source the greater the count rate will be through all angles between the two detectors.

$^{60}$Co $\gamma$-$\gamma$ Correlation

**Theory**

The angular distribution of a $\gamma$ emission from an excited nuclear state with initial angular momentum $I_i$ decaying to a final state $I_f$. (See Figure 33) depends on the total angular momentum $L$. From conservation of momentum $L$ is defined by a closed vector triangle. Thus

$$ L = I_f - I_i, \quad (7) $$

with a $z$-projection of

$$ M = m_f - m_i \quad (8) $$

for the allowable spin substates $2I + 1$.

The total angular momentum is therefore restricted by the selection rule

$$ |I_i - I_f| \leq L \leq I_i + I_f \quad (9) $$

![Figure 33. Energy level diagram for the transition $I_i \rightarrow I_f$](image)
It is important to note that under normal circumstances the angular distribution is isotropic or their is an equal population of the \( m \) substates, i.e.

\[
W(\Theta) = \text{constant}
\]

There are two ways to produce an equal population of \( m \) substates. One is to align the nuclei in a strong magnetic field (~10T) and cooling them to ~ 0.01K, which is not practical here. The other is to observe a previous radiation from a \( \gamma \)-cascade of two radiations \( \gamma_1 \) and \( \gamma_2 \). This is known as Angular correlation.

Exercise 1

The distributions for the decay \( I_{c}=2 \rightarrow I_{f}=1 \), with \( L=2 \) and \( M=0, \pm 1 \) are

\[
W_{2}^{0}(\Theta) = \frac{1}{4\pi} \left( 1 + \frac{5}{7} L_2(\cos \Theta) - \frac{12}{7} P_4(\cos \Theta) \right)
\]

\[
W_{2}^{+1}(\Theta) = \frac{1}{4\pi} \left( 1 + \frac{5}{14} L_2(\cos \Theta) + \frac{8}{7} P_4(\cos \Theta) \right)
\]

\[
W_{2}^{-2}(\Theta) = \frac{1}{4\pi} \left( 1 - \frac{5}{7} L_2(\cos \Theta) - \frac{2}{7} P_4(\cos \Theta) \right)
\]

where the Legendre Polynomials are defined as

\[
P_2(\cos \Theta) = \frac{1}{2} (3\cos^2 \Theta - 1)
\]

\[
P_4(\cos \Theta) = \frac{1}{8} (35\cos^4 \Theta - 30\cos^2 \Theta + 3)
\]

For \( M=0 \) we have:

\[
W_{2}^{0}(\Theta) = \frac{1}{4\pi} \left( 1 + \frac{5}{7} \left( \frac{1}{2} \right)(3\cos^2 \Theta - 1) - \left( \frac{12}{7} \right) \left( \frac{1}{8} \right)(35\cos^4 \Theta - 30\cos^2 \Theta + 3) \right)
\]

\[
= \frac{1}{4\pi} \left[ 1 + \frac{15}{14} \cos^2 \Theta - \frac{5}{14} - \frac{420}{56} \cos^4 \Theta + \frac{360}{56} \cos^2 \Theta - \frac{36}{56} \right]
\]
The angular distributions for $W_2^0(\Theta)$, $W_2^{\pm 1}(\Theta)$ and $W_2^{\pm 2}(\Theta)$ calculated from the equations 10, 11 and 12 (pages 45 and 46) are shown in Figure 34. These illustrate that the three allowed m-transitions ($\Delta m = 0$, $\Delta m = \pm 1$ and $\Delta m = \pm 2$) exhibit distinctly different angular distributions. This is to be expected since the allowed m-transitions correspond to distinct z-projections of the total angular momentum $L=2$.

Figure 34. The angular distributions ($W(\Theta)$) for the allowable m-transitions for the decay $I_i=2$ to $I_f=1$ when $L=2$.

The angular distributions of Figure 34 cannot be resolved by a x-ray detector as the splitting is typically of the order of $10^{-8}$ eV in a strong magnetic field. What is observed is the sum of all the allowed m-substates, which has the form

$$W_L(\Theta) \propto \sum_{m_i,m_f} \rho(m_i) G(m_i,m_f) W_L^M(\Theta)$$
where $p(m_i)$ is the relative population of the $m_i$ substate and $a(m_i, m_f)$ is the probability of the $m_i \rightarrow m_f$ decay. $p(m_i)$ is defined as

$$p(m_i) = \frac{1}{2I_i + 1} \quad (14)$$

for the case where all the $m$ substates are equally populated.

