Nuclear and Particle Physics

Lecture notes and figures

Objective:

Comprehend the modern picture of the physics of the atomic nucleus
To be able to apply the theory to the solution of problems involving the quantum mechanical properties of the nucleus,
as well as its structure, stability, instability, and decay mechanisms.

These lecture notes are copies of my originals. They are not meant to be definitive, nor are they guaranteed to be free of error.

They are made available on request, with the hope that they may assist you in this course.

I have also put in the reading room several copies of introductory Nuclear Physics texts. These are the property of either my graduate students, or myself.

We would appreciate it if they were not removed from the reading room.

These notes are on the web at
http://www.ph.unimelb.edu.au/~max/

Max Thompson
September 2002

Nuclear and Particle Physics 354

Lecture times

Mon., Wed., Fri. @ 3.15 pm

Hercus Theatre

Tutorial

? **?**

Lecturer (lectures 19-36) Max Thompson Room 407

-

There will be an assignment, worth 7.5% of the final mark

Text Books

(18% discount at bookroom)

? Nuclear and Particle Physics — *W. S. C. Williams*

$96 (Paper Bk)

Introductory Nuclear Physics** — ***Kenneth Krane

~$120 (Hard Bk)

♥**Concepts of Nuclear Physics** — ***Bernard Cohen***

(Hard to Find)

♣**Introduction to Nuclear Physics** — ***Harald Enge***

~$100

- ? Recommended text for the entire course
- * **Very thorough**
- ♥ **More descriptive**
- ♣ **More experimental**

Lecture 1 (2002)	Krane	Enge	Cohen	Williams
Introduction				1.1
Rutherford scattering	11.6			1.2/3
Nomenclature	1.2	1.2		1.6
Constituents		1.10	1.1	1.6
Nuclear size	3.1	1.3	1.2	3.1

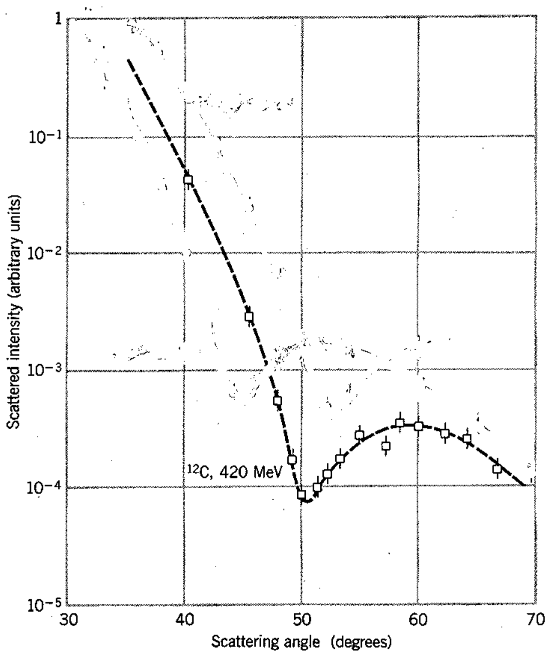
Problems Lecture 1

1 **Calculate the distance of closest approach of a α-particle of 6 MeV to a nucleus of gold.**

2 **If the mass of a nucleon is 1.7 x 10^{-27} kg, what is the mass density of the nucleus ^{16}O? What is the mass density of the nucleus ^{208}Pb? Compare both of these with the atomic mass density of the same material.**

3. **From the uncertainty principle $\Delta p.\Delta x \approx \hbar$, and the fact that a nucleon is confined within the nucleus, what can be concluded about the energies of nucleons within the nucleus?**

4. **Calculate the approximate diameter of the ^{12}C and ^{16}O nuclei from the data in Fig. 3.1 of Krane.**



Lecture 1

Why study Nuclear Physics?

Basically because understanding of the nature of the force between the most fundamental components of matter that are directly accessible is not understood. Unlike the case of atomic physics, where quantum electrodynamics provides an exact description of the EM force, the nuclear force is as yet not completely understood.

For that reason there are still very fundamental experiments being undertaken in an effort to clarify the nature of the force between the nucleons (protons and neutrons). In particular the effect of placing these nucleons in close proximity to many others, that is within the nucleus.

For those of you who are going to proceed to research in physics, this is a sufficient reason for studying nuclear physics, or indeed any branch of physics. For all of you, including those who will go into the real world and make money, the impact of nuclear physics in many areas of life is immense, and as physicists you should know more about it than the average Herald-Sun reader.

These areas include

- the environment: the continuing debate about the safety and environmental cleanliness of nuclear power, the feasibility of fusion power.
- medical applications: radiation therapy, diagnostic tools such as radioactive scans, CAT, PET. And total body composition studies.

