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EINSTEIN AND THE JUMBO JET: THE GLOBAL POSITIONING SYSTEM

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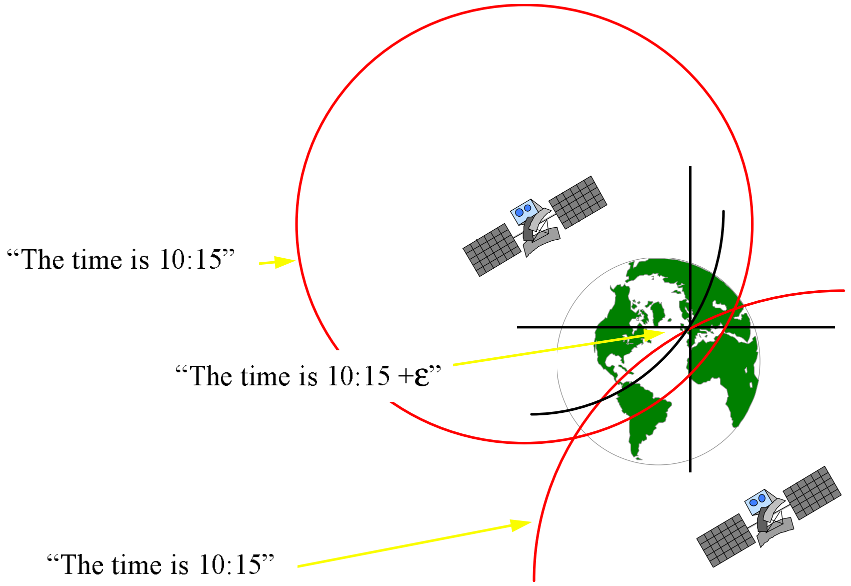
Introduction

A constellation of 24 satellites orbits 20,000 km overhead: the NavStar or Global Positioning System (GPS). With a hand-held receiver, it is possible to pick up signals from these satellites and locate your position on the surface of the Earth to an accuracy of a few hundred metres. With some additional signals from a local ground station, this accuracy can be improved to a few metres or perhaps even centimetres. This is accurate enough to potentially allow an aircraft to make a safe landing on a fog-bound runway, guided only by the GPS. Or to measure the sluggish drift of the continents. This whole system works because of the remarkable legacy of Albert Einstein and his two revolutionary theories about space, time and gravity: The theory of Special Relativity and the theory of General Relativity. This lecture aims to describe these theories and how they are used in the GPS.

The constellation of Global Positioning System satellites.

The GPS works by each satellite sending a very accurate time signal to your hand held receiver. From this, the receiver is able to work out how far it is from the satellite. With this information from four or more satellites, the receiver is able to calculate its position to high accuracy on the Earth. Information from four or more satellites is needed because there are four unknowns: the latitude, longitude, altitude and the correct time. The correct time is needed because the in-built clock in the hand-held receiver is not as accurate as the clocks in the satellites. To calculate the distance from the receiver to the satellite, the time signals must be exceptionally accurate. An error of 1 billionth of a second (or $1\text{ns}=1\times10^{-9}$ s) leads to an error in position of $\delta x=\delta t\times c=30$ cm, where $c=3\times10^{8}$ ms^{-1}, the speed of light. The satellite clocks used for this work are called atomic clocks and they are so accurate that they would loose only one second of time every million years.



Time signals from accurate clocks allow the distance to any satellite to be accurately determined.

But this is not enough. Isaac Newton assumed that the universe is governed by a majestic clockwork, where the flow of time is everywhere smooth, uniform and in synchronisation for everything, everywhere. Einstein showed that this is not true. The fabric of space and time itself distorts this uniform flow. Two clocks, each keeping perfect time, moving separately from each other, will get out of synchronisation. The GPS cannot be made to work accurately enough to land an aeroplane, or measure the continental drift, with an understanding of Newtonian Physics alone.

