Suggested Solutions for Tutorial 4

Mechanical sensors

Force sensors

Q1: *(a)* The weight is given by

$$\begin{aligned} W &= 2.3 \times 9.81 \\ &= 22.563 \text{ N} \end{aligned}$$

Q1: *(b)* Since the restoring force is given by $F = -kx$, the weight of bananas is $W = kx$, so solving this for k and plugging in the numbers we get

$$\begin{aligned} W &= kx \\ \Rightarrow k &= \frac{W}{x} \\ &= \frac{22.563}{5.8 \times 10^{-2}} \\ &= 389.017 \text{ N/m} \end{aligned}$$

Q1: *(c)* The new extension is given by solving $W = kx$ for x and using the new weight given by the additional mass of bananas.

$$\begin{aligned} W &= kx \\ \Rightarrow x &= \frac{W}{k} \\ &= \frac{(2.3 + 1.2) \times 9.81}{389.017} \\ &= 0.0882609 \text{ m} \end{aligned}$$

Pressure sensors

Q2: *(a)* An absolute pressure sensor is one where one chamber is evacuated and sealed, the other is ported to the unknown pressure P.

Q2: *(b)* A gauge pressure sensor is one where one chamber is vented to the atmosphere, the other is ported to the unknown pressure P.

Q2: *(c)* A differential pressure sensor is one where one chamber is ported to an unknown pressure P_1, and the other is ported to an unknown pressure P_2, the differential pressure is $\Delta P = P_1 - P_2$.

Q3: *(a)* An open-tube manometer measures pressure by comparing the unknown pressure to atmospheric pressure added to the pressure of a height of mercury. Put mathematically this is

$$P = P_{atm} + \rho g h$$

to find the height of mercury we solve this equation for h and put the numbers into it

$$\begin{aligned} h &= \frac{P - P_{atm}}{\rho g} \\ &= \left(\frac{12.34 \times 101325 - 101325}{13.595 \times 10^3 \times 9.81} \right) \times 1000 \\ &= 8615.52 \text{ mmHg} \end{aligned}$$

Q3: *(b)* No, it will need a mercury column more than 8 m high!

Q4: *(a)* For a U-tube barometer the relationship between the unknown pressure is directly related to the height of mercury measured.

$$P = \rho g h$$

To find the height of mercury for the maximum pressure, we solve this for h and plug in the numbers

$$\begin{aligned} h_{max} &= \frac{P_{max}}{\rho g} \\ &= \left(\frac{102 \times 10^3}{13.595 \times 10^3 \times 9.81}\right) \times 1000 \\ &= 764.807 \text{ mmHg} \end{aligned}$$

Q4: *(b)* Similarly for the minimum pressure

$$\begin{aligned} h_{min} &= \frac{P_{min}}{\rho g} \\ &= \left(\frac{99 \times 10^3}{13.595 \times 10^3 \times 9.81}\right) \times 1000 \\ &= 742.313 \text{ mmHg} \end{aligned}$$

Q4: *(c)* Yes, it is definitely able to distinguish between the maximum and minimum values, there being a difference of about 20 mm and hence about 20 graduations between minimum and maximum.

Q4: *(d)* The sensitivity of the apparatus is the derivative of h with respect to P, this is

$$\begin{aligned} \frac{dh}{dP} &= \frac{1}{\rho g} \\ &= \frac{1}{13.595 \times 10^3 \times 9.81} \\ &= 7.49811 \times 10^{-6} \text{ m/Pa} \end{aligned}$$

Q4: *(e)* use a substance instead of mercury with a smaller density.

Medical pressure measurement

Q5: *(a)* From the notes we have

$$P = FR$$

solving this for R we get

$$\begin{aligned} R &= \frac{P}{F} \\ &= \frac{13.3 \times 10^3}{0.8 \times 10^{-3}} \\ &= 1.6625 \times 10^7 \text{ Pa.s/m}^3 \end{aligned}$$

Q5: *(b)* Again, from the notes we have Poiseuille's law

$$F = \frac{P\pi R^4}{8\eta L}$$

solving this for L and plugging in the numbers gives

$$\begin{aligned} L &= \frac{P\pi R^4}{8\eta F} \\ &= \frac{13.3 \times 10^3 \times \pi \times (0.0005)^4}{8 \times 4 \times 10^{-3} \times 0.8 \times 10^{-3}} \\ &= 0.00010201 \text{ m} \end{aligned}$$

Flow rate sensors

Q6: *(a)* From the notes we know that

$$R = \frac{\rho \bar{v} D}{\mu}$$

solving this for $\bar{v}$ and plugging in the numbers gives

$$\begin{aligned} \bar{v} &= \frac{R\mu}{\rho D} \\ &= \frac{1600 \times 4 \times 10^{-3}}{1060 \times 2 \times 10^{-2}} \\ &= 0.301887 \text{ m/s} \end{aligned}$$

Q6: *(b)* The Reynolds number is given above and plugging the relevant bits and pieces into it gives

$$\begin{aligned} R &= \frac{\rho \bar{v} D}{\mu} \\ &= \frac{4 \times 1060 \times 3.01887 \times 2 \times 10^{-3}}{4 \times 10^{-3}} \\ &= 6400 \end{aligned}$$

the Reynolds number is greater than 4000, hence we expect the flow to be turbulent.

Q7: *(a)* The Venturi effect gives

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

where P_1 and v_1 are the pressure and velocity respectively of the pipe initially, and P_2, v_2 are the pressure and velocity respectively of the pipe after the pipe narrows, and ρ is the density of the petrol. We wish to solve for v_2 so...

$$\begin{aligned} P_1 + \frac{1}{2}\rho v_1^2 &= P_2 + \frac{1}{2}\rho v_2^2 \\ \frac{2(P_1 - P_2)}{\rho} &= v_2^2 - v_1^2 \end{aligned}$$

$$\begin{aligned} v_2^2 &= v_1^2 + \frac{2(P_1 - P_2)}{\rho} \\ v_2 &= \sqrt{v_1^2 + \frac{2(P_1 - P_2)}{\rho}} \\ &= \sqrt{(4.3)^2 + \frac{2(208 \times 10^3 - 182 \times 10^3)}{0.68 \times 10^3}} \\ &= 9.74477 \text{ m/s} \end{aligned}$$

Q7: *(b)* We can write the statement in part *(a)* mathematically as

$$A_1 v_1 = A_2 v_2$$

where A_1 and v_1 are the cross sectional area and velocity initially and A_2 and v_2 are the cross sectional area and velocity finally. The radius of the narrow part of the pipe will be obtained from A_2, so we solve the above equation for this

$$A_2 = \frac{A_1 v_1}{v_2}$$

knowing that the cross sectional area is given by $A = \pi r^2$ we have

$$\pi r_2^2 = \frac{\pi r_1^2 v_1}{v_2}$$

solving for r_2

$$\begin{aligned} r_2^2 &= \frac{r_1^2 v_1}{v_2} \\ \Rightarrow r_2 &= r_1 \sqrt{\frac{v_1}{v_2}} \\ &= 1 \times \sqrt{\frac{4.3}{9.74477}} \\ &= 0.664276 \text{ m} \end{aligned}$$

Q8: From the notes we have

$$\Delta f = \frac{2 f v \cos\theta}{c_s}$$

we want v so

$$\begin{aligned} v &= \frac{\Delta f c_s}{2 f \cos\theta} \\ &= \frac{(1.05 \times 10^6 - 1 \times 10^6) \times 1500}{2 \times 1 \times 10^6 \times \cos(15^\circ)} \\ &= 38.8229 \text{ m/s} \end{aligned}$$