Krane	Cohen	Enge	Williams	Topic
ch 1	ch 1	ch1	ch 1/2	The nuclear system
				Terminology
				Properties: size, mass, stability, spin, moments etc
				The stability chart
3.3	7.2	4.6	ch 4	Binding energy, Semi-empirical mass formula
3.1	1.2	1.3	ch 3	potential scattering, measurement of nuclear radius
4.2	3.10	3.2		nuclear scattering
4.1	3.2	2.2	9.8/9	The deuteron
4.4	3.11	2.5/6	9.7	the nuclear force
4.5	3.7	3.11	9.10	Exchange nature of nuclear force
5.1	ch 4	6.1-5	ch 8	Shell Model
5.2/3	ch5/6	6.6-10		Complex nuclei, collective models
11.3	6.3	6.7	10.11	Isospin
			ch 7	Nuclear Reactions
11.10	ch 13	ch 13	7.7	i compound nucleus
11.11	ch 14	ch 13	7.9	ii direct
				iii photonuclear reaction
ch 8	ch 10	ch 10	ch 6	Alpha decay
ch 9	ch 11	ch 11	ch 5	Beta decay
ch 10	ch 12	ch 9	ch 5	gamma decay
				applications of nuclear physics
				i CAT and PET
				ii NMR
				iii TNAA
				iv PIXIE

You can see that the range of topics is quite extensive. The degree of theoretical rigor will vary from topic to topic. Overall I hope that you might at the end of the course be able to say that the course objective has been met.

Basic

By definition the course will study "the nucleus"

1 What it is:

its components
its size
its density

2 What holds it together:

the nature of the nuclear force
the properties of the nuclear quantum states.
its stability and instability

3 The reaction mechanisms:

reactions that allow us to probe its properties
natural decay reactions

4 Some applications

Nomenclature

Nuclear components:

proton and neutron (**NUCLEONS**)

Nucleus: stable collection of nucleons.

Size: ~ 10^{-15} - 10^{-14} m (cf atom ~10^{-10} m)

A particular nuclear species is called a **NUCLIDE.**

A particular nuclide consists of N neutrons and Z protons

The charge on a proton is +e so the nuclide $^{A}_{Z}X$ has Z protons, A= N+Z nucleons and N neutrons.

A is the nuclear mass number
Z is the charge or isotopic number
N is the neutron number

Note that not all combinations of Z and N are stable. To first order those with N=Z are. The reason for this is directly relatable to the nature of the nuclear force, and this lecture course.

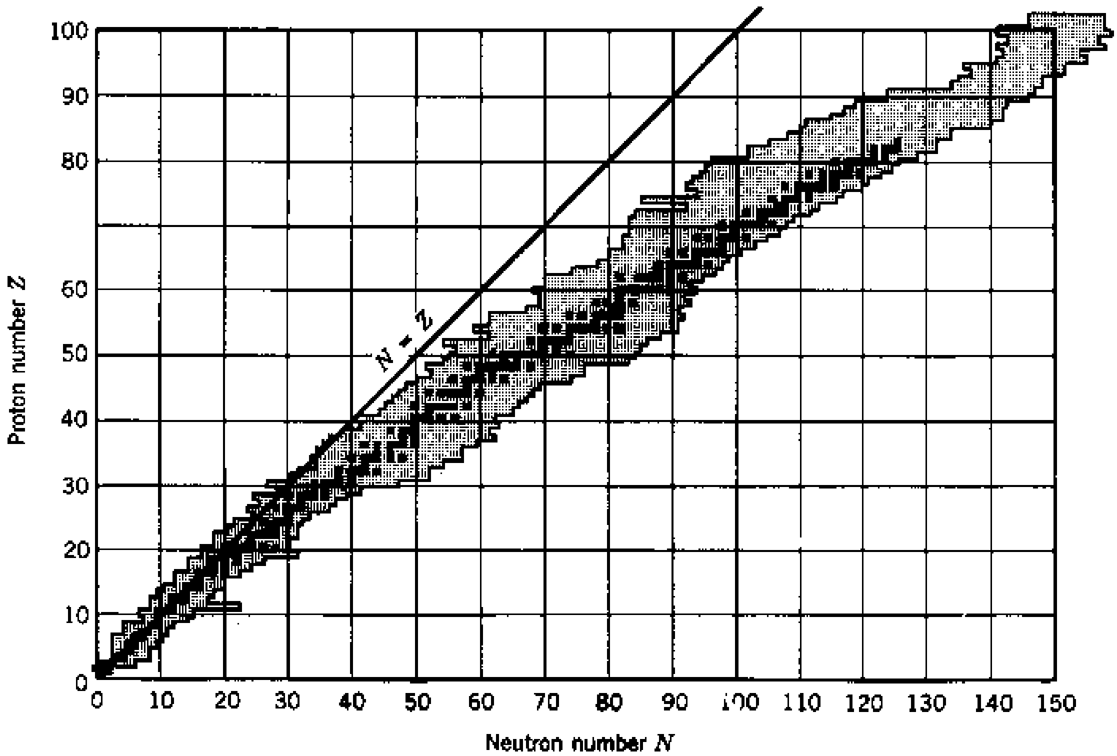


Figure 1.1 Stable nuclei are shown in dark shading and known radioactive nuclei are in light shading.