I will first discuss the contribution of Einstein to our understanding of the flow of time for two clocks in relative motion (the theory of Special Relativity) and then introduce the effects of gravity (the theory of General Relativity). Both effects are important for the GPS, the satellites move fast relative to the ground, and orbit high overhead in the Earth’s gravitational field. I will focus on three major effects: (1) Time Dilation as a result of relative motion, (2) Relativity of simultaneity as a result of relative motion and (3) Gravitational blue shift in a gravitational field. The first two are accounted for by the theory of Special Relativity and the last by the theory of General Relativity.

Moving Objects

In 1905, the young Albert Einstein, aged 26 and working as a clerk in the Swiss patent office, changed forever the way we see the world. With one breathtaking radical new idea, Einstein cleaned up a lot of unresolved problems in physics and proposed a whole set of strange new results that had never been thought of before, but have all now been verified by careful experiments. To see where Einstein’s idea came from, it is necessary to look at a few facts about moving objects.

Galileo and Newton sorted out the untidy state of Physics in the seventeenth and eighteenth centuries. They overturned the old ideas, left over from the ancient Greeks, ideas over a thousand years old, by simply doing some careful experiments that showed the world does not work as it first appears, owing to the many complex interacting effects that control the way real things behave. For example, Aristotle thought that the natural state of motion was for things to be at rest on the Earth's surface, but we now know that this is simply a result of frictional forces that will eventually slow down and stop an object originally set in motion. In controlled experiments, where the force of friction can be greatly reduced, we see that an object once set in motion will continue indefinitely on its way unless something acts on it to slow it down or deflect it on a new path. Indeed the Pathfinder space probe to Mars coasted all the way after its rockets provided the initial boost in just the first few minutes of its seven month journey.

Measurements of the position of the "Crown" in the court of the School of Physics and of the Fountain at the front of the Exhibition Buildings using the GPS on successive days in July 1997.

It is a remarkable result that if you are sealed inside a closed car with a perfectly smooth ride and are zooming along a freeway at 100 km/hr, there is no experiment you can do to measure your speed relative to the road without looking outside. You can drop a ball into your lap in perfect safety, the ball does not immediately get left behind as soon as it leaves your grasp and hit you in the stomach! You can also pour yourself a drink without missing the glass since you, the bottle, the (non--alcoholic) fluid and the car all move along with the same speed.

You may have noticed an illusion when sitting in a stopped train in the railway yards waiting for a signal to change. If another train pulls up along side, after which either your train or the other train moves off, it is very difficult to determine which of you is really moving. You need to look out the other window to check the scenery (unless of course you move off with a rapid jerk!).

With the effect of friction identified and allowed for, all sorts of new physics was discovered. For example, Newton was able to deduce that if you push on something, that thing pushes back on you with an equal and opposite force. For example, a model train on a circular track that is mounted on the rim of a bicycle wheel held horizontal will set the wheel in motion when it starts off. The train wheels push on the track as they propel the train forward, which causes the track to move in the opposite direction. In this experiment, we say that momentum has been conserved. Initially neither the train nor the track was moving. Afterwards, the forward momentum of the train was exactly balanced (and cancelled out) by the backward momentum of the track. The train moves forward *relative* to the track. But to an external observer, the overall system stays where it is. No absolute motion can be produced pushing on anything: for any object seen moving forwards, there must be another object moving backwards.

Towards the end of the last century, people were trying to find a way of measuring the *absolute velocity* of moving objects relative to something that was *really at rest*. For example you know that the planet Earth orbits the sun. Can we consider therefore that the Earth is really moving and the sun itself is at rest? Well, we also know that the sun itself is in orbit around the centre of our galaxy, the Milky-Way galaxy, so perhaps we should consider that the centre of our galaxy is really at rest. But we also know that our entire galaxy is speeding in some kind of orbit amongst the galaxies that make up our local cluster of galaxies. Perhaps we should seek some point in outer space that is really at rest? No matter where that point might be, we in our complex series of motions around the Sun, the galaxy and our local cluster, will surely be moving relative to it!

