Suggested Solutions for Tutorial 5

Chemical sensors

Catalytic sensors

Q1: Take a platinum coil imbedded in a pellet of ThO_2/Al_2O_3 and coated with a porous catalytic metal (e.g. palladium). When combustible gas reacts with the catalytic surface (which is hot as the platinum coil acts as a heater as well as resistive temperature sensor) and the heat from the reaction increases the temperature of the pellet and thereby increases the resistance of the platinum coil, which one detects, the heating being related to how much flammable gas exists in the vicinity.

Electrochemical sensors

Q2: *(a)* We know that

$$I = \frac{4FA\alpha_m D_m P_0}{x_m}$$

and that

$$S = \frac{I}{P_0}$$

so this means that

$$S = \frac{4FA\alpha_m D_m}{x_m}$$

We wish to find the thickness x_m, so we solve this equation for it and plug in the numbers

$$\begin{aligned} x_m &= \frac{4FA\alpha_m D_m}{S} \\ &= \frac{4 \times 9.6 \times 10^4 \times 2.6 \times 10^{-6} \times 1.5 \times 10^{-6} \times 1.3 \times 10^{-9}}{7.5 \times 10^{-9}} \\ &= 2.59584 \times 10^{-7} \text{ m} \end{aligned}$$

Q2: *(b)* Since F, α_m and D_m are constants, we can vary only A and x_m, hence having a smaller thickness and/or a larger surface area will increase the sensitivity

Q2: *(c)* The area, since the thickness is very hard to control.

Concentration sensors

Q3: *(a)*

$$c = \frac{-\log \frac{I}{I_0}}{\epsilon l}$$

Q3: *(b)* Since we have

$$c = \frac{A}{\epsilon l}$$

this implies that the absorbance is given by

$$A = \epsilon c l$$

the sensitivity is the derivative of this with respect to the concentration

$$\frac{dA}{dc} = \epsilon l$$

Q3: *(c)* Since ϵ is a constant, the only parameter one can play with is l which would need to be increased to increase the sensitivity.

Bomb detection

Q4: We know that the energy of a spring is given by

$$E = \frac{1}{2} k x^2$$

Since x_0 is at the equilibrium position of the spring, then the energy stored in the spring there is zero, so we can say that the energy required to get to point x_f is

$$E = \frac{1}{2} k x_f^2$$

We need at least 2 TNT molecules to get to x_f, this amount of energy is $2 \times 10^{-3} \times 1.6 \times 10^{-19}$ J. So, we solve the above equation for k and use the amount of energy released from exploding two TNT molecules and that $x_f = 10\ \mu$m. This gives

$$\begin{aligned} k &= \frac{2E}{x_f^2} \\ &= \frac{2 \times 2 \times 10^{-3} \times 1.6 \times 10^{-19}}{(10 \times 10^{-6})^2} \\ &= 6.4 \times 10^{-12} \text{ N/m} \end{aligned}$$