An **ISOTOPE** is one of a set of nuclides with the same Z and consequently different A. *ie* isotopes (as atoms) have the same chemical properties but different masses.

An **ISOBAR** is one of a set of nuclides with the same A but different N and Z. e.g $^{14}_{8}O$, $^{14}_{7}N$, $^{14}_{6}C$.

Nuclear Size

Let's look first at the quantitative evidence as to the size of the nucleus.
Historically (actually from ancient Greek times) the atom was regarded as the ultimate particle. However the discovery of the electron, in the late 1890's, was probably the beginning of the end of that notion.

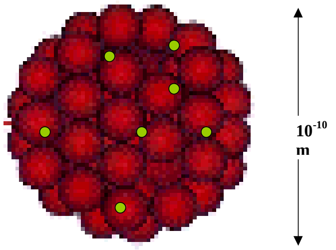
Size cf atom. Marble at centre of oval

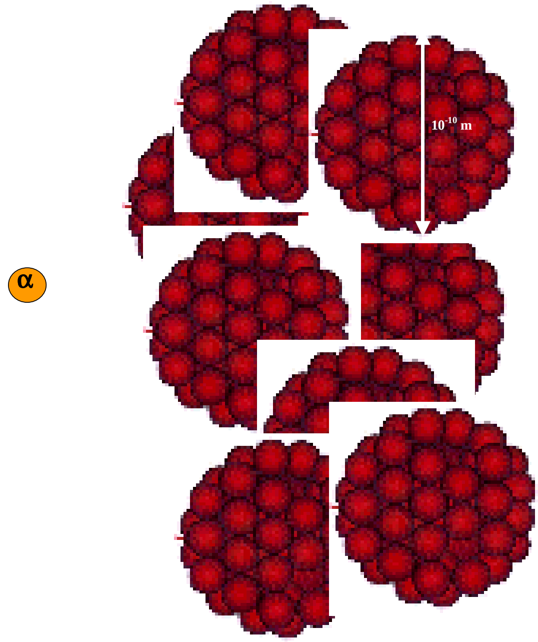
The real demise of the atoms inviolability came as a result of studies of the new fangled radio-active decay. α-particles were emitted from atoms, and these were shown by Rutherford and others to be in essence helium. Where did they come from?

Thomson model of nucleus: The centre of the atom was a mix of protons and electrons, so as to give the observed Z. However this was large and the mix was like a "Plum pudding"

It was Rutherford in 1911 who postulated that the atom was essentially empty space consisting of a massive minute nucleus with the appropriate numbers of electrons surrounding it.

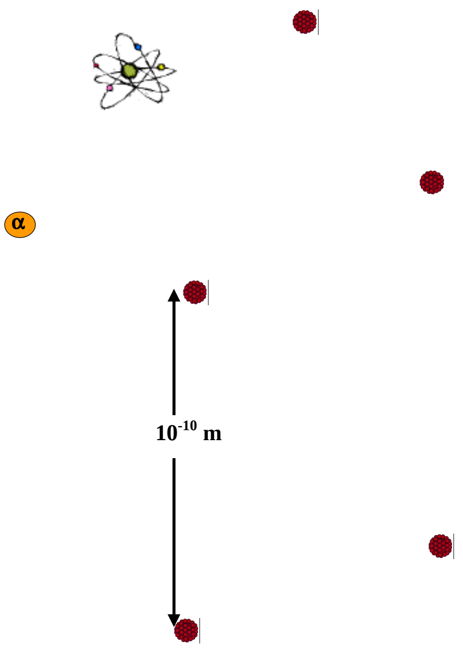
He proved this in a classic experiment, of which you are aware, where high-energy α-particles were fired through thin foils of different materials, and the deflection of the αs was measured.





Thomson's plum-pudding model was the size of an atom, and consisted of A protons (determining the mass) and A-N electrons determining the known Z of the atom.

In essence, if the plum pudding model were correct, the 10^{-10} m atoms would be cheek to jowl and the α's would scatter successive scatterings as they penetrated the foil. Thus there would be a relatively small scattering angle[1]. On the other hand if the nucleus was tiny but had a large (Z) charge, the scattering would be classical electrostatic scattering. In reality he found that essentially all the α's got through, with few deflected more than 1^o, and essentially none were reflected.



