Solutions to problem sheet 2

640-381: Sensors

September 2, 2004

1 Question 1

1.1 Problem

An 80 kg mass is hung from the end of a 30 cm long aluminium cylinder of raqdius 1 cm. The diameter of the Al cylinder is monitored by a piezoresistive sensor, consisting of an insulated 0.5 mm radius copper wire wrapped once around the cylinder.

- What is the resistance change of the Cu sensor?
- What change in diameter of the Al cylinder does this represent?
- What is the sensitivity of the Cu Sensor?
- What is the resistance change in the Al cylinder?
- What is the sensitivity of the Al cylinder?
- Would you use the resistance change in the Al cylinder to measure the dimensional changes instead of the Cu piezorestivie sensor?

Data:

$$Y_{Al} = 70 \times 10^9 N.m^{-2}$$
$$\rho_{Al} = 2.75 \times 10^{-8} \Omega.m$$
$$\rho_{Cu} = 1.69 \times 10^{-8} \Omega.m$$

1.2 Solution

The first thing to do, as always, is draw a picture. The cylinder is stretched by an 80 kg mass. The volume is constant because no material is added or removed, and we assume that the density doesn't change. However, we are told that L increases. We know from lectures that

$$\frac{\Delta L}{L} = strain$$

Is the definition of strain. In this case, the stress and strain are in the same direction, which makes the maths (strictly, geometry) much simpler. Y=stress/strain is the definition of Youngs modulus, which is quoted as 70109 N.m in the original versions of the tute sheet, but should actually read $Y = 7 \times 10^{10} N.M^{-2}$ So,

$$\frac{\Delta L}{L} = \frac{stress}{Y}$$

Stress is defined as

$$\begin{aligned} stress &= force/area \\ &= \frac{mg}{A} \\ &= \frac{80 \times 9.8}{\pi(.001)^2} \\ &= 2496815 \end{aligned}$$

$$\begin{aligned} \frac{\Delta L}{L} &= \frac{mg}{Y} \\ &= \frac{2.49 \times 10^5}{7 \times 10^{10}} \\ &= 3.55 \times 10^{-6} \end{aligned}$$

Let L_1 be the length before stretching, and L_2 the length after stretching.

$$\begin{aligned} L_2 &= L_1 + \delta L \\ &= L_1 + \frac{mg}{Y} L_1 \end{aligned}$$

$$
\begin{aligned}
&= L_1(1 + \frac{mg}{Y}) \\
&= (1 + 3.55 \times 10^{-6})(0.3) \\
&= 0.300001065
\end{aligned}
$$

Let r_1 and r_2 be the radii of the Al cylinder before and after stretching, respectively.

Because the volume is constant,

$$\pi r_1^2 L_1 = \pi r_2^2 L_2$$

$$
\begin{aligned}
r_2 &= r_1 \sqrt{\frac{L_1}{L_2}} \\
&= r_1 \sqrt{\frac{1}{1 + \frac{mg}{Y}}}
\end{aligned}
$$

The change in the radius of the cylinder is

$$
\begin{aligned}
\Delta r &= r_2 - r_1 \\
&= r_1(1 - \sqrt{\frac{1}{1 + \frac{mg}{Y}}}) \\
&= (.3)(1 - \sqrt{\frac{1}{1 + \frac{800}{7 \times 10^{10}}}}) \\
&= 1.7 \times 10^{-9} m
\end{aligned}
$$

The length of copper wire around the Al cylinder becomes fatter and shorter as the Al cylinder stretches. Let us assume that the change in area of the copper wire has a negligible effect on the resistance compared with the change in length (we will justify this assumption later). Let us also assume that the restivity of the copper doesn't change. The length of the wire changes from $2\pi r_1$ to $2\pi r_2$. Let the old resistance be R_1 and the new resistance be R_2.

