= 0.743 \text{ m Hg}$$

$$= 743 \text{ mmHg}$$

(i)

$$h_{max} = \frac{102 \times 10^3}{(1.36 \times 10^4)(9.8)}$$

$$= 765 \text{ mmHg}$$

①(b) continued

(iii) Yes - good sensor because 20mm difference can easily be seen by eye.

(iv) Sensitivity $= \dfrac{dh}{dP} = \dfrac{1}{\rho g}$

$$= 7.5 \times 10^{-6} \text{ m/Pa}$$

(v) Improve sensitivity by using fluid with less density, e.g. water.

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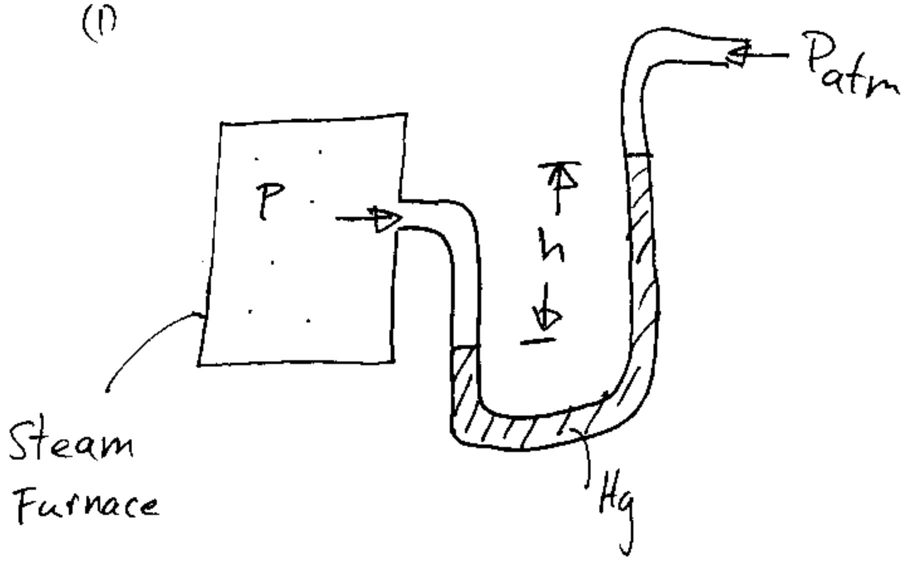
(2)

(a)

(i)



$$P - P_{atm} = \rho g h$$

$$h = \frac{P - P_{atm}}{\rho g}$$

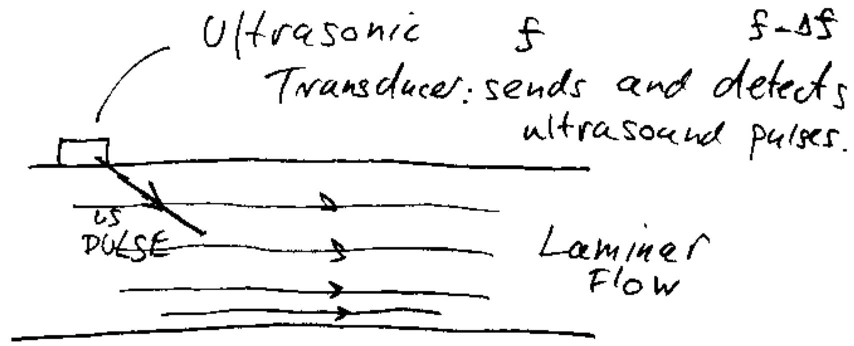
$$= \left[\frac{(12.34)(101325) - (101325)}{13.595 \times 10^3 \times 9.81}\right](1000)$$

$$= 8616 \text{ mm Hg}$$

(ii) 8 metres of mercury? Dangerous and cumbersome. Not good.

② (b)

(i)



note that with Laminar flow the velocity is greatest at the middle and zero at the walls. We will get a range of Doppler shifts – we are interested in the maximum i.e. Δf_{max}.