A series of experiments were thought up to measure our speed relative to this proposed place of rest. Most of these were based on electromagnetism. The reason for this is that we have seen already that no sort of mechanical experiment will do.

Electromagnetism

Electromagnetism involves charged particles, electric and magnetic fields. We can see the results of the electric force, most familiar to us as the force that pushes electrons out of power points and through wires that allow us to do many useful things. But in its simplest possible form it makes the gold leaf in an electroscope deflect and two charged pith balls, carrying the same sign charge, repel each other. The electric force from a charge Q on another charge q, distance r away is:

$$F_E = kqQ/r^2 = qE$$

where k is a constant and E is the electric field generated by charge Q.

The magnetic force, generated by currents of moving charged particles, also acts on charged particles, but they have to be moving. The magnetic force on a charge q moving with speed v in a magnetic field B is:

$$F_B = qv\times B$$

In this case the magnetic force results in a deflection of the speeding electrons sideways to their direction of motion. We find speeding electrons in wires or as beams in space like in electron microscopes.

In fact, any two *moving* charged particles will exert both an electric force and a magnetic force on each other. We can see the result of this by just putting two wires parallel to each other. When electrons go zooming down the wires, the wires are either pulled together or pushed apart depending on if the currents are parallel or not.

The reason why electromagnetism experiments were proposed as a way of finding our speed relative to whatever point in space was at rest was that light was found to consist of a wave of electric and magnetic fields. Just as sound waves were known to be waves in air, scientists supposed that light was electromagnetic waves in remarkable hypothetical stuff that they called *the Aether.*

The Hypothetical Aether

This Aether had to have astonishing properties. It had to fill all space, since we know light gets here from distant stars, it had to weigh very little and it had to be very springy, since we know light travels very quickly. Indeed you and I had to be full of the stuff without knowing it! The scientists thought that this Aether stuff would be "at rest" and all the planets, stars, galaxies, etc, would just plough through it without being noticeably affected by it.

Properties of air	*Properties of the Aether*
It carries sound waves	It carries light waves
It has a certain density, weight and chemical composition	Nothing else
You can breath it	

People were soon trying to do physics experiments to measure the speed of the Earth through the Aether. Even in our sealed, smooth car zooming along the freeway, we could do an experiment to show that we were really moving by checking the result of an electromagnetism experiment, designed to detect the hypothetical *Aether wind* blowing through the car. Or so people thought. There were a few niggling worries, though, about the extraordinary properties required of the aether.

Pretty soon some of the results of the proposed experiments started coming in. An early attempt to measure the magnetic force between two charged objects as the orbiting Earth carries them through the Aether failed completely. Scientists were perplexed. A definitive experiment was called for. The Michelson--Morley experiment was such a definitive experiment. The idea of this experiment is very simple. If the Earth moved through the Aether, then there should be an Aether wind blowing past in the same way that you feel a wind if you put your hand out of the window of a speeding car. The experiment aimed to compare the time it took light, supposed to be travelling in the Aether, to travel out and back to a distant mirror into the Aether wind, with the round trip travel time to a distant mirror placed at right angles to the Aether wind. It was expected that the round trip time into the wind would be longer compared to the trip at right angles, since it is never possible to make up time travelling with the wind that was lost travelling into the wind.

The two round trip times were compared in a very sensitive way by combining the light in an interference pattern. The result of the experiment took everyone by surprise. *No effect on the round trip travel times of the two light journeys of the Aether wind could be detected.*

No effect from the hypothetical Aether wind could be observed.

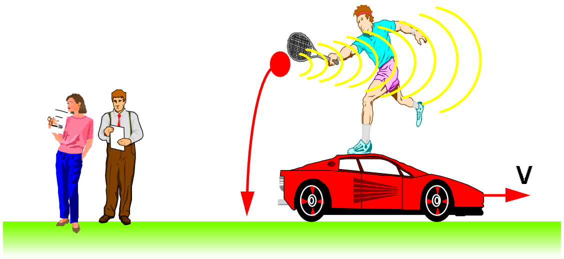
The hypothetical Aether did not exist!

