THE JULY LECTURES IN PHYSICS – 1991

Lecture 2

LIGHT IN FOCUS – 600 MILLION YEARS OF PROGRESS

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In the shallow murky seas of the early paleozoic era, a tremendous variety of crab–like organisms scuttled about on the sea floor searching for food. Competition for the limited resources must have been fierce because we see in the fossil record of the era that some of the organisms, called trilobites, evolved remarkable eyes. These eyes probably evolved to help trilobites see better to find food in the dim murky light of the sea floor. The eyes were remarkable because they included a complicated extra lens that corrected the defects of the main eye lens. Trilobites with these eyes could see sharper, brighter images compared to their less well endowed cousins.

We will be taking a closer look at the marvellous eyes of the trilobites later in the lecture. Before we do, we need to take a close look at lenses themselves. How do they work? When don't they work too well? We are going to look at simple lenses, investigate the main reason why simple lenses need fixing up and review some of the ingenious techniques used in nature and science to do this.

To see some of the problems that arise when tying to use lenses to form images it is helpful to look at an easy way of forming an image that does not use a lens. An image can be formed with nothing more complex than a simple pinhole in a piece of cardboard. How this is done is not hard to see. First, look at the object of which an image is to be formed. An object that is well lit reflects light in all directions. If this reflected light falls directly on a screen (such as a retina, light sensitive film or charge coupled device (CCD) array), light coming from all the different points on the object overlaps and no image is formed.

If now a piece of cardboard containing a pinhole is placed between the well lit object and the screen, there is only one possible way the light can get from the object to the screen, that is through the pinhole. Each point on the screen can be illuminated by only one point on the object. Light on the screen from one point on the object no longer overlaps with light from different points on the object so an image is formed. This image will be very dim because only a tiny amount of light reflected from each point on the object actually makes it onto the screen!

It is not hard to see that the image formed by a pinhole is upside down, since light from the lowest part of the object, travelling in a straight line, must rise to pass through the pinhole and then hits the top part of the screen. The trick of making images with pinholes has been known for a long time. Early painters made use of an invention called a *Camera Obscura* where a sunlit scene was projected onto a wall of a dark room by a pinhole in a covered window. The resulting image could be used as a guide to painting the sunlit scene.

Figure 1: Light from an object falls onto a screen. Light from one point on the object overlaps with light from all the other points. The screen is uniformly illuminated by an infinite number of overlapping images.

Figure 2: If a piece of cardboard with a pinhole is placed between the object and the screen the overlapping light rays are cut out, an inverted image of the object is formed on the screen.

There is a suprising place where pinholes can form images. This is underneath trees on a sunny day! The overlapping leaves in the tree's canopy mostly block out the sun's rays. But, here and there, gaps exists where light from the sun can travel straight through to the ground. These gaps in the canopy, which act like pinholes, produce an array of circular sun's images on the shadowed ground underneath the tree. There is nothing particularly distinctive about this and most people probably don't notice it. But during an eclipse of the sun by the moon, the images of the sun formed by the tree *camera obscura* are dramatic. Now, instead of the images being circular, the image of the sun has a 'bite' out of it as the moon passes between us and the sun. An array of banana shaped images of the sun can be seen on the ground under the tree!

A pinhole is a remakably simple way of forming an image. Not suprisingly the image quality leaves a lot to be desired. For a start the image is very dim because only a tiny fraction of the light coming from the object actually gets through the pinhole, most is thrown away through being blocked out by the screen. The image is also not

Figure 3: A lens replacing the pinhole gives a substantial improvement in the image by making it a lot brighter. The light rays are shown bending at the centre of the lens as in the thin lens approximation. The light rays actually bend only at the entrance and exit interfaces.

very sharp, since a small detail on the object will be imaged to a fuzzy blob on the screen. The size of the blobs on the screen is related to the diameter of the pinhole and the distance between the pinhole and the screen. If you attempt to sharpen things up by making the pinhole smaller, the image gets dimmer and gets fuzzier! This is because when the pinhole diameter gets as small as about a wavelength of light, the light diffracts and forms an interference pattern on the screen instead of a nice localised point!

To get around the problem with pinholes we need a lens. A lens has the ability to make use of some of the light that we were previously throwing away on the piece of cardboard around the pinhole. A simple biconvex lens will allow light to pass through its centre, following the same path as if it was a pinhole, but will bend the other rays of light, that originated from the same point on the object, so that they all eventually reconverge at the same point on the other side of the lens.

The shape of the lens is critical to determining the quality of the image. A simple biconvex lens may be nothing more than two spherical surfaces placed back–to–back. Spherical surfaces are chosen because they are very easy to make. Unfortunately they are not ideal for forming images.

