640-245
Electromagnetism
&
Special Relativity

General Relativity - Part 1
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Introduction to General Relativity

- The Equivalence principle
  - what is the difference between gravity and acceleration?
- Rotating disks
  - "Artificial gravity" fields
  - Curved space

The General Theory of Relatvity

- Gravity & Acceleration
- Gravitational charge and Inertial Mass
- Weightlessness when falling
- The equivalence principle
- Flat space
- Curved space
- Experimental tests
- Black Holes

General Relativity

- " ... remained, for forty years after its discovery (by Einstein), an austere intellectual monument - a somewhat sterile topic isolated from the mainstream of physics and astronomy - whose practitioners were 'magnificent cultural ornaments' "
- In the late 20thC, GR is an engineering subject!

General Relativity: Step-by-step

Forces of Nature

Inertia (pushing)

Inertia & Gravity

Weightless when falling

Equivalence Principle

Light and gravity

General Relativity: Step-by-step

Curved space

Rotating disks

Stars during eclipse

Perihelion of Mercury

The Global Positioning System

Massive objects (Black Holes)

The Cosmos

⚠ Summary of forces in nature

- Gravity

$$F_G = \frac{KMm}{r^2}$$

- Electromagnetism

$$F_E = \frac{kQq}{r^2}$$

- Weak Nuclear
- Strong Nuclear

Inertia $\quad F_I = m_I a$

Gravitational force binds the solar system

Electromagnetic force binds atoms

Weak force in radioactive decay

Strong force binds the nucleus

Gravity and Inertia: Newton

- Inertia

m_I $\quad F$
a

Newton: $F = m_I a$
"Force equals (inertial) mass times acceleration

- Gravity

Newton:
$F = GMm_G/r^2$
hence
$F = m_G g$

"Gravitational force law"
with acceleration due to gravity
$g = GM/r^2 = 9.8 m/s^2$

Gravity and Inertia: Eötvös

- The Eötvös experiment
- On a rotating Earth, the direction "down" depends on latitude, m_G and m_I

$m_G \ne 0$
$m_I \ne 0$ (crossed out)

$m_G \ne 0$
$m_I \ne 0$

$m_G = 0$
$m_I = 0$ (crossed out)

Result: "Down" is the same for all substances to an accuracy of 5 in a billion:
$m_G = m_I$

Equivalence Principle: Introduction

- Inertial mass: $F = m_I a$
- Gravitational mass: $F = GMm_G/r^2$
- Newton knew $m_I = m_G$
- Eötvös experiment confirmed this to
- Why?

Arghhh!

Einstein: 1912

"The effects of a gravitational field are equivalent to those of an upward acceleration"

This is the *Equivalence Principle*

Equivalence Principle: Falling

- Can eliminate gravity by appropriate change of reference frame!

These two are the same!

On Earth

Falling

In outer space

g

Summary

Forces of Nature

Inertia (pushing)

Inertia & Gravity

Weightless when falling

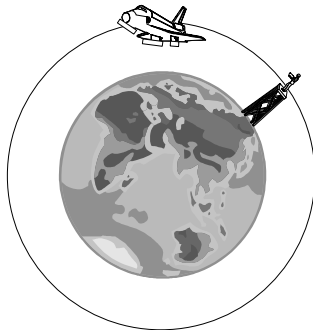We are here → Equivalence Principle

Light and gravity

- Unique among the forces of nature: Gravity is connected to inertia
- Hence you are weightless when you fall
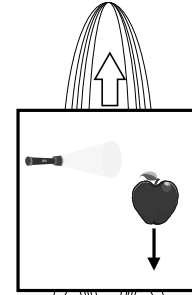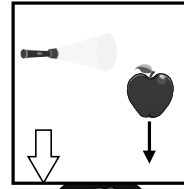- Has profound implications for the cosmos!

Aside: Why are astronauts weightless?

- Radius of Earth 6,350 km
- Height of tower 600 km
- Gravity at top of tower:
  - almost same as on surface!
- Why are astronauts weightless?
- Answer: Because they are falling!

Equivalence Principle

What's the difference?

Equivalence Principle: Light and gravity
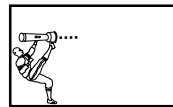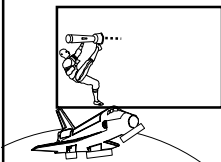
In orbit around Earth

In deep space (no gravity)

Equivalence Principle: Light and gravity

In orbit around Earth

In deep space (no gravity)

Equivalence Principle: Light and gravity
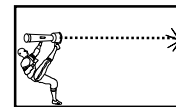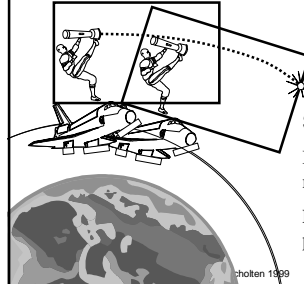
In orbit around Earth

In deep space (no gravity)

DIFFERENT RESULTS!

But both are weightless, so must be the same!

Equivalence Principle: Light and gravity

In orbit around Earth

In deep space (no gravity)

SAME RESULTS!

But both are weightless, so must be the same!

Light must follow a curved path.