The Rutherford model

I want to go through this measurement with you. The theory of Coulomb or Rutherford scattering is in your text (Krane p 396+). First the experimental equipment:

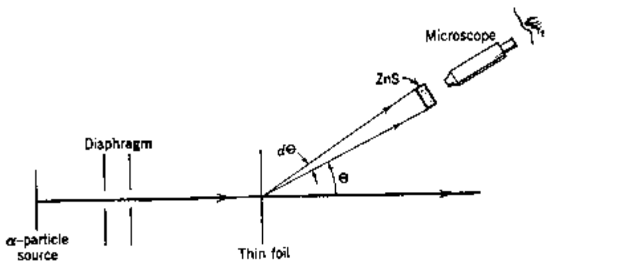


Figure 4-2 Arrangement of an α-particle scattering experiment. The region trav the α particles is evacuated.

---

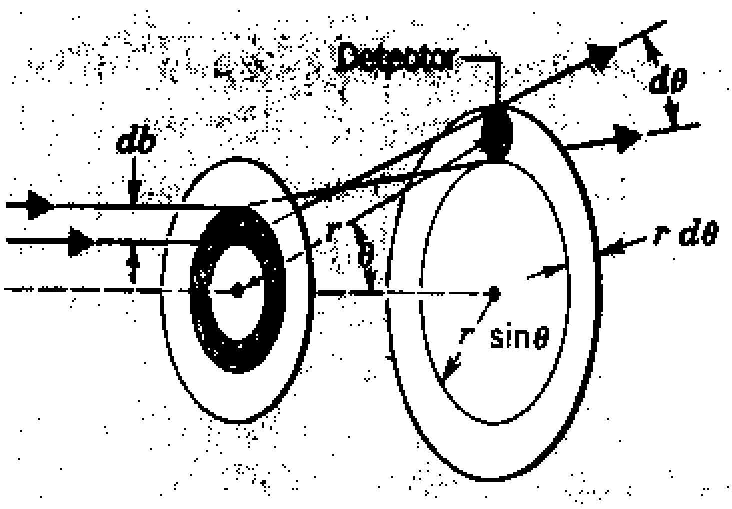
[1] See Fowlers (U of Virginia) lecture http://www.phys.virginia.edu/classes/252/Rutherford_Scattering/Rutherford_Scattering.html

- A source of high-energy charged particles…α-particles from Ra with a kinetic energy of about 7+ MeV (the same as used in the Part 3 nuclear prac.
- A very thin foil to produce the scattering. The classic foil is gold, since it can be made extremely thin (400 atoms was used) , ensuring that multiple scatterings are unlikely. In fact Rutherford used materials with a range of different Z to check the theory.
- A detector of α-particles. Today we would use a SS detector such as in the Part 3 experiment. However in 1911 they were some 90 years too early, and had to rely on scintillations of the α's on a layer of ZnS. This detector system was incomplete, since the recording apparatus was missing. Hence
- A number of research students, whose job it was to observe the individual scintillations at the different angles, and record the results. This particular piece of apparatus has not changed in 90 years. Graduate students are still essential.

Now let's look at the microscopic level to understand the physics.

Rutherford assumed that the coulomb scattering was the result of an infinitely heavy, point charge. The scattering is a classical collision problem where, as shown in the figure some of the KE of the incident α is converted into PE at the point of minimum approach. Naturally for a conservative force, after the interaction the final KE is restored. The locus of the trajectory is a hyperbola.

The angle of scattering θ, depends on how close is the line of approach of the incident α to the point scatterer. This is measured by the impact parameter *b*. The smaller *b* the larger will be the angle of scattering.



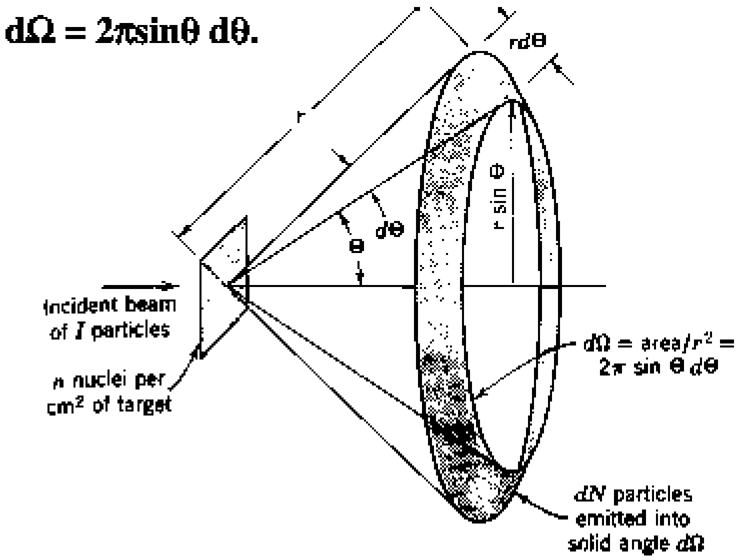
The relation between θ and *b* is $\cot\frac{\theta}{2} = \frac{2b}{d}$, where d is the distance of closest approach, i.e. when we have a head-on collision when $b = 0$.

Exercise: calculate the distance of closest approach of an α of energy 6 MeV to a nucleus of Gold.