Before continuing with a discussion of lenses, it is worthwhile looking for a moment at mirrors. Mirrors can be used to do some of the same jobs as lenses, and have the advantage for our discussion that they are easier to understand because we don't have to worry about how light bends in passing from air into glass. We just have to know that light reflects off each bit of a mirror with the same angle as it came in with. We can use a curved mirror to make an image of an object.

Take, for example, a concave mirror, as also used in all of the world's most powerful astronomical telescopes. As with a simple biconvex lens, the type of concave mirror simplest to make is one that is the segment of a sphere. A concave mirror can be made by simply starting with two cylindrical glass disks, placing a film of grinding paste between them, then rotating the disks together with a sweeping circular motion. Eventually, one of the blanks will be ground to a spherical concave shape, while the other is ground to a spherical convex shape. You may have seen people doing this on the ABC program 'The Astromomers' last week. The glass blank can be coated with a thin layer of aluminium to make the reflective surface. This simple method of making a concave mirror only works because two spherical surfaces will always fit together, regardless of their orientation.

The spherical shape works very well as a focusing device, provided not too much of the surface is used. If a parallel bundle of light rays is reflected by a spherical mirror, all of the rays which strike the mirror towards the centre will be reflected to one point. This is just what we want. Unfortunately, as more and more of the mirror is used to make the image brighter and brighter, the rays no longer converge to the same point as the rays striking the mirror towards the centre. The mirror is said to suffer from *spherical aberration*.

Figure 4: (Left) Spherical aberration from a spherical mirror. (Right) Focusing light with a parabolic mirror. From a notebook of Leonado da Vinci.

A very simple illustration of spherical aberration can be seen by a casual glance into the bottom of a coffee cup illuminated by a strong light. The cylindrical shape of the wall of the cup (a cross section of a cylinder is a circle which is the two dimensional equivalent of a sphere!) focuses the light into a symmetrical shape called a *caustic*. The caustic is formed by the light coming into a 'focus' on the bottom of the cup. The caustic shows that not all of the light reflected off the wall of the cup comes to a focus at the same point. Instead it is spread over a large area of the bottom of the cup. This can be a problem if you are trying to make a sharp, bright image!

This problem with spherical mirrors has been known for thousands of years. Indeed, all through the scientific literature since the time of the ancient Greeks, the classic diagram showing the spherical aberration caustic can be found. Some of this literature describing the spherical aberration caustic is accompanied by a description of the mirror shape required to form a true focus. This shape, discovered by ancient Greek mathematicians, is a parabola.

In a scientific work called *Diocles on Burning Mirrors*, written down over 2000 years ago, a new and potentially deadly 'Star Wars' directed energy weapon is described. Diocles was a Greek mathematician who lived about 200 BC. His work has come down to us as an Arabic translation of a lost Greek original. In *Diocles on Burning Mirrors*, a spherical mirror is described and it is pointed out that such a mirror is not much good at focusing the suns rays to cause burning. This is because of spherical aberration. Instead, a new, improved, shape is described, said to be formed from a 'conic section', a parabola. It is described how this has a greatly superior power for burning because all of the sun's rays are focused to a point, instead of being scattered around as in a spherical mirror.

Legend has it that these burning mirrors were used by Archimedes (287–212 BC) in the coastal defence of Syracuse against attack from Roman trireme ships. The mirrors were used to focus the sun's rays onto the ships to set them on fire. If this was actually done, then considerable technical problems would need to have been overcome. At the latitude of the Mediterranean, sunlight at noon has an intensity of about 1390 W/m^2. A mirror 20 m in diameter with a focal length of 100 m (roughly a 'bow shot' in length) would concentrate a power of over 400 kW onto the ship, surely enough to set a wooden ship on fire! However the radius of curvature, together with the essential perturbation into the parabolic shape, would have to be accurately maintained. A difficult problem for such a large mirror! An intriguing comment by a Byzantine architect, Anthemius of Tralles, writing in the 6^{th} century states that Archimedes did the job with a large number of small flat mirrors. Perhaps Archimedes trained a squad of mathematically inclined soldiers to hold up their polished shields to form the shape of a parabola. Most modern large mirrors for astronomical telescopes are made in this way, with the modification that computers are used to 'hold' the individual mirror segments.