The resistance change is

$$\begin{aligned}
\Delta R &= R_2 - R_1 \\
&= \frac{\rho}{A} 2\pi (r_2 - r_1) \\
&= \frac{\rho}{A} 2\pi \Delta r \\
&= \frac{2\pi 1.69 \times 10^{-8}}{\pi (.0005)^2} 1.7 \times 10^{-9} \\
&= 2.28 \times 10^{-10} \Omega
\end{aligned}$$

The change in the diameter of the cylinder is simply

$$2\Delta r = 3.56 \times 10^{-8}$$

The restance of the copper wire is

$$\begin{aligned}
R &= \frac{\rho 2\pi r_1}{A} \\
&= \frac{2\pi 1.69 \times 10^{-8}}{\pi (.0005)} \\
&= 1.35 \times 10^{-3} \Omega
\end{aligned}$$

The sensitivity is

$$\frac{\Delta R}{R} = 1.79 \times 10^{-6}$$

The change in resistance for the Al cylinder is

$$\begin{aligned}
\Delta R &= \frac{\rho \Delta L}{A} \\
&= \frac{2.75 \times 10^{-8} (0.3)}{\pi (.01)^2} \\
&= 9.322 \times 10^{-11} \Omega
\end{aligned}$$

the resistance of the Al cylinder is

$$\begin{aligned}
R &= \frac{\rho L}{A} \\
&= \frac{2.75 \times 10^{-8} (0.3)}{\pi (.01)^2} \\
&= 2.626 \times 10^{-5} \Omega
\end{aligned}$$

The sensitivity of the Al cylinder is

$$\frac{\Delta R}{R} = 3.55 \times 10^{-6}$$

Measuring the length has twice the sensitivity, but it is very difficult to measure such a small absolute resistance, so I would use Cu.

2 Question 2

2.1 Problem

What is the sensitivity of a 1 mm thick BaTiO3 piezoelectric force sensor with an electrode area of 1 cm2? How much less sensitive would the sensor be if it were made of PVDF? Of Quartz? For quartz, sensitivity drops with a slope of -0.016 %/C. Assuming the above sensitivity was measured at 21 C, what voltages would be measured if a 100 N force were applied to the quartz sensor at 21 C and - 15 C?

Data:

$$\begin{aligned}
\epsilon_r^{BaTiO_3} &= 1700 \\
d_{11}^{BaTiO_3} &= 78pC/N \\
\epsilon_r^{quartz} &= 4.5 \\
d_{33}^{quartz} &= 2.3pC/N \\
\epsilon_r^{PVDF} &= 12 \\
d_{33}^{PVDF} &= -30pC/N
\end{aligned}$$

2.2 Solution

$$Sensitivity = \frac{\partial V}{\partial F}$$

Induced charge is

$$Q = d_{11}F$$

Capacitance links charge and voltage

$$C = \frac{Q}{V}$$

so

$$\begin{aligned} V &= \frac{Q}{C} \\ &= \frac{d_{11}F}{C} \end{aligned}$$

Now

$$\begin{aligned} C &= \frac{\epsilon A}{d} \\ &= 10^{-4}\epsilon_r\epsilon_0 10^{-3} \end{aligned}$$

so

$$V = \frac{d_{11}F(10^{-3})}{10^{-4}\epsilon_r\epsilon_0}$$

and the sensitivity is

$$\frac{\partial V}{\partial F} = \frac{(10)d_{11}}{\epsilon_r\epsilon_0}$$

grinding on,

$$\begin{aligned} \frac{\partial V}{\partial F}_{BaTiO_3} &= \frac{78 \times 10^{-9}(10)}{8.8 \times 10^{-12}(1700)} \\ &= 52.1V/N \end{aligned}$$

$$\begin{aligned} \frac{\partial V}{\partial F}_{PVDF} &= \frac{-30 \times 10^{-9}(10)}{8.8 \times 10^{-12}(12)} \\ &= 2840V/N \end{aligned}$$

$$\begin{aligned} \frac{\partial V}{\partial F}_{quartz} &= \frac{2.3 \times 10^{-9}(10)}{8.8 \times 10^{-12}(4.5)} \\ &= 580V/N \end{aligned}$$

We are told that the sensitivity changes by -0.016 parts in 100 for every one degree rise in temperature. The wording of the question is a little ambigious, but in fact quartz sensors become less sensitive as they get hotter. The temperature difference is 36 degrees, so the sensitivity changes by $36 \times 0.016 = 0.576\%$. Thus

$$V_{21^\circ} = \frac{\partial V}{\partial F}_{quartz} \times 100 = 58000V$$

$$V_{-15^\circ} = 580 \times (1 - 0.00576) \times 100 = 577000V$$