The probability of an α hitting a ring of width *db* distance *b* from the zero impact line is P(*b*) *db* = ρ t 2π *b db* where ρ is the number of atoms per unit volume and t is the thickness of the foil.

This is thus the probability of the α being scattered between angle θ and dθ, and since we know b as a function of θ, this can be written as:

$$P(b)db = \frac{\pi}{8}\rho t\, d^2 \sin\theta \frac{d\theta}{\sin^4\frac{\theta}{2}}$$



Note that this is the probability of an α incident with impact parameter *b* being scattered into an angle between θ and θ +Δθ. For a beam of α's of intensity *I*, incident uniformly on the foil, the solid angle dΩ subtended at angle θ is: dΩ = 2πsinθ dθ. So in the above equation we substitute for dθ, and find that the probability of an α ending up at angle θ per unit solid angle (dΩ) is:

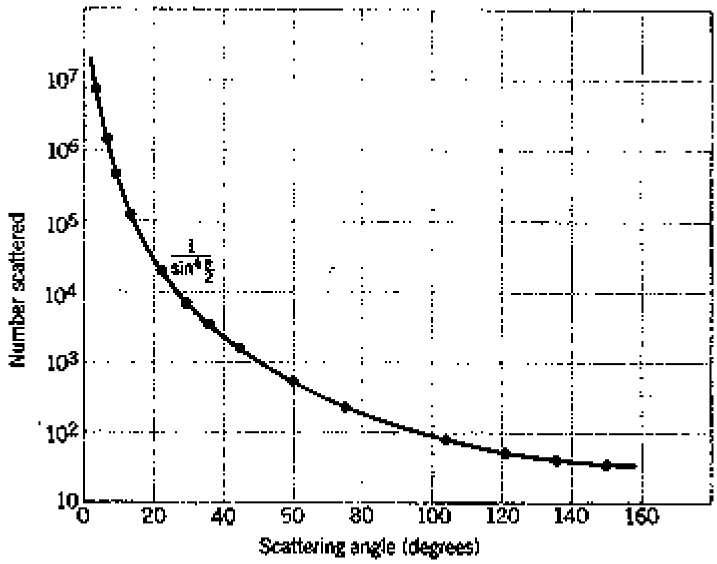
$$\frac{dN}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_o}\right)^2 \left(\frac{Ze^2}{Mv^2}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}} I\rho\, t$$

In the above an expression for d (the distance of closest approach) has been found by you as an example, for an α of KE= $1/2mv^2$, and charge 2e, scattering off a charge Ze. I leave this as an exercise for you to derive this, and to evaluate it.

At a later lecture we will express this in terms of the "differential cross section" However for the moment it is sufficient to see that in essence for a given foil (Z) and given α energy, the probability of detecting an α at a given θ is given by:

$$\frac{dN}{d\Omega} \propto \frac{1}{\sin^4\frac{\theta}{2}} .$$

I was this extreme dependence on scattering angle that told Rutherford that the nucleus was essentially of point size.



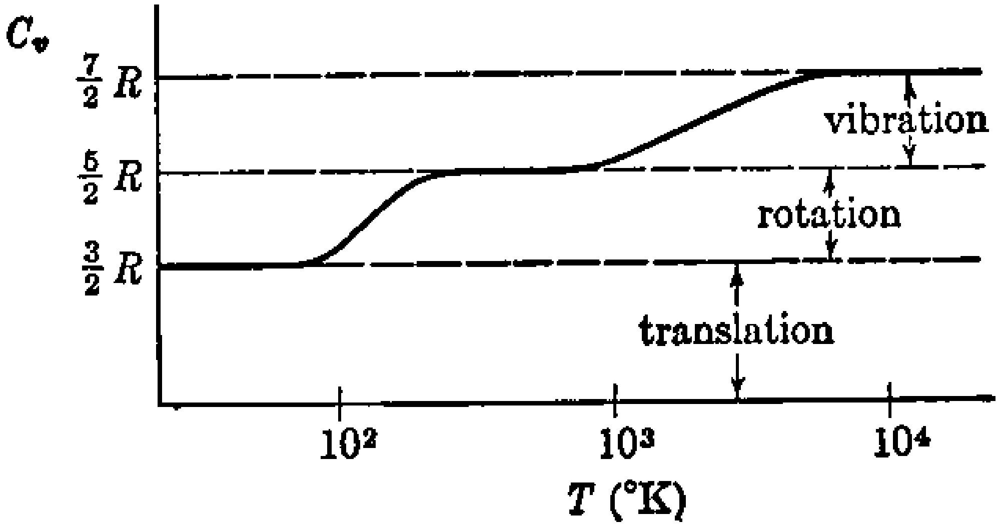
(d)

Figure 11.10 *(d)* The dependence of scattering rate on the scattering angle θ, using a gold foil. The $\sin^{-4}(\theta/2)$ dependence is exactly as predicted by the Rutherford formula.