The parabolic burning mirror appears to have been rediscovered by the Muslim scientist and philosopher, known in the west as Alhazen (c965–c1039). Born in Basra, Alhazen was notable because he was able to secure a job with the Caliph al-Hakim (996–1031) in Egypt early in the 11^{th} century by promising a scheme to regulate the flow of the Nile. The scheme proved to be unworkable so Alhazen had to feign madness for the rest of his life in order to be free from his royal patron. Alhazen gives details of his burning mirror invention in his book *On the Spherical Burning Mirror*. Alhazen also published one of the very first books on the focusing of light with lenses. This book, the *Optical Thesaurus*, translated into Latin, provided the foundations for optical thought in the west.

Much later, the notebooks of Leonardo da Vinci (1452–1529) also reveal the diagram of the spherical aberration caustic, along with mathematical proofs of the superior focusing properties of the parabolic shape. It is not clear if Leonardo knew how to make a parabolic shape, generally not possible until the advent of modern computer controlled milling machines, but his manuscripts contain diagrams of many ingenious

Figure 5: An artist's impression, from the Middle Ages, of Archimede's use of burning mirrors in the defence of Syracuse. A lot of other optical phenomena are shown, some of it inaccurately portrayed! This illustration is from the *Opticae Thesaurus*.

Figure 6: Leonardo da Vinci's machine for grinding spherical mirrors.

machines that could be used to grind spherical mirrors.

Galileo Galilei (1564–1642) also knew about spherical aberration and its detrimental effects on images seen through his latest invention, the telescope. Constructed from lenses made with spherical faces, the images of stars would appear as bright points surrounded by a halo caused by spherical aberration. Johann Kepler (1571–1630), learning of Galileo's sensational astronomical discoveries, wrote a commentary *Conversations with Galileo's Sidereal Messenger* which described the problems caused by spherical aberration. Kepler suggested that a correction lens with a hyperboloid shape be included in the telescope. But he then goes on to lament the lack of suitable techniques for machining such an aspheric shape. An aspherical lens was beyond the technology of the 17$^{\text{th}}$ century.

In typical style, Isaac Newton (1642–1727) describes how *'The imperfection of Telescopes is vulgarly attributed to the spherical Figures of the Glasses and therefore Mathematicians have propounded to Figure them by the Conical Sections'*[1]. Newton showed that in fact *chromatic aberration*, arising from the different focal length of a lens for the different colours was actually a more severe problem. So, *'Seeing therefore the improvement of Telescopes of given lengths by refractions is desperate; I contrived heretofore a Perspective by reflexion, using instead of an Object Glass a concave Metal'*[2].

A spherical mirror is free from chromatic aberration, but not spherical aberration. In describing his experiments with mirrors for telescopes, Newton includes a paragraph complaining about the difficulties of getting decent mirrors made by the London mirror grinders, and how he had to make the mirrors himself. Unfortunately, Newton's mirrors tarnished rapidly and so did not produce superior images to glass lens telescopes.

Since Newton's time, mirror grinding and polishing machinery has become increasingly sophisticated (and expensive!). Now it is possible to grind any required shape.

[1] From Prop. VII, Theor. VI of *Opticks: or a Treatise of the Reflexions, Refractions, Inflexions and Colours of Light*, Isaac Newton, London, 1704, p59

[2] ibid., p75

Figure 7: Newton's telescope with a spherical mirror. From *Opticks*.

For example, the Hubble space telescope was ground to a special aspheric shape to a precision of only a few atomic diameters! Unfortunately it was ground very carefully to slightly the wrong shape, as recent reports in the media have indicated. There was no problem with the mirror grinding machinery, it made the mirror to have exactly the shape that was demanded, to the required precision. The mirror grinding machine computers were simply given the wrong shape. The error occurred in the system used to provide the computers with feedback information on how close they were getting to the correct shape. Despite this problem, Hubble's mirror is still very good compared with a simple spherical surface and has already made several important discoveries thanks to its ability to focus very bright images from its large mirror in the ideal seeing conditions of space. A future space mission will install a small correction mirror at the primary focus of the main mirror. This will compensate almost exactly for the accurately known defects.

We can now return to our discussion of eyes. Eyes of organisms that live in air, unlike those of organisms in the sea, have a relatively easy job of focusing light. To focus light, a lens must bend diverging rays of light into a focus. The bending occurs as the light passes across the interface between the air and the glass of the lens, then again as the light passes out of the lens back into the air. The amount by which an interface bends light depends on the difference between the refractive indices of the lens material and of the medium in which the lens is immersed. The bigger the difference between these two refractive indices, the stronger the lens is at focusing light. Organisms in the air (refractive index = 1) could use a lens made from water (refractive index = 1.33), enclosed by a membrane, to make a satisfactory eye. On the other hand, fish or other organisms in the sea, live in water already and must therefore make use of optically more dense materials in their eyes, for example calcite (refractive index = 1.66).