Exercise 2
(i) \hspace{1cm} J_i = 2 \rightarrow J_f = 1 \hspace{1cm} \lambda = 2 \hspace{1cm} a(m_i, 0) = 1

$$p(m_i) = \frac{1}{(2)(1) + 1} = \frac{1}{5}$$

Thus

$$w_L(\theta) \propto \sum_{m, m_f} \frac{1}{5} w_L^M(\theta) \quad (15)$$

To determine the weighting of the substates we must find the number of $m$-substate transitions allowed. This is achieved by consulting the transition diagram shown in Figure 35 (page 48). This illustrates that there are 8 states of $\Delta m = +2$

2 states of $\Delta m = +2$

2 states of $\Delta m = -2$

3 states of $\Delta m = +1$

3 states of $\Delta m = -1$

3 states of $\Delta m = 0$

Thus we have 4 states for $\Delta m = \pm 2$, 6 states for $\Delta m = \pm 1$ and 3 states for $\Delta m = 0$.

$$\Rightarrow w_L(\theta) \propto \frac{1}{5} \left[ 3w_2^{0}(\theta) + 6w_2^{+1}(\theta) + 4w_2^{+2}(\theta) \right]$$
\[ W_{2}(\theta) = 0.9 \left[ \frac{15}{8\pi} \cos^{2}\theta \sin^{2}\theta \right] + 0.1 \left[ \frac{5}{16\pi} \left( 1 - 3\cos^{2}\theta + 4\cos^{2}\theta \right) \right] \]
Figure 36. The angular distribution $W_2(\Theta)$ for equally populated $m$-substates.

The angular distribution $W_2(\Theta)$ for an unequal population of $m$-substates is shown in Figure 37. This illustrates that when the substates are unequally populated there is significant variation in the intensity of the angular distribution.

Figure 37. The angular distribution $W_2(\Theta)$ for unequally populated states.
One of the methods to create an unequal population of $m$-substates is to observe a $\delta$-$\gamma$ cascade. By observing the directional correlation between $\delta$, and $\gamma$, and unequal population of states can be created.

The experimental set-up for this measurement is shown in Figure 38 (page 51). If we take the direction of $\delta$, to be the $z$-axis and assume all of the $m$-substates to be equally occupied the population of the intermediate state $I_i$ is

$$P(m) \propto \sum_{m_i} G(m_i, M) W_{z, i}^M(\theta) \quad (\theta = 180^\circ)$$

where $M_i$ is

$$M = m_i - m$$

and $m$ is the magnetic substate of the intermediate level.

As a result, when a photon propagates in a definite direction $M_i$ is restricted. This leads to the angular correlation function

$$W(\theta) \propto \sum_{m_i, m} G(m_i, m) W_{z, i}^{*}\theta(\theta = 0^\circ) G(m, m_f) W_{z, f}^{M_f}(\theta)$$

This is generally written using the following formalism:

$$W(\theta) = 1 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta)$$

with

$$A_2 = A_2^{(1)} A_2^{(2)} \quad \text{and} \quad A_4 = A_4^{(1)} A_4^{(2)}$$

The superscripts denote the coefficients for $\delta_i$ and $\gamma_i$. 
Figure 38(b) Schematic diagram of the experimental apparatus for the angular correlation measurement and (b) the $s-s'$ cascade.

Taking into consideration the particular measurement regime the angular distribution can be written as

$$W(\theta) = 1 + A_2 Q_2^2 P_2(\cos \theta) + A_4 Q_4^2 P_4(\cos \theta) \quad (21)$$

where the coefficients $Q_k$ take into account the finite size of the detector.