When you evaluate the distance of closest approach, you will find that the nucleus of Au is~10^{-14} m in radius.

Before going on to discuss how to measure the size of nuclei, I want to consider the nature of the physics of the system. Firstly confirm that it is a quantum system we are studying, and then to see what order of magnitude quantum states we might expect.

From your early studies of statistical mechanics you are aware that the kinetic theory of gases relied on treating atoms classically, as if they were solid little balls. It worked. The size of the system was macroscopic, and classical physics sufficed. You may however remember that the model predicts the molar specific heat for a diatomic gas as c_v = 7/2RT; which it is, at relatively high temperatures. This figure comes from an average energy per molecule of 1/2kt per degree of freedom. For H_2 there are 7 degrees of freedom (3 translational, 2 vib. And 2 rot). However at lower temperatures it becomes 5/2RT then 3/2RT. You may recall that the explanation of this phenomenon was that the rot and vibrational energies were "frozen" (i.e.quantised), and at low temperatures 1/2kT is less than one quantum. So even in this fairly large scale system quantisation was evident.



2-18. Molar specific heat of molecular hydrogen (I

More importantly, if we were to take these solid little balls and examine them by say scattering electrons off them, as long as the electron energies were low, the scattering would be elastic, and we could consider them to be just solid balls. However you know that as soon as Frank and Hertz fired electrons of a few 10s of ev at them, the electrons lost energy: the atoms had absorbed energy from the incident particle. Not only that but the energy was in discrete values, and was emitted subsequently as photons. **These little balls had internal structure that was quantised.** The spacing of the quantum states is as you recall, in the region of the energy of a visible photon (~ev). So when probed with electrons with energies of this order, the internal structure can be seen.

Quantisation and size

Molecules

Quantisation in the Kinetic theory of gases

Atoms

Quantisation as revealed by Frank and Hertz
Substructure due to electrons

Nuclei

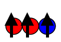
Rutherford showed elastic scattering
Is the nucleus quantised?
Heizengerg uncertainty principle $\Delta x.\Delta p \equiv \hbar$

For atom $\Delta x{\sim}10^{-10}$ m ➔$\Delta E{\sim}$eV
For nucleus $\Delta x{\sim}10^{-14}$ m ➔ $\Delta E{\sim}$several MeV
Quantum states of order MeV
Substructure due to nucleons

Nucleon

Assume $\Delta x{\sim}10^{-15}$ m ➔ $\Delta E{\sim}$ 100 MeV
Quantum states of order 100 MeV
(Δ-resonance at about 300 MeV
substructure due to quarks

p **n** **s=1/2**



Δ **s=3/2** **E~300 MeV**

We are now at the next stage down: we have considered the nucleus as a classical ball, and in Rutherford scattering observed elastic scattering of MeV α particles. If this nucleus has substructure, what order of magnitude is the spacing of the quantum states? We may then be able to probe any quantum substructure by using suitably energetic probes.

We can get some idea using Heizenberg's uncertainty relation. $\Delta x.\Delta p \sim \hbar$.

For an atom $\Delta x{\sim}10^{-10}$ m ➔ Δp ➔ $\Delta E{\sim}$ ev

For a nucleus $\Delta x{\sim}10^{-14}$ m ➔ Δp ➔ $\Delta E{\sim}$ MeV

The quantum states of a typical nucleus are of order MeV. So if we want to study the structure (that is the protons and neutrons that constitute it) by scattering we need a probe of energy >>MeV.

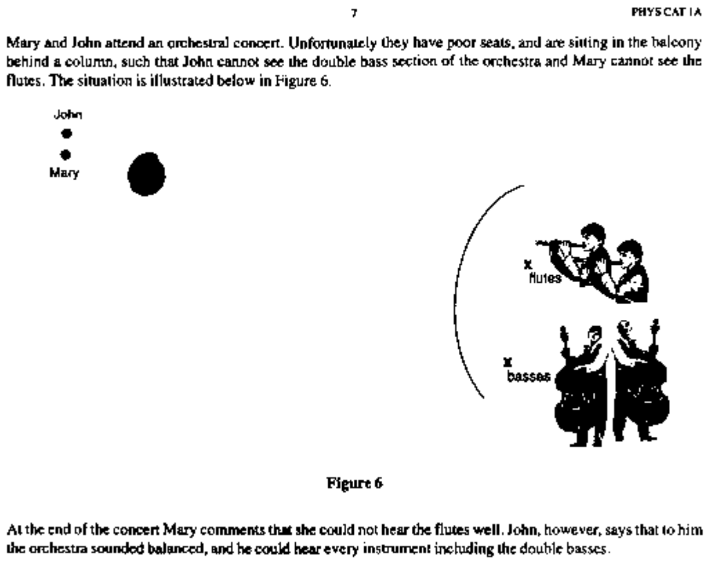
Just out of interest, we might assume that the nucleons are structureless. But again if we consider their size as 10^{-15} m, we expect quantum states at ~ 100 MeV. Indeed the first excited state of the nucleon is at about 300 MeV and is called the Δ resonance. It is, in terms of the substructure of the nucleon, a rearrangement of the quarks that form the substructure of nucleons (the GS of the nucleon is p=uud, n = ddu u has q=+2/3e, d has q = -1/3e. The Nucleon GS has the spin of the quarks coupled to s= ½ , and the Δ has one d or one u flipped to give s=3/2)