In the case of the human eye, most of the focusing is done by the interface between the front of eye itself and the outside air. The front surface of the eye, the *cornea*, has a refractive index of about 1.38. The cornea encloses a watery fluid called the *aqueous humour* which has a refractive index of about 1.34. Behind the cornea is the lens of the eye which has a refractive index that varies between 1.41 in the inner core to 1.39 in the outer layers. Since there is little difference between the the refractive indices of the lens and the aqueous humour in which it is immersed, the lens does not focus the light very much. Behind the lens is the centre of the eyeball filled with a gelatinous substance called the *vitreous humour*. This has a refractive index of about 1.34, about the same as water and the aqueous humour. Since the retina lines the inside of the back of the eyeball, the image must be formed inside the medium of the vitreous humour, on the surface of the retina. None of the interfaces between the internal components of the eye have as large difference in refractive index as that between the cornea and the outside air. Therefore, to a good approximation, the eye is nothing more than a single refracting interface.

The cornea has just the right degree of curvature to bring light (almost) to a focus on the retina. The cornea looks like a spherical interface, but on closer examination is not. A spherical interface would produce a blurry image because of the effect of spherical aberration.

Figure 8: Parallel light from the left focused by a spherical interface.

The exact shape of the refracting interface required to focus parallel light is an ellipse. The shape parameters of the ellipse depend on the refractive index of the material from which it is made.

Figure 9: Parallel light from the left focused by an elliptical interface.

While an elliptical interface works well for objects located dead ahead and a long way off, it is not ideal for imaging objects up close or at the edges of our field of view. Other shapes, including hyperbolae and Cartesian Ovals are required for these jobs. In fact the cornea of the human eye is a compromise shape dictated by the uses to which it is put. This shape turns out to be a flattened hyperbola. Fine tuning is done by some other refracting interfaces inside the eye, including those of the lens in the eye. The lens of the eye gives us the ability of focus on far and near objects. This is because we can voluntarily change the degree of curvature of the eye's lens, and hence its strength.

The lens of the eye also contains some subtle refractive index gradients that further help to reduce aberration. When the performance of the components of the eye is matched to the optical sensitivity and resolution of the retina the result is remarkable. The resolution of the human eye, at best, approaches the quantum limit.

The eyes of organisms living in the sea must be made from optically quite dense media and typically the focusing is done by a lens alone, rather than by a refracting interface as was the case for the human eye. For example, the eye of a squid can be dissected and the lens itself removed. This will still function as a lens, focusing light to the position of the retina, even when used in isolation, provided it is immersed in water! A squid lens would be too strong for use in air, images would be formed ahead of the retina. Similarly, the human eye is too weak when immersed in water. There is very little difference between the refractive index of the water and the cornea, hence light passes into the eye without being focused. Indeed, it functions more like a pinhole camera, the pupil forming the pinhole with the consequent degradation in performance!

We can now return to our discussion of the trilobites and their remarkable eyes swimming in the murky paleozoic seas. A trilobite eye is a compound eye which means that is has a myriad of small lenses packed together. Each small lens consists of a spherical dome of a calcite–like material with a refractive index of about 1.66. This does most of the focusing, but owing to the spherical shape, produces images blurred by spherical aberration. This works well in bright light, where the spherical surface can have a shallow curvature and only a little light can be allowed through just the centre of each lens. In this case spherical aberration is not important.

However, evolutionary forces acting on trilobites competing for food in murky water, drove several improvements. To see better in murky water it is necessary to gather more light. This can be done by making the lenses bigger. However, as I have shown, light passing through the edge regions of a spherical interface will not come to a focus in the same place as light that passed through the centre of the lens. To get around this problem, trilobites evolved an extra refracting interface at the back of their eyes. The extra refracting interface was between the back of the calcite front of the eye and a bowl–shaped layer of chitin (refractive index = 1.53). The shape of the extra refracting interface was exactly right to correct the spherical aberration of the spherical front of the eye. A trilobite equipped with such a corrected eye could see brighter, sharper images and was more able to survive in the dim murky water.

Remarkably, the same shape seen for the correcting interface seen in the trilobite eyes was derived completely independently by René Descartes (1596–1650) who just

Figure 10: (Left) Two individual lens units from a trilobite compound eye. In this diagram, light enters the eye units from above, through the upper lens unit, and is focused onto the bottom of the intralensar bowl. (Right) A description by Descartes of a thin spherical lens corrected for spherical aberration, the light rays are shown entering the lens from the left and being focused at F. From *Trilobites – A Photographic Atlas.*

made use of what he knew about how light behaved when it travelled from one medium to another, along with a bit of mathematics. The diagrams he published in his work *La Géometrie* in 1637 show an astonishing similarity to the structure of the trilobite eye.