Equivalence Principle

Since $m_I = m_G$ these are equivalent
(same laws of Physics, everything)

Equivalence Principle: How to model it?

Summary, continued
- You are weightless when you fall
- Inertial mass and gravitational mass are the same thing
- Gravity can cancelled by falling (accelerating)
- Therefore gravity and acceleration are equivalent
- In orbit (falling), light must follow a curved path to be equivalent to deep space (no gravity)
- What does this imply about the structure of space with gravity?
- Answer: Curved space!

Experimental test of C=2πr

Radius, r	Measured Circumfrence, C	C/r
10 cm	62.8 cm	6.28

Experimental test of C=2πr

Radius, r	Measured Circumfrence, C	C/r
10 cm	62.8 cm	6.28
50 cm	314.2 cm	6.28
100 cm	628.0 cm	6.
50 m	314.2 m	6.2
10,000 km	40,0000 km	4.00

Yikes!!
C<2πr

Why??

Drawing the Big Circle (on Earth)

C<2πr
for circles drawn on curved surfaces

## Drawing the Big Circle (off Earth)

Curved space

- The geometry of curved surfaces (like the surface of the Earth) is different to flat surfaces
- On a flat surfaces $C = 2\pi r$
- On a spherical curved surface $C < 2\pi r$
- How can we use this to introduce gravity?
- Answer: Use the equivalence principle and introduce acceleration instead!
- Also use special relativity....

Apply Equivalence Principle

- Side view of rotating cylinder
- Twelve 1 metre rulers laid down around rim by observer on ground
- Circumfrence: $C = 2\pi r = 12$ m

r

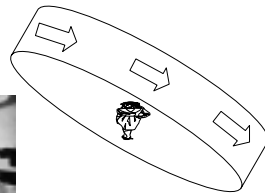Velocity of rim

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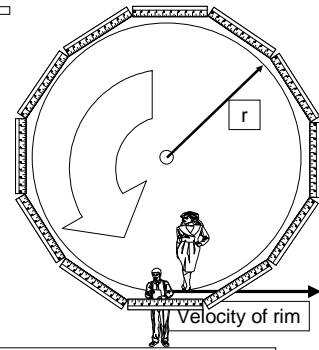
Apply Equivalence Principle

- A rotating cylinder provides "artifical gravity"

Observer held onto wall by centripetal force

"Artificial gravity" in the control room of the "Discovery" (2001- A space Odyssey)

Apply Equivalence Principle

- Observer on rim sees rulers going past at high speed
- They are shrunken to less than 1 m by Lorentz contraction!
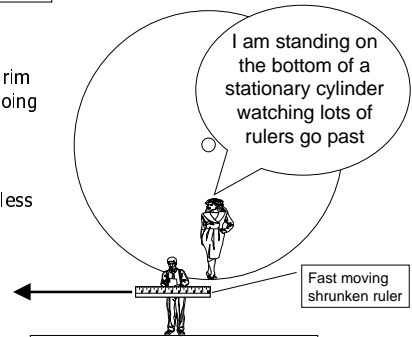
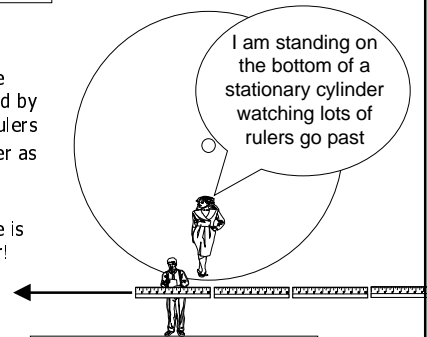I am standing on the bottom of a stationary cylinder watching lots of rulers go past

Fast moving shrunken ruler

Lorentz contraction makes observer on ground thin, observer on cylinder fat

Down
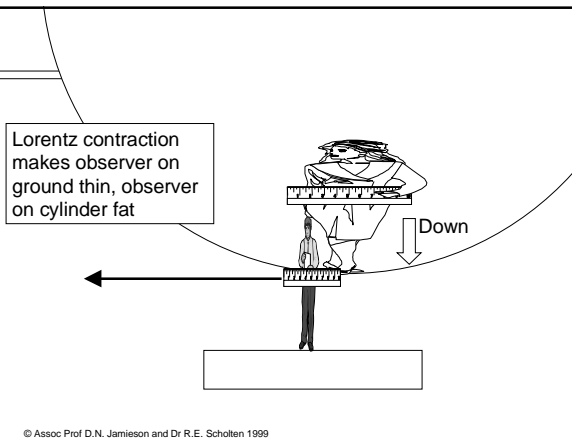
Apply Equivalence Principle

- Circumfrence now measued by contracted rulers
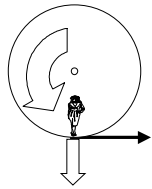- Same number as before, but shorter
- Circumfrence is less than $2\pi r$!

I am standing on the bottom of a stationary cylinder watching lots of rulers go past

Apply Equivalence Principle

- "Gravity" points outwards on rim of rotating disk
- $C < 2\pi R$ there

- Gravity points inwards on Earth
- $C > 2\pi r$ there
- Earth's gravity makes more space!
- Well tested by experiments and daily use of the GPS