The Special Theory of Relativity

Einstein did not wait for the results of the Michelson--Morley experiment. For him, the earlier experiments were enough. First he reaffirmed what we already knew from Newton's time, that all laws of physics apply regardless of the speed of the “laboratory” where the experiment takes place, provided the laboratory moves along at constant speed. Then, he took some ideas about symmetry and equality and, in a single bold stroke, abolished the Aether and introduced a radical idea that cleaned up the whole situation.

Einstein proposed that the speed of light was always the same, regardless of who measures it and how fast they are going relative to the source of the light. In other words, there is no Aether.

- Summary



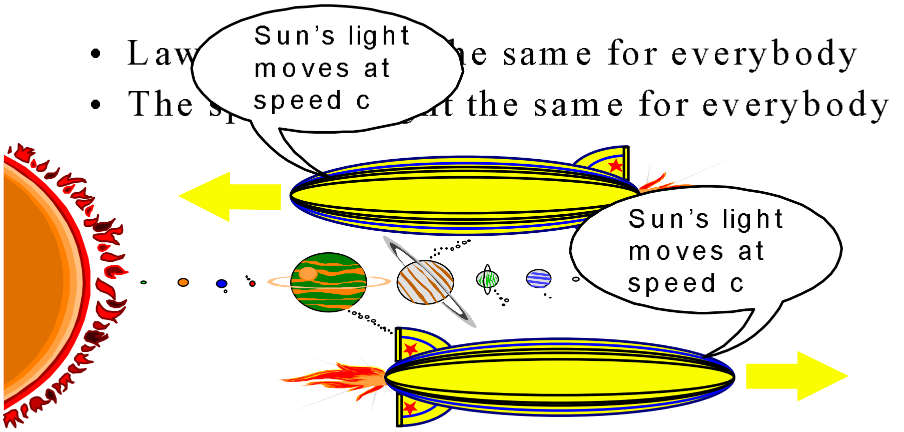
Object	Us	Them
Us	0	−v
Them	v	0
Tennis Ball	0	−100km/hr
Sound Wave	speed of sound	speed of sound −v
Light	c	c

The reason why this idea is radical is not hard to see. First, imagine throwing a ball from the roof of a speeding car. If you throw the ball forwards, someone on the ground sees the ball fly off with a speed equal to the speed of the ball as you threw it, plus the speed of the car. This is not what Einstein says happens with light. Since there is no Aether, the person on the ground would see a beam of light shone on them from a car still travel at the same speed, regardless of the speed of the car! Even if the car was replaced with a fast spaceship that zipped past at half of the speed of light, a person on the ground would still see a beam of light travelling at the same speed.

Second, imagine shouting from near the tail of a speeding Concorde aircraft. Your sound waves travel at the speed of sound in air, so if the aircraft is travelling faster than the speed of sound, your shout is