Descartes' result applied to small interfaces with a relatively shallow curvature. Christian Huygens (1629–1695) published a somewhat different shape for larger, more curved, interfaces in his *Treatise on Light* in 1690. Sure enough, virtually identical trilobite eyes have been found in the fossil record.

To give you a feeling of the excellence of the trilobite eyes, it is helpful to look at their f–number. The f–number of a lens is a measure of how good it is at gathering light. It is defined as:

$$f/\# = \frac{focal\ length}{Lens\ aperture\ diameter}$$

Lenses with a small f–number have a large light gathering capacity compared to lenses with a large f–number. A typical inexpensive camera lens may, at maximum aperture, be f/3.5 or f/4. An expensive single lens reflex (SLR) camera may have a lens of f/2, f/1.8 or even f/1.4. A change in f–number from f/2 to f/1.4 doubles the amount of light reaching the film and hence halves the exposure time.

Lenses with f–numbers below f/1.4 are very expensive. For example, in 1991 at a camera shop near the University, a Nikon standard lens with a focal length of 50 mm (the 'standard lens') and a f–number of f/1.8 cost \$160. For a Nikon lens with the same focal length but superior light gathering power of f/1.4 cost \$486. Increasing the light gathering power still further to f/1.2 results is a lens costing \$672. If you really want to take photographs in very low light conditions then Cannon make a 50 mm focal length lens with a f–number of f/1.0. This will set you back over \$3000

Figure 11: (Left) Two individual lens units from a trilobite compound eye, in the same orientation as figure 10. This trilobite had thicker eyes compared to the one shown in figure 10. (Right) A description by Huygens of a thick spherical lens corrected for spherical aberration, the light enters the lens from the top and is focused at F. From *Trilobites – A Photographic Atlas.*

and you need very strong arms to be able to wield such a huge lump of glass on your camera!

The human eye has a focal length of about 17 mm and the iris can be dilated to a maximum diameter of about 8 mm. Hence the f–number of a human eye is about f/2.1. The trilobite eye was able to reach f/1.1, which is about 3.6 times the light gathering power of the human eye. Evolutionary forces drove trilobite eyes to be nearly as good as the best camera lens available commercially today!

Evolutionary forces in the form of economic competition has driven lens making technology to ever greater levels of sophistication. Modern lenses, like the Nikon and Cannon lenses described above, for video or photography, are likely to contain a large number of separate pieces of glass, all with a carefully chosen shape and refractive index. These lenses have very small spherical and other aberration considering their light gathering power. Some compact zoom lenses, like the one fitted to the Kodak Disc camera, may also contain aspheric glass lenses to keep spherical aberration to a minimum. Some inexpensive cameras may actually contain just a single aspheric lens made from injection moulded plastic. The design for these modern lenses is invariably handled by a computer.

All of the lenses and mirrors discussed so far actually used lumps of carefully shaped matter placed into the light path to force the light into a focus. Are there alternative methods? The answer is yes. In 1915 Albert Einstein (1879–1955) published his landmark paper on the General theory of Relativity. In this paper, Einstein predicted that gravity could act on light in the same way that it acts on matter. Einstein's theory was confirmed in 1919 when the astrophysicist Sir Arthur Eddington (1882–1944) observed bending of starlight passing through the sun's gravitational field.

In 1936 Einstein showed it was possible for a distant star in our galaxy, lying directly behind a foreground star, could be imaged by the gravitational field of the foreground star. The image would be in the form of a ring of light in the sky. Einstein thought that the required alignment was unlikely to occur in practice, and indeed this seems to be the case. However, in 1979, two apparently identical quasars were discovered, right next to each other, located out towards the edge of the visible universe, well outside our galaxy. These two quasars turned out to be two images of a *single* quasar. A massive galaxy lying between the quasar and Earth, on our line of sight, is most likely responsible for the twin image. The massive galaxy is acting as a gravitational lens thanks to its strong gravitational field. Recently a peculiar giant ring, seen in the constellation of Leo, may be the first observation of what is now called an Einstein ring, the image of a single bright galaxy lying immediately behind a massive foreground object.

The light from the quasars, that we see as images formed by gravitational lenses, started on its journey to Earth around 3 billion years ago, long before the trilobites evolved. The trilobites lived around 600 million years ago and eventually became extinct. The basic structure of their eyes has been passed down today to insects and crustacea. This is surely evidence for the success of the design!

So we have seen that focusing of light is not a straight forward task, but is nevertheless accomplished extremely well by many living organisms and even massive galaxies in deep space.

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