PHYS2180: Suggested Solutions for Tutorial 7

Optical sensors

Photoelectric effect

Q1: photodiode/photomultiplier/avalanche photodiode

Q2: *(a)* Since $c = f\lambda$, the frequency of the light is

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{5 \times 10^{-7}} = 6 \times 10^{14} \text{ Hz}$$

The energy of each photon is therefore

$$E = hf = 6.63 \times 10^{-34} \times 6 \times 10^{16} = 3.98 \times 10^{-19} \text{ J}$$

Since 1 eV $= 1.6 \times 10^{-19}$ J,

$$E = \frac{3.98 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.49 \text{ eV}$$

This is the maximum energy that can be given to an electron by a photon of this light. Because 2 eV is needed to remove an electron, the maximum energy of the photoelectrons in this situation is 0.49 eV.

Q2: *(b)* A shorter wavelength means a higher frequency and hence more energy to be imparted to the photoelectrons.

Q3: We know from the notes that

$$K_{max} = hf - w$$

The maximum wavelength will correspond to the *minimum* frequency, which occurs when the energy of the incident photon ($E = hf$) is equal to the work function of the metal, i.e. when $K_{max} = 0$, so now we have

$$\begin{aligned} hf &= w \\ \Rightarrow \frac{hc}{\lambda} &= w \qquad (c = f\lambda) \\ \Rightarrow \lambda &= \frac{hc}{w} \\ &= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{2.3 \times 1.6 \times 10^{-19}} \\ &= 5.4 \times 10^{-7} \text{ m} \end{aligned}$$

Q4: From the notes we have

$$K_{max} = hf - w$$

Since we know the minimum frequency where photoelectrons are emitted from the copper surface, then we know that $K_{max} = 0$ and hence we have sufficient information to calculate the work function, which is given by

$$\begin{aligned} w &= hf \\ &= 6.626 \times 10^{-34} \times 1.1 \times 10^{15} \\ &= 7.2886 \times 10^{-19} \text{ J} \end{aligned}$$

Since we now know the work function of the metal, we can calculate the maximum energy of the photoelectrons when light of a higher frequency is directed at the surface, which we do now by plugging the relevant numbers into the formula

$$\begin{aligned} K_{max} &= hf - w \\ &= 6.626 \times 10^{-34} \times 1.5 \times 10^{15} - 7.2886 \times 10^{-19} \\ &= 2.6504 \times 10^{-19} \text{ J} \end{aligned}$$

Q5: *(a)* The work function is equal to the energy of the incoming photons since they are at the threshold frequency f_t.

$$w = hf_t = \frac{hc}{\lambda_t} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{558 \times 10^{-9}} = 2.22 \text{ eV}$$

Q5: *(b)* The energy of a photon of wavelength 400 nm is

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}} = 3.1 \text{ eV}$$

The maximum kinetic energy of the emitted electrons is then

$$\left(\frac{1}{2}mv^2\right)_{max} = hf - w = 3.10 \text{ eV} - 2.22 \text{ eV} = 0.88 \text{ eV}$$

The stopping potential is therefore 0.88 eV.

X-rays

Q6: Since $hf = eV$, here we have

$$f = \frac{eV}{h} = \frac{1.6 \times 10^{-19} \times 10^4}{6.63 \times 10^{-34}} = 2.4 \times 10^{18} \text{ Hz}$$

Q7: The frequency of the X-rays is

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{2 \times 10^{-11}} = 1.5 \times 10^{19} \text{ Hz}$$

Since $hf = eV$,

$$V = \frac{hf}{e} = \frac{6.63 \times 10^{-34} \times 1.5 \times 10^{19}}{1.6 \times 10^{-19}} = 6.2 \times 10^4 \text{ V} = 62 \text{ keV}$$