The Special Theory of Relativity

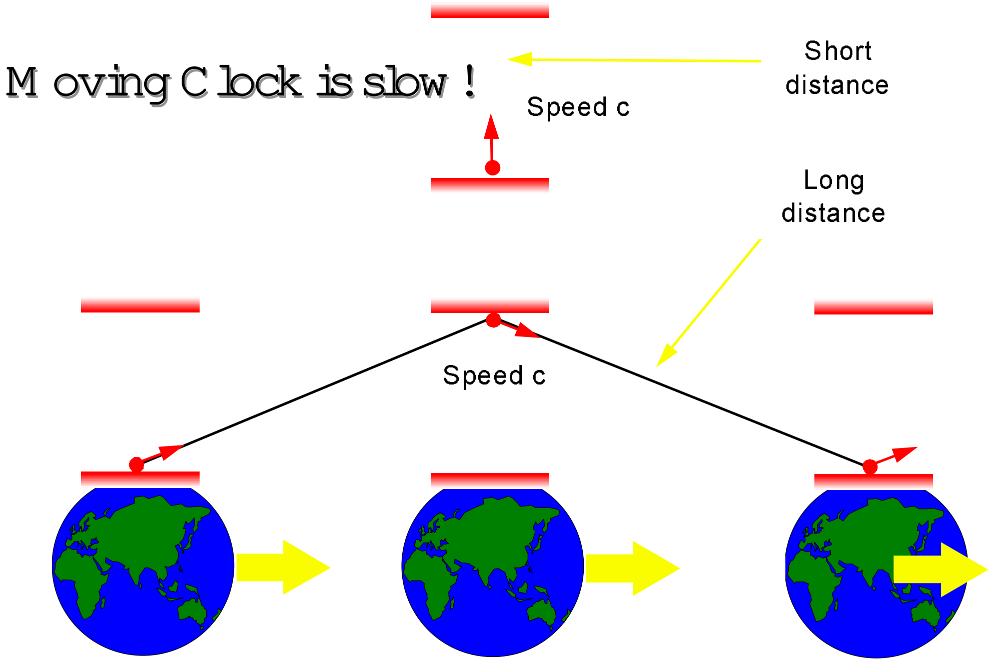
- Law e same for everybody
- The s t the same for everybody



blown backwards and someone standing on the nose of the aircraft will never be able to hear you! Light does not do this. Regardless of how fast a spaceship travels, it is always possible for people standing on either end to communicate with light beams which they both see travelling exactly at the speed of light. Furthermore, for example, if you measure the speed of light coming from the Sun, you always get the same answer, even if you are speeding towards the Sun at half the speed of light (or any other speed) in a fast spaceship!

Double Stars

We can check the constancy of the speed of light by simply looking through a telescope at a double star system. Since many stars orbit very close to each other, they can be travelling at very high speeds. If the speed of light sent out by the stars depended on how fast the stars were going, light from the stars when they were moving away from the Earth would take a lot longer to get here compared to light from the stars when they were moving towards us. The motion of the stars would appear very chaotic as the light got out of order on its way to the Earth! We don't see chaotic behaviour. Instead we see that the stars orbit each other in the way we expect. The fast stars have not given their light output the orbital speed.



Light clock on Earth observed from space runs slow owing to the long travel path, but same spped for light.

The Light clock

But there is obviously something peculiar going on here. How can everyone measure the same speed for light? To see the peculiar property of our universe that makes this so, consider the operation of a simple clock which we can use to tell the time. This simple clock simply consists of two ideal mirrors, parallel and facing each other. A photon bouncing backwards and forwards between the mirrors times one click of the clock. Let us assume the two mirrors are mounted one above each other on the Earth. Someone outside the Earth, stationary in space with respect to the Sun sees the Earth go zooming past on its orbit around the Sun. In the time it takes our bouncing photon to cross the space between the mirrors, this observer sees the Earth move a little way along the path of its orbit. Hence this observer sees the photon has to travel a diagonal path in order to keep hitting the mirrors.

This diagonal path is longer than the distance between the mirrors. *However this observer sees the photon travelling at the same speed as we see it, hence times a longer interval between ticks of our clock!*

The outside observer therefore concludes that our clock is running slow! It is this result that allows both us on the Earth and the outside observer to get the same result for the speed of light despite our relative motion. Since we would see the same effect in a light clock held by the outside observer, who we see zip past us during the experiment we conclude:

Moving clocks run slowly compared to our clocks.