Ultrasound pulses shifted by

$$\Delta f = \frac{2 f v \cos\theta}{c_s}$$

(ii) $c_s = 1500\ m/s$

$\Delta f = (1.05 - 1) \times 10^6 = .05 \times 10^6$

$\theta = 15^\circ$

plug in to get $v = 38.8\ m/s$

(3) This problem is a bit trivial: it's job is to remind us of the properties of Hookes Law springs, which we will need later.

(a) $\text{weight} = 2.3\,\text{kg} \times 9.8\,\text{m/s}^2$

$= 22.6\ \text{N}$

note the distinction between weight and mass.

(b) $F = -kx \Rightarrow k = \dfrac{22.6}{5.8 \times 10^{-2}}$

$= 390\ \text{N/m}$

(c) $x = \dfrac{(2.3 + 1.2) \times 9.8}{390}$

$= 8.8\ \text{cm}$

notice that we are keeping everything to 2 or 3 significant figures.

(4)

Reynolds number: if less than 4000, laminar
otherwise turbulent.

$$R = \frac{\rho \bar{v} D}{\mu}$$

mean velocity. The blood will be flowing faster than this at the center of the artery.

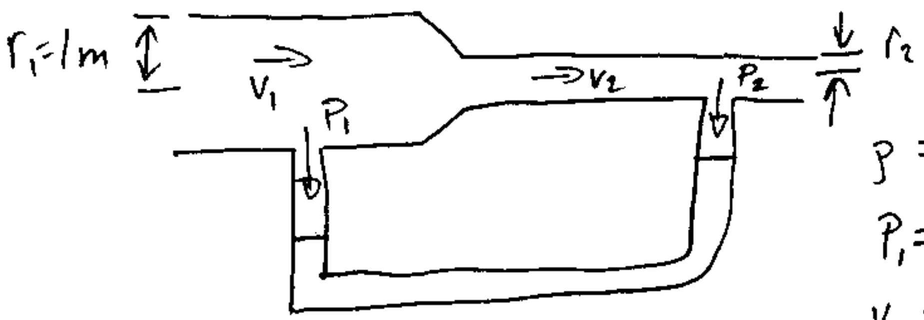
(a) $\bar{v} = \frac{R\mu}{\rho D}$

$\mu = 4 \times 10^{-3}$ Pa.s
$\rho = 1060$ kg/m^3
$D = 2 \times 10^{-2}$ m
$R = 1600$

$\bar{v} = 0.3$ m/s

(b) now $\bar{v} = 0.3 \times 4 = 1.2$ m/s
plug in to get $R = 1600$. Turbulent! not laminar.
(or see that $R = (\text{const})\,\bar{v}$ and multiply by 4 directly.)

(5)



$\rho = 0.68 \times 10^3 \ kg/m^3$

$P_1 = 208 \ kPa$

$v_1 = 4.3 \ m/s$

$P_2 = 182 \ kPa$

$$P + \frac{\rho v^2}{2} = \text{const}$$

(a) $$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$\left[(P_1 - P_2) + \frac{\rho}{2} v_1^2\right] = \frac{1}{2}\rho v_2^2$$

$$v_2 = \sqrt{\frac{2}{\rho}\left[(P_1 - P_2) + \frac{\rho}{2} v_1^2\right]}$$

plug in to get

$$v_2 = 9.74 \ m/s$$

(b) $A_1 v_1 = A_2 v_2$ because the amount of fluid flowing per second must be the same. [Conservation of mass flow].

units:

$A_1 v_1 = m^2 . m/s$

$= m^3/s$ = volume/time.

$A_2 = \pi r_2^2$

$A_2 = \frac{A_1 v_1}{v_2}$

$r_2 = r_1 \sqrt{\frac{v_1}{v_2}}$

$= 0.66 \ m$