(i) The transition $0^+ \rightarrow 2^+ \rightarrow 0^+$

$$A_2 = A_2^{(1)} A_2^{(2)} = (-0.598)^2 = 0.357604$$

$$A_4 = A_4^{(1)} A_4^{(2)} = (-1.069)^2 = 1.142761$$

$$Q_2^2 = 0.93026 \quad Q_4^2 = 0.78367$$

$$W(\theta) = 1 + \left(\frac{1}{2}\right) A_2 Q_2^2(3\cos^2 \theta - 1) + \left(\frac{1}{8}\right) A_4 Q_4^2(35\cos^4 \theta - 30\cos^2 \theta + 3)$$

N.B. The values of $A_2^{12}$ and $A_4^{12}$ are found in Table 4 on page 53 - (22)
(ii) For the transition $2^+ \rightarrow 2^+ \rightarrow 0^+$

\[ A_2 = A_2^{(1)} A_2^{(2)} = (0.128)(-0.598) = -0.076544 \]

\[ A_4 = A_4^{(1)} A_4^{(2)} = (-0.305)(-1.069) = 0.326045 \]

\[ Q_2^2 = 0.93026 \quad Q_4^2 = 0.78367 \]

\[ W(\theta) = 1 - 0.03561(3\cos^2\theta - 1) + 0.03914(35\cos^4\theta - 30\cos^2\theta + 3) \]  

(23)

(iii) For the transition $4^+ \rightarrow 2^+ \rightarrow 0^+$

\[ A_2 = A_2^{(1)} A_2^{(2)} = (-0.171)(-0.598) = 0.102258 \]

\[ A_4 = A_4^{(1)} A_4^{(2)} = (-0.008)(-1.069) = 0.00855 \]

\[ Q_2^2 = 0.93026 \quad Q_4^2 = 0.78367 \]

\[ W(\theta) = 1 + 0.04756(3\cos^2\theta - 1) + 0.000838(35\cos^4\theta - 30\cos^2\theta + 3) \]  

(24)

The angular correlation functions defined by equations 22, 23 and 24 (pages 51 and 52) are shown in Figure 39. These demonstrate that it may be possible to distinguish the different spin sequences.

To calculate the values of $A^{(r)}_R$, we will assume pure E2 radiation. Thus

\[ A^{(r)}_R = F_R (I, I', 2, 2) \]
where the superscript $i$ refers to the $\gamma$-ray number. The values of $A_k^{(i)}$ are given in table and can be found in reference [3].

Table 4. The $A_k^{(i)}$ coefficients for the transitions studied.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$A_2^{(1)}$</th>
<th>$A_2^{(2)}$</th>
<th>$A_4^{(1)}$</th>
<th>$A_4^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+ \rightarrow 2^+$</td>
<td>-0.5976</td>
<td>-0.5976</td>
<td>-1.069</td>
<td>-1.069</td>
</tr>
<tr>
<td>$2^+ \rightarrow 2^+$</td>
<td>0.128</td>
<td>0.128</td>
<td>-0.305</td>
<td>-0.305</td>
</tr>
<tr>
<td>$4^+ \rightarrow 2^+$</td>
<td>-0.171</td>
<td>-0.171</td>
<td>-0.08</td>
<td>-0.08</td>
</tr>
</tbody>
</table>

Figure 39. The angular correlation functions for the transitions $0^+ \rightarrow 2^+ \rightarrow 0^+$, $2^+ \rightarrow 2^+ \rightarrow 0^+$ and $4^+ \rightarrow 2^+ \rightarrow 0^+$. 
<table>
<thead>
<tr>
<th>( \Theta )</th>
<th>( W(\Theta) )</th>
<th>( W(\Theta) )</th>
<th>( W(\Theta) )</th>
<th>( W(\Theta) )</th>
<th>( W(\Theta) )</th>
<th>( W(\Theta) )</th>
<th>( W(\Theta) )</th>
<th>( W(\Theta) )</th>
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<td>0.000000</td>
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</tr>
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<td>1.770954</td>
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<td>0.000000</td>
<td>0.000000</td>
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</tr>
<tr>
<td>1.607944</td>
<td>0.000000</td>
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<td>0.000000</td>
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<td>0.000000</td>
</tr>
<tr>
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<td>0.000000</td>
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<tr>
<td>1.817944</td>
<td>0.000000</td>
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<td>1.875944</td>
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</tr>
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<td>1.933944</td>
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<td>1.991944</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
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<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Table 5: The calculated values of the angular correlation functions \( W_{\Theta}(\Theta), W_{\Theta}(\Theta), W_{\Theta}(\Theta), W_{\Theta}(\Theta) \) and for the transitions 0\( \rightarrow \)2\( \rightarrow \)2. 1.77\( \rightarrow \)2\( \rightarrow \)2 and 1.77\( \rightarrow \)2.
The $^{22}$Na source was replaced by the $^{60}$Co source and the threshold values of the TSCAs were adjusted. To achieve a higher count rate the TSCA windows were initially set wide open. The number of counts were measured over a 10 minute interval whilst varying the angle between the two detectors in 10° steps. The results are given in Table 6 and Figure 40.