Simple algebra shows that the amount by which moving clocks (speed v and time interval t') run slow compared to our clock (time interval t) is just:

$$t' = t(1-v^2/c^2)^{-1/2}$$

This result is known as time dilation. Clearly this is an outrageous prediction from the point of view of traditional Newtonian physics. Fortunately there are some very sensitive and accurate experimental tests of this result. An important thing to keep in mind is that the effect is insignificant at most speeds we are familiar with. It only becomes important at very high speeds, approaching the speed of light itself. We must look at fast sub atomic particles, or out into space to find things moving fast enough to show an effect. Or. in the case of the Global Positioning System, use very

Object	*Speed*	*Speed as a fraction of light speed*
Running person	20 km/hr	0.00000002
Car on freeway	100 km/hr	0.00000001
Jumbo jet	1000 km/hr	0.0000001
Rotation of Earth	1800 km/hr	0.0000002
Earth orbiting Sun	30 km/s	0.0001
Sun orbiting galaxy	250 km/s	0.001
Orbiting neutron stars	400 km/s	0.01
Speed at which the Coma galaxy cluster recedes from us	7000 km/s	0.02
Cosmic ray protons	2 $\times10^8$ m/s	0
Light	3×10^8 m/s	1

accurate clocks.

Muon experiment

Cosmic rays produce unstable sub--atomic particles called muons in the uppermost layer of Earth's atmosphere. We can also make them in the laboratory. When we do that, then examine them with particle detectors, we find they decay away in a very short time. In fact, muons live on average for only 2.2 millionths of a second. On the top of a high mountain, just below the layer where the muons are being produced, we can measure the number of muons arriving in our detector. We can then remeasure the number of muons arriving at sea level. We don't expect to see that much! Even if the muons were zooming down as fast as the speed of light, which they aren't, most of them should decay before they get down to sea level. Instead we find an enormous number of muons arriving down at sea level! Somehow the muons are able to live longer than the ones we measured in the laboratory. As they zip down from the upper atmosphere to sea level at high speed, the muon's internal clock is running slow! Indeed, we find that the muons are living about ten times longer than in the laboratory, indicating that they must be zipping down at about 0.996 times the speed of light.

However, we have a problem. How do the muons explain how it is that they are able to live long enough to reach sea level from the top of the high mountain? Their clocks still run at the proper speed as far as they are concerned! The answer is in yet another strange consequence of the constant speed of light. The muons see that the mountain appears to have shrunk as it zips past! If this were not so, there is no way someone travelling along with the muons could explain how they were able to make it to the bottom of the mountain in the incredibly short time allowed by the 2.2 millionths of a second half life.

The Lorentz Contraction

The shrinkage of the mountain is the complement of the time dilation formula. It is a general result of the Lorentz contraction that must occur for objects we see zipping past at high speed in order for everyone to see light travelling at the same speed. The Lorentz contraction is given by:

$$L = L'(1-v^2/c^2)^{-1/2}$$

where L is the contacted height and L' is the proper height. Once again, the effect of the Lorentz contraction is small unless the velocity of the object is large. We don't expect to see fast cars contract as they speed up!

The Speed of Light as the Limiting Speed

The time dilation formula tells us that when a clock zips past at the speed of light, we would see that time on it had slowed down so much that it had stopped completely! This is the strange world inhabited by a light photon. As a photon launches itself cross space from a light source to the place where it is absorbed, for example from a star to the retina of your eye, time stops completely. As far as the photon is concerned, it makes the crossing in literally no time at all! The source and destination are in the same place, the distance has been Lorentz contracted down to zero!

What happens now if we try to push something to a speed faster than the speed of light? The time dilation formula tells us that time for that object will slow down to zero, then start running backwards! The object would begin to move backwards in time! There are serious problems here. Running backwards in time would let things happen before their causes. People could experience their 2 st birthday before they were born. Clearly this is absurd. For related reasons, similar problems occur if even something as insubstantial as information is sent faster than the speed of light.

Perhaps because of this absurdity, something happens to prevent any object being pushed to a speed faster than that of light. It turns out that as you accelerate an object it begins to get heavier. The faster you get it going, the heavier it becomes. Even if you could get it going to the speed of light, you would find that its mass had become infinite. Consequently, there is not enough energy in the entire universe to accelerate even the smallest particle of matter up to the speed of light. We recognise that an object gets heavier as it gets faster because it is being made to absorb kinetic energy. Evidently this kinetic energy contributes to the object's inertia and makes it harder to accelerate. We have here a practical demonstration of the mass--energy equivalence represented by Einstein's famous equation: $E=mc^2$ where m is now a function of velocity:

$$m(v) = m_o/(1 - v^2/c^2)^{1/2}$$

and m_o is the rest mass of the object, that is to say the mass of the object we would measure before it begins to move.

