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Suggested Solutions for Tutorial 6

Blackbody radiation and the Stefan-Boltzmann law

Q1: To calculate the effective area we use the Stefan-Boltzmann law.

$$P_{net} = e\sigma A(T^4 - T_0^4)$$

Since we have a blackbody, $e = 1$, σ is a known constant and is equal to 5.67×10^{-8} W.m^{-2}.K^{-4}, P_{net} is 1000 W, the temperature of the blackbody $T = 900 + 273$ K and the reference temperature (room temperature) is $T_0 = 293$ K. Solving the above equation for the area we have

$$A = \frac{P}{e\sigma(T^4 - T_0^4)}$$

and now we just plug in the numbers

$$\begin{aligned} A &= \frac{1 \times 10^3}{1 \times 5.67 \times 10^{-8} \times ((900 + 273)^4 - 293^4)} \\ &= 0.0094 \text{ m}^2 \end{aligned}$$

Q2: The surface area of the ball is

$$A = 4\pi r^2 = (4\pi)(0.02 \text{ m})^2 = 0.005 \text{ m}^2$$

and its absolute temperature is $T = 400\ ^\circ\text{C} + 273 = 673$ K. Hence

$$P = e\sigma AT^4 = 0.3 \times 5.67 \times 10^{-8} \times 0.005 \times (673)^4 = 17.4 \text{ W}$$

Q3: The emissivity of a blackbody is $e = 1$. From $R = \frac{P}{A} = \sigma T^4$ we have

$$T = \sqrt[4]{\frac{R}{\sigma}} = \sqrt[4]{\frac{6.5 \times 10^7}{5.67 \times 10^{-8}}} = 5800 \text{ K}$$

Q4: The emissivity of skin is the same at both temperatures, so

$$\frac{R_1}{T_1^4} = \frac{R_2}{T_2^4}$$

Here $T_1 = 34\ ^\circ\text{C} + 273 = 308$ K. Hence

$$\frac{R_2 - R_1}{R_1} = \frac{T_2^4 - T_1^4}{T_1^4} = \frac{308^4 - 307^4}{307^4} = 0.013 = 1.3\ \%$$

which is large enough to detect.

Q5: The person will lose energy from 33 °C to 20 °C so one needs to calculate the power lost from the higher temperature to the lower temperature. This is described by

$$P_{net} = e\sigma A(T^4 - T_0^4)$$

Using this equation ant $T = 306$ K and $T_0 = 293$ K, we have

$$P_{net} = 1 \times 5.67 \times 10^{-8} \times 1.4 \times (306^4 - 293^4) = 111 \text{ W}$$

This is a large energy loss. It is approximately equal to the basal metabolic rate of about 120 W for a food intake of 2500 kcal per day. We protect ourselves from such a great energy loss by wearing clothing, which, because of its low thermal conductivity, has a much lower outside temperature and therefore a much lower rate of thermal radiation.

Q6: *(a)* Since we have

$$2.82 = \frac{hf_{max}}{kT}$$

and we know that

$$c = f_{max}\lambda_{max}$$

we get the relation

$$2.82 = \frac{hc}{\lambda_{max}kT}$$

Solving for λ_{max} gives

$$\lambda_{max} = \frac{hc}{kT \times 2.82}$$

plugging in the temperature we have

$$\begin{aligned}\lambda_{max} &= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23} \times 3 \times 2.82} \\ &= 0.00170264 \text{ m}\end{aligned}$$

Q6: *(b)* reusing the equation above we have

$$\begin{aligned}\lambda_{max} &= \frac{hc}{kT \times 2.82} \\ &= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23} \times 300 \times 2.82} \\ &= 1.70264 \times 10^{-5} \text{ m}\end{aligned}$$

Q6: *(c)* reusing the equation again we have

$$\begin{aligned}\lambda_{max} &= \frac{hc}{kT \times 2.82} \\ &= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23} \times 3000 \times 2.82} \\ &= 1.70264 \times 10^{-6} \text{ m}\end{aligned}$$

Q7: *(a)* Since we have

$$2.82 = \frac{hf_{max}}{kT}$$

and we know that

$$c = f_{max}\lambda_{max}$$

we get the relation

$$2.82 = \frac{hc}{\lambda_{max}kT}$$

Solving for T gives

$$T = \frac{hc}{\lambda_{max} k \times 2.82}$$

plugging in the wavelength we have

$$\begin{aligned} T &= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{700 \times 10^{-9} \times 1.38 \times 10^{-23} \times 2.82} \\ &= 7297.04 \text{ K} \end{aligned}$$

Q7: *(b)* reusing the above equation we get

$$\begin{aligned} T &= \frac{hc}{\lambda_{max} k \times 2.82} \\ &= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{3 \times 10^{-2} \times 1.38 \times 10^{-23} \times 2.82} \\ &= 0.170264 \text{ K} \end{aligned}$$

Q7: *(c)* using the equation again we get

$$\begin{aligned} T &= \frac{hc}{\lambda_{max} k \times 2.82} \\ &= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{3 \times 1.38 \times 10^{-23} \times 2.82} \\ &= 0.00170264 \text{ K} \end{aligned}$$

Q8: *(a)* Given we know that

$$2.82 = \frac{hf_{max}}{kT}$$

we wish to merely solve for f_{max} and plug the relevant numbers in like so

$$\begin{aligned} f_{max} &= \frac{2.82kT}{h} \\ &= \frac{2.82 \times 1.38 \times 10^{-23} \times 300}{6.626 \times 10^{-34}} \\ &= 1.76197 \times 10^{13} \text{ Hz} \end{aligned}$$

Q8: *(b)* From the relation

$$c = f\lambda$$

we can find λ_{max} corresponding to f_{max}. This is

$$\begin{aligned} \lambda_{max} &= \frac{c}{f_{max}} \\ &= \frac{3 \times 10^{8}}{1.76197 \times 10^{13}} \\ &= 1.70264 \times 10^{-5} \text{ m} \end{aligned}$$

Q9: *(a)* We have that

$$2.82 = \frac{hf_{max}}{kT} = \frac{hc}{\lambda_{max}kT}$$

hence the temperature is

$$T = \frac{hc}{2.82\lambda_{max}k} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{2.82 \times 589 \times 10^{-9} \times 1.38 \times 10^{-23}} = 8672.2 \text{ K}$$

Q9: *(b)* Using the equation for temperature again we have

$$T = \frac{hc}{2.82\lambda_{max}k} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{2.82 \times 16.4 \times 10^{-6} \times 1.38 \times 10^{-23}} = 311.459 \text{ K}$$

Q9: *(c)* *(a)* The sun, *(b)* a human

Q10: That's where most of the electromagnetic energy from the sun is.