### Table 6: Angular correlation measurements of $^{60}$Co with the TSCA windows open.

<table>
<thead>
<tr>
<th>Angle (±0.5°)</th>
<th>N  (Counts)</th>
<th>dN</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>10662</td>
<td>103</td>
</tr>
<tr>
<td>100</td>
<td>11250</td>
<td>106</td>
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<tr>
<td>110</td>
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<tr>
<td>120</td>
<td>11585</td>
<td>108</td>
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<tr>
<td>130</td>
<td>11164</td>
<td>106</td>
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<td>140</td>
<td>11915</td>
<td>109</td>
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<td>150</td>
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<td>160</td>
<td>11375</td>
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<tr>
<td>170</td>
<td>11685</td>
<td>108</td>
</tr>
<tr>
<td>180</td>
<td>12284</td>
<td>111</td>
</tr>
</tbody>
</table>

To determine whether the results confirm the $4^+ \rightarrow 2^+ \rightarrow 0^+$ transition the theoretical angular correlation functions were normalised so that theoretical and observed values could be plotted on the same graph. The normalisation was performed by:

$$
\chi = \frac{N_0}{W_c(0)}
$$

$$
d\chi = \chi \left( \frac{dN_0}{N_0} \right)
$$

where $\chi$ = scale factor

$N_0$ = number of counts at angle $\theta = 90^\circ$

$W_c(0)$ = value of the angular correlation function at angle $\theta = 90^\circ$
Figure 40. Angular correlation of $^{60}$Co with the TSCA window open.

The normalised angular correlation functions are given in Table 7 and Figure 41 (pages 57 and 58). The figure shows that the observed angular correlation does not match either the $0^+ \rightarrow 2^+ \rightarrow 0^+$ or the $2^+ \rightarrow 2^+ \rightarrow 0^+$ transitions. However, the observed trend does not match the $4^+ \rightarrow 2^+ \rightarrow 0^+$ transition. There seems to be a large degree of noise in the observed angular correlation. This is most likely due to random coincidences, which cause large variations in the number of counts observed at a particular angle.

To ascertain whether isolating the $^{60}$Co $\gamma$-ray peaks would improve the observed angular correlation, the TSCA windows were adjusted and the number of counts were measured in 10-minute intervals as the angle between the two detectors was varied. The new TSCA settings are given on page 59. The results are found in Table 8 (pages 59) and Figure 42 (pages 59).
Table 7. The normalised angular correlation functions for the transitions 0→2⁺, 2⁺→2⁺ and 4→2⁺.
Figure 4: The observed and normalised theoretical angular correlation functions with the TSOA window open.
TSCA1 (1173.237 MeV X-ray peak)

ULD = 3.14 V, LLD = 1.81 V

TSCA2 (1332.501 MeV X-ray peak)

ULD = 3.05 V, LLD = 2.0 V

Table 8: Angular correlation measurements of $^{60}$Co with the TSCA windows set to isolate the $^{60}$Co X-ray peaks.

<table>
<thead>
<tr>
<th>Angle (±0.5°)</th>
<th>N (Counts)</th>
<th>dN</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>1435</td>
<td>38</td>
</tr>
<tr>
<td>110</td>
<td>1512</td>
<td>39</td>
</tr>
<tr>
<td>120</td>
<td>1500</td>
<td>39</td>
</tr>
<tr>
<td>130</td>
<td>1519</td>
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<td>140</td>
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<td>160</td>
<td>1568</td>
<td>40</td>
</tr>
<tr>
<td>180</td>
<td>1663</td>
<td>41</td>
</tr>
</tbody>
</table>

Figure 42: Angular correlation of $^{60}$Co with the TSCA windows set to isolate the $^{60}$Co X-ray peaks.
The theoretical angular correlation functions were again normalized. The results are given in Table 9 (page 61) and Figure 43 (page 62). This illustrates the same variation from the theoretical $4^+ \rightarrow 2^+ \rightarrow 0^+$ transition as the results when the TSAC windows were open (see Figure 41 on page 58). The only difference is the counting statistics. It does not matter whether you isolate the X-ray peaks as the coincident requirement should reject the background events and register real X-X cascade events.