Once again this does not result in appreciable changes to the masses of ordinary objects moving at everyday speeds, but is very significant for sub--atomic particles in nuclear accelerators. It is found that the fast protons in the Fermilab synchrotron, moving with a kinetic energy of 500 GeV (billion eV), are 533 times heavier than when they are at rest! All of the special magnets designed to keep the protons in orbit in the machine have had to be designed for this.

The topics of the Lorentz contraction and the limiting speed being the speed of light are diversions from our discussion of the GPS. We are most concerned only with the phenomenon of time dilation. For the GPS, time dilation in their orbits amounts to slow running of about 7 billionths of a second per day. This corresponds to a position error of .8 m per day and this must be compensated.

Relativity of Simultaneity

The second relativistic effect of concern here is the phenomenon of the relativity of simultaneity. Like time dilation, this also arises from the constant speed of light for all observers. Imagine an experiment in a lecture theatre where the lecturer stands equidistant between two students. A flash of light from the position of the lecturer would then reach the two students *simultaneously*, since they are equidistant.

A light flash seen simultaneously for two equidistant students in a lecture theatre.

An observer at rest relative to the Sun sees the two students receive the light flash at different times because of the relativity of simultaneity.

But now visualise the same experiment seen from outside the lecture theatre from a spaceship parked in space stationary relative to the Sun. The spaceship sees the Earth zoom past during the experiment. The spaceship will see the trailing student moving towards the light flash as it propagates out from the original flash and the leading student moving away from the flash as they are carried by the speeding Earth. Remember that the spaceship observer sees the light flash propagating at the speed of light uniformly in all directions from the position of the original flash. Clearly now the trailing student will encounter the flash first, followed some time later by the leading student who is being carried away from the light flash by the speeding Earth. The times when each student see the flash will be different when seen by the spaceship. The two students will not see the flashes simultaneously as was the case for the lecturer standing on the Earth. Simultaneity is relative.

For the GPS this means that signals from the satellites themselves cannot be used to synchronise all adjacent satellites in orbit. For a chain of three satellites in a particular orbit, labelled A, B, C, then A can be synchronised with B and B can be synchronised with C, but then C will not be synchronised with A. The error arising from this effect, called the Sagnac effect, for a ring of clocks around the equator of the Earth, is about 200 billionths of a second. The clock hardware in the satellites needs to be programmed to take this effect into consideration.

General Relativity

In 9 7 Einstein took his Special Theory of Relativity one step further. He, along with Newton before him, knew a remarkable fact. The gravitational force between two objects, masses M (for example, the Earth) and m_g (for example, an object on the Earth), a distance r apart is:

$$F_G = GMm_g/r^2$$

where G is a constant. An object in the gravitational field of the Earth, for example, experiences an acceleration due to gravity. By analogy with the very similar formula for the electric force, we can call M and m the *gravitational charge* of the two objects.

Newton discovered that the force needed to push an object along with a particular acceleration, a, is given by:

$$F = m_i a$$

where m_i is called the inertial mass of the object. Now here is the remarkable fact. If we adjust the force F we apply to an object so that it produces the same acceleration, a, as we get in a gravitational field, we find that $F=F_G$, and that to very great accuracy:

$$m_g = m_i$$

A direct consequence of this is that we become weightless when we are in free fall, since all masses fall with the same acceleration.

This is astonishing! Why should the gravitational charge accurately equal the inertial mass? Newton never was able to figure out an answer to this question.