Now we might ask how to determine the size of the nucleus with some certainty. Naturally we are probing something we can't see, and we need to probe with an external probe and interpret the results. Scattering from the nucleus is a standard tool. Since the nucleus contains charges we can use coulomb scattering. The first response is to note that this was done this nearly 100 years ago, by Rutherford. However if you recall he assumed that there was a point charge. If the probe is to get close enough to see the size of the nucleus we must incorporate this into the analysis.

Scattering of high energy (200-500-MeV) electrons was the preferred method, and we will shortly look at the experimental methods. However you might well have already worked out how this will work and what the experimental results will look like.

Harking back to Rutherford, I would expect that if the electron doesn't make it close to the nucleus (that is for small scattering angles) the results are simply coulomb scattering from a point charge. It is only when the electron is close enough to discern that the nucleus is not a point that differences might occur. To interpret these deviations, we can no longer consider scattering of particles in a classical way. The incident beam of electrons, for example, must e considered as a deBroglie wave.

A beam of 200-MeV electrons has a deBroglie wavelength of about 10 fm (< diam of a moderate nucleus). (you should check this). So we have here a typical problem of a wave scattering around a small object. (1997 VCE question).


7 PHYSCAT IA

Mary and John attend an orchestral concert. Unfortunately they have poor seats, and are sitting in the balcony behind a column, such that John cannot see the double bass section of the orchestra and Mary cannot see the flutes. The situation is illustrated below in Figure 6.

Figure 6

At the end of the concert Mary comments that she could not hear the flutes well. John, however, says that to him the orchestra sounded balanced, and he could hear every instrument including the double basses.

So I would expect a scattering cross section to look like:

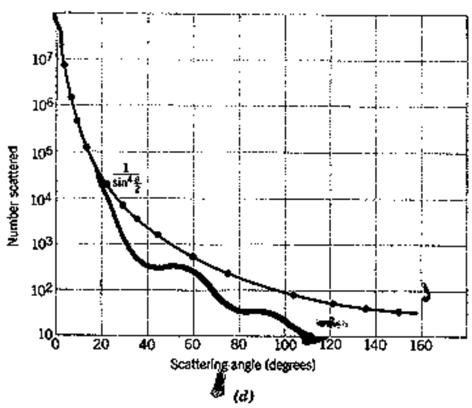


Figure 11.10 (d) The dependence of scattering rate on the scattering angle θ, using a gold foil. The $\sin^{-4}(\theta/2)$ dependence is exactly as predicted by the Rutherford formula.