From Figure 41 (page 58) and Figure 43 (page 62) the observed angular correlation is relatively weak. This suggests that the transition is $4^+ \rightarrow 2^+ \rightarrow 0^+$ as the other two transitions can be rejected. However, there is still significant variation from the theoretical distribution. This is probably due to random fluctuations. If the data was acquired for a longer period of time or a number of samples were taken the deviation from the theoretical distribution could be reduced.
<table>
<thead>
<tr>
<th>Angle $\theta$ (±0.5°)</th>
<th>Angle $\theta$ (rads)</th>
<th>d$\theta$ (rads)</th>
<th>N (counts)</th>
<th>dN</th>
<th>$W(\Theta)$ for the transition 0→2→0</th>
<th>$W(\Theta)$ for the transition 2→2→0</th>
<th>$W(\Theta)$ for the transition 4→2→0</th>
<th>Normalised $W(\Theta)$ for the transition 0→2→0</th>
<th>Normalised $W(\Theta)$ for the transition 2→2→0</th>
<th>Normalised $W(\Theta)$ for the transition 4→2→0</th>
<th>dW(\Theta) for 0→2→0</th>
<th>dW(\Theta) for 2→2→0</th>
<th>dW(\Theta) for 4→2→0</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.0</td>
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<td>1.13143</td>
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<tr>
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Figure A3. The observed and normalised theoretical angular correlation function when the TAC windows were set to isolate "x-ray peaks.

- **Observed**
- **Normalised $W(\Theta)$ for the transition 0->2->0**
- **Normalised $W(\Theta)$ for the transition 2->2->0**
- **Normalised $W(\Theta)$ for the transition 4->2->0**
Conclusions

In the first part of the experiment the techniques utilised in γ-Spectroscopy were investigated. It was found that to produce a γ-ray energy spectrum the raw signal from the PMT the signal is required to be amplified and shaped. Using $^{137}$Cs, $^{22}$Na and $^{60}$Co we were able to determine a linear relationship between the energy of the incident γ-ray and the channel number on the MCA. This was found to be

$$\text{MCA} = 0.5668E_γ - 10.515,$$

where $E_γ$ is in units of KeV.

We were also able to determine the energy resolution of the detector using the $^{60}$Co source. This was found to be $6.6 \pm 0.6\%$ for the 1173.237 KeV γ-ray and $6.6 \pm 0.7\%$ for the 1332.501 KeV γ-ray.

In the second part of the experiment the coincidence electronics necessary for the angular correlation measurements was studied using the $^{22}$Na source. It was found that the amplifier should be operated in bipolar mode, the TSCA window must be set to isolate the energy peak of interest and a delay is required to be added between the signals from the two detectors for the TOF to acquire a time spectrum. The time spectrum of $^{22}$Na with the detectors 180° apart was measured and the time resolution of the coincidence system was found to be $41 \pm 7\,\text{ns}$.
Finally the angular correlation of $^{22}$Na and $^{60}$Co was investigated. In the case of $^{22}$Na it was determined that the intensity was at a maximum when the detectors were 180° apart and at a minimum when the detectors were 90° apart. This was expected since the two $\gamma$-rays produced following electron-positron annihilation are emitted back-to-back (180° apart).

The angular distribution of $^{60}$Co was found to be significantly different to that of $^{22}$Na, which was due to measuring a $\gamma-\gamma$ cascade. It was found that the angular correlation was weak and most closely approximated the $4^+ \rightarrow 2^+ \rightarrow 0^+$ transition. Thus the spin sequence of the $\gamma-\gamma$ cascade of $^{60}$Co was found to be $4^+ \rightarrow 2^+ \rightarrow 0^+$. The angular distribution exhibited some deviation from the theoretical distribution. However, either taking the data for a longer period of time or taking a number of samples should minimise this deviation.

REFERENCES