Equivalence principle

Einstein decided that the reason why the inertial mass was the same as the gravitational mass was because gravity and acceleration were equivalent. That is, it is always possible to cancel out a gravitational field by just going into free fall, or accelerating. But this idea is more powerful than this, Einstein had the idea that if nature allows this to happen, then the laws of physics must be the same in a laboratory that is being accelerated through space at 9.8 m/s^2 by a powerful rocket engine as a laboratory that is resting on the surface of the Earth. This is known as the *Equivalence Principle.*

We can use the results of experiments in accelerated laboratories to understand what happens in gravitational field. In every test of this principle over the ninety years since it was put forward it has been found to be true.

A laboratory on Earth gives the same results as a laboratory in an accelerating rocket.

The Equivalence principle can be used to understand how clocks behave in the gravitational field of the Earth. We will do this by studying a situation where clocks experience different accelerations, then use the equivalence principle to see what happens in a gravitational field. Imagine a turntable is equipped with two clocks. One clock is located near the centre of the turntable and the other is located out near the rim. As the turntable rotates the clocks have a tendency to be thrown off so we

Time dilation of clocks on a rotating turntable.

have to provide them with a ledge on the turntable to prevent this happening. The ledge prevents the clocks from sliding off the turntable as it rotates. The ledges push against the clocks and keep them moving in a circle.

The clock nearest the rim is moving fastest so it gets pushed hardest by its ledge. The clock near the centre is moving slower so it does not get pushed so much by its ledge. An external observer sees the outer clock moving faster and hence, by time dilation, it ticks more slowly than the inner clock which does not move so fast. An observer standing on the outer clock's ledge feels the ledge pushing against their feet to keep them moving in a circle. For this observer, anything they drop off the ledge "falls" away from them as it is flung off the turntable. "Down" appears to be away from the centre of the turntable and out towards the rim. The inner clock appears to be "above". By the Equivalence principle, the same thing must be true in a gravitational field. We have the result:

Clocks high in a gravitational field run fast compared to low clocks.

This effect is sometimes called a gravitational blue shift because photons emitted from the high clock towards the low clock gain energy (but not speed!) as they "fall" through the gravitational field towards the low clock. A gain in energy causes the photon frequency to shift towards the higher frequency end of the electromagnetic spectrum. This is called a blue shift, even though it may not involve visible light.

The Global Positioning System

So now we can collect all these results from the Special and General theories of relativity and apply them to the Global Positioning system. We have three important effects that affect the clocks in the global positioning system satellites: ( ) a time dilation effect owing to the fact that the satellites move in a twelve hour orbit and hence move at high speed relative to the surface of the Earth, (2) the relativity of simultaneity and ( )a gravitational blue shift as the GPS clocks are higher in the Earth's gravitational field compared to clocks on the surface of the Earth.

In the case of effect (2), the GPS clocks are programmed to be synchronised as if they were in a non-rotating position. The other two effects concern the clock rates and they are in opposite directions. In the case of time dilation from special relativity, the GPS clocks run slow by about 7 billionths of a second per day. The gravitational blue shift causes the clocks to run fast by about 45 billionths of a second per day. Thus the net effect is for the GPS clocks to run fast by about 8 billionths of a second per day compared to identical clocks on the surface of the Earth. If this effect was not allowed for, very major errors in the positions deduced from the satellite time signals would result, amounting to more than 0metres per day. In practice, the clocks on the GPS satellites are made to run slightly slow to compensate for the net relativistic effect. The satellite ground station also send periodic corrections to allow for other more subtle effects caused by inevitable wobbles in the satellite orbits and perturbations from the pressure of sunlight and outgassing of components within the satellite.

When Einstein's theories were first introduced, very few people understood them. But, having stood the test of time, they have passed from being a curiosity of an arcane bunch of theoretical physicists, into textbooks of modern engineering. Indeed, a recent article in a journal of the *Institution of Electrical and Electronic Engineers* ("I-triple-E"), was devoted to a discussion of the role of relativity theory in the future of engineering! Think about that as Einstein helps you make a future safe landing on a fog-bound runway that is invisible to the pilot, who is guided by the signals from the Global Positioning System!

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