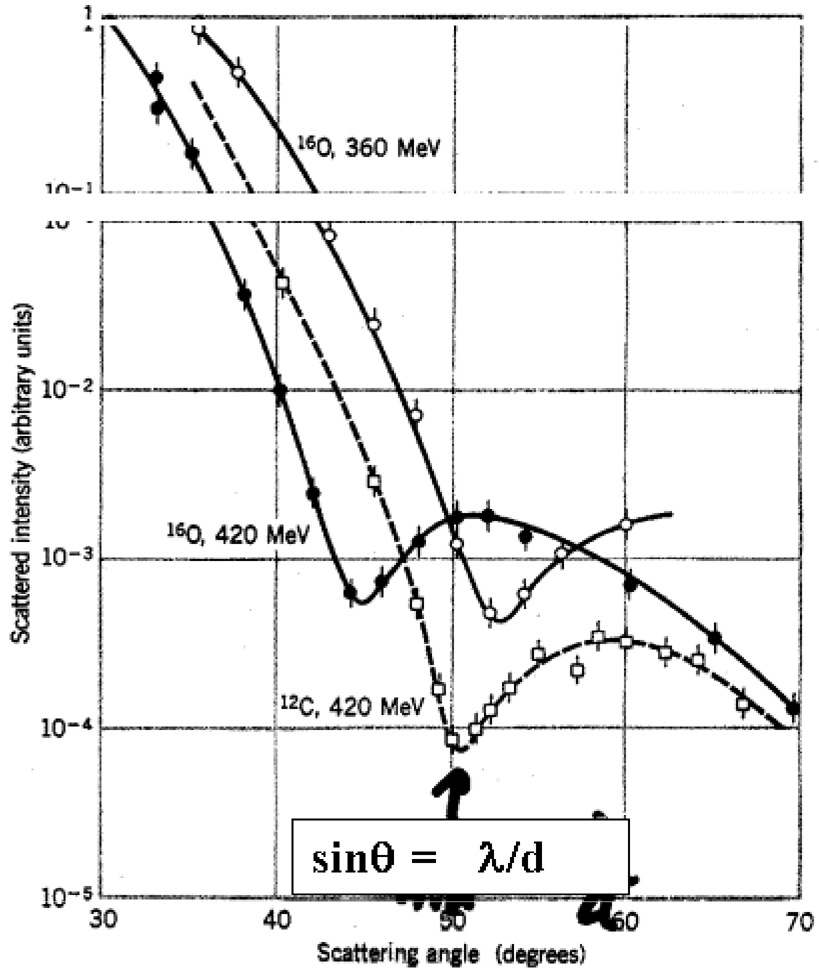


Figure 3.1 Electron scattering from ^{16}O and ^{12}C. The shape of the cross secti is somewhat similar to that of diffraction patterns obtained with light waves. T data come from early experiments at the Stanford Linear Accelerator Center (H. Ehrenberg et al., *Phys. Rev.* **113**, 666 (1959)).

ere are some real ones:

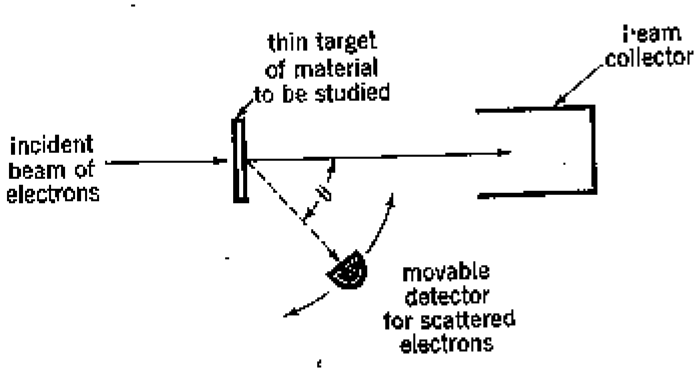
Note for ^{16}O the first maximum moves to a smaller angle as the wavelength decreases (energy increases).

However there is a lot more that can be got from these data, and it is necessary to look in a little more detail.

Before we do that let's see how these measurements are made. Our research group is actually involved in electron scattering experiments, with one PhD student currently at Mainz University. We also work at Tohoku University where elastic electron scattering was a major research program.

What we need is:

a source of high-energy (~100+ MeV) electrons
a thin target containing the nuclei
a means of measuring the angle and energy of the scattered electrons.



What is an electron linear accelerator?

A means of transferring EM power to electron KE.

Components: injector (100keV)
Klystron power
Drift tube

Electron magnetic spectrometer

Need to be able to change angles

Need to measure energy of scattered electrons.

We want to consider only elastically scattered ones, not those that have given energy to the nucleus.

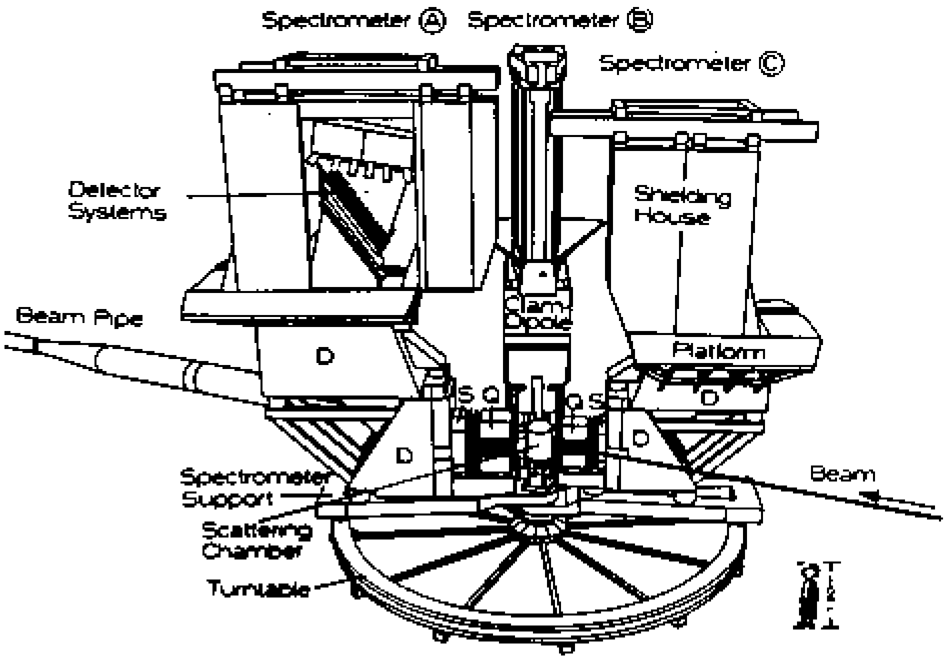


Figure 1: *The three-spectrometer setup at MAMI*