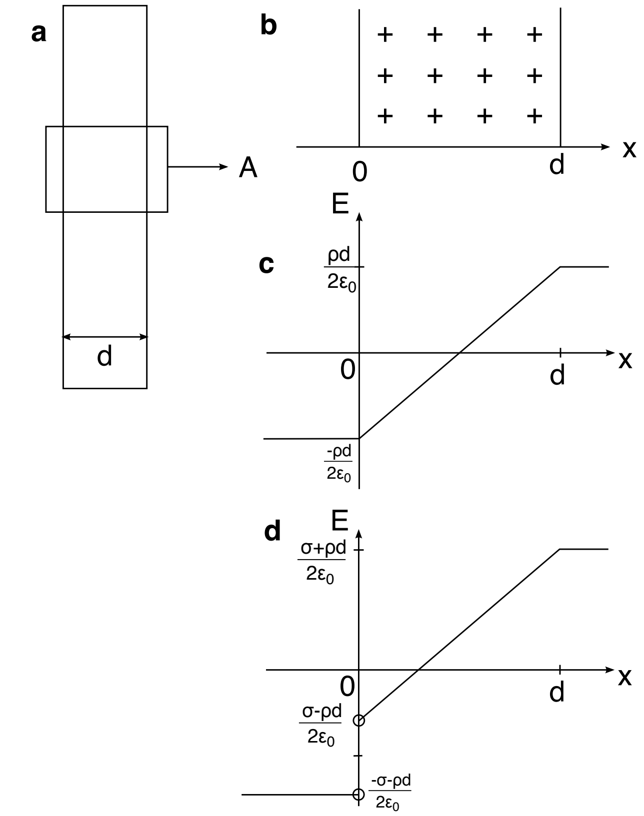
PHYC100004 Exam 2010 Q3ab solution

November 22, 2010



a)

We begin with a Gaussian surface (Fig **a**)

(Elizabetta taught that a cylinder is best, but many kids realized that the shape of the area didn't matter).

Gauss' Law:

$$\oint \vec{E} \cdot \vec{dA} = \frac{q_{enc}}{\varepsilon_0}.$$

The enclosed charge is $q_{enc} = \rho A d$.

Due to symmetry, only the ends of the Gaussian volume matter, and by performing the integral we get:

$$E \cdot 2A = \frac{\rho A d}{\varepsilon_0}$$

$$\Rightarrow E_{A,C} = \frac{\rho d}{2\varepsilon_0}$$

This gives the field in Regions A and C. The direction will be perpendicularly away from the sheet of charge (if ρ is positive).

In Region B, the fields partially cancel out (Fig **b**). To find the field, we use the same approach as before, but with Gaussian surfaces face-to-face inside the conductor at the position of interest. The fields will be the same as before, but with d replaced by x for the first volume and $-(d-x)$ for the second volume. The net field is thus (Fig **c**)

$$E_B = \frac{\rho\,(x-(d-x))}{2\varepsilon_0} = \frac{\rho}{\varepsilon_0}\left(x - \frac{d}{2}\right)$$

Less than five students got this right.

b)

The additional plane offsets the field in all three regions (Fig **d**):

$$E_{plane} = \frac{\sigma}{2\varepsilon_0}$$

$$E_{net} = E_{sheet} + E_{plane}$$

$$E_{net} = \vec{x}\begin{cases} \frac{-\rho d-\sigma}{2\varepsilon_0} & , \quad x < 0 \text{ (Region A)} \\ \frac{\rho}{\varepsilon_0}\left(x-\frac{d}{2}\right)+\frac{\sigma}{2\varepsilon_0} & , \quad 0 < x < d \text{ (Region B)} \\ \frac{\rho d+\sigma}{2\varepsilon_0} & , \quad d < x \text{ (Region C)} \end{cases}$$

Comments

Many students had a lot of trouble with this question, with less than 30±15% making a credible attempt. The diagram was not particularly helpful – it would be better to include coordinates. In order to better test all the concepts required to answer this question, it would be more useful to lead the student through it and test each concept individually.

Less than a quarter of students found $E_{A,C}$, and of those, many forgot the 2, indicating that they did not understand the integration step.

More than 95% of students incorrectly said that the field in Region B was $E_B = 0$, with about 10 explicitly stating that the field inside a conductor is zero. (The question makes no reference to a conductor.) The words "fixed charge" or "non-conducting region" would help with reading comprehension.

Quite a few students also mentioned *magnetic* fields.