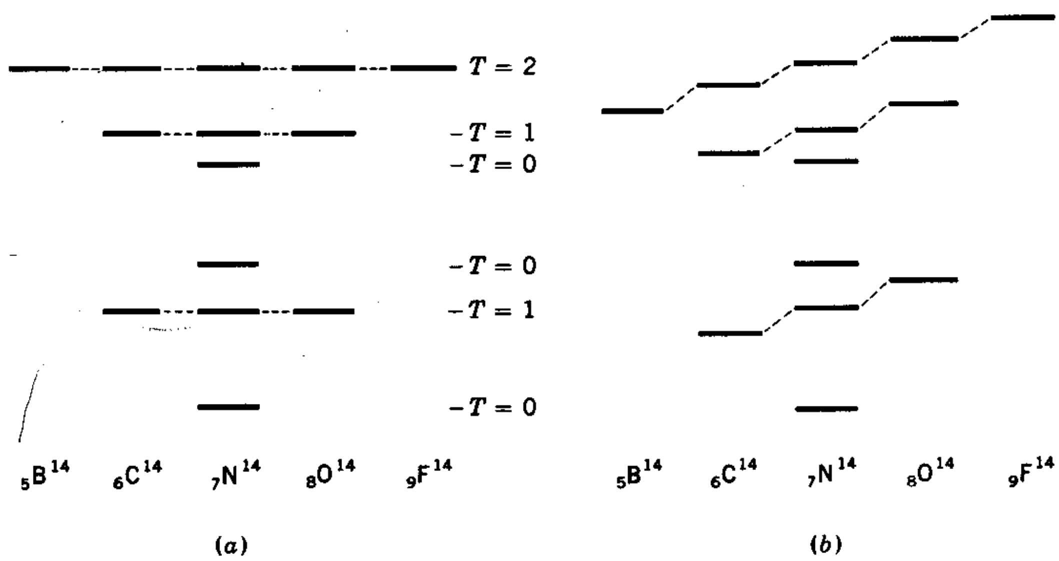
Lecture 10	Krane	Enge	Cohen	Williams
Collective models				
Spherical nuclei.	**5.2**	**6.9-10**	**Ch 6**	**Ch 8**
Isospin	**11.3**	**6.7**	**6.3**	**8.10**

Problems Lecture 10

1 What are the values of T for the GS of ^{63}Cu, ^{109}Ag, ^{238}U?

2 Write down the configurations (including j^{π}) the isobaric analoges of the ground state of ^{14}B that are shown in the diagram below.

FIGURE 6-12 Isobaric analog states in $A = 14$ nuclei: *(a)* without coulomb forces; *(b)* the effect of coulomb forces. States are classified according to the T quantum numbers.



Review Lecture 9

1 If nuclei are deformed the shell model potential is not spherically symmetric

2 Nillson model of deformed shell model takes this into account. You should understand that for prolate nucleus (football) the orbit for largest m_j intercepts fewer nucleons, and hence the potential is less deep, and the high m_j states higher in energy. Conversely for –ve deformation. Example ^{19}F, on simple shell model expect j^{pi} = 5/2, in practice it is $½^{+}$.

Deformed even-even nuclei. Especially in region A= 140-200 or so.

4 Observe lovely sequence of states 0,2, 4, 6, 8, etc all +ve parity
The GS is 0^+since even – even. What about the 2^+ 4^+ etc?

5 The excited states correspond to rotation of the nucleus as a whole about one of the axes other than the symmetry axis. (Rotation about the symmetry axis cannot be seen, but rotation ofabout the other axes can.) We can see if a football is rotating end over end.

6 Classically AM L= $\mathbf{I}\omega$ and E = ½ $\mathbf{I}$ ω^2 = $\dfrac{\boldsymbol{L}^2}{2\mathbf{I}}$
In a q mech system $\boldsymbol{AM} = \sqrt{\boldsymbol{I(I+1)}}\hbar$

So energy of states is E $\dfrac{\mathbf{L}^2}{2\mathbf{I}}$ = $\dfrac{\hbar^2}{2\mathbf{I}}\boldsymbol{I(I+1)}$ Where I = 0, 2, 4, 6, etc

Even values of I only, since if GS is even parity, and its config. does not change, then all the rotn. states must have even parity, and since $\pi = (-1)^I$, odd values of I are forbidden.

7 So in units of $\dfrac{\hbar^2}{2\mathbf{I}}$, the energies of the states are separated by 6, 20, 42, etc almost exactly what is seen.

NB ***I*** **=** nuclear spin QN**.**
I = moment of inertia

8 You can actually quantify this. Assume it is a rigid body, so

$$\mathbf{I}_{rigid} = 2/5\, MR^2(1 + 0.31\beta)$$

For typical mass nucleus(A= 170) this gives $h^2/2\mathbf{I}$ = 6 keV, right order of magnitude, but too small.

In fact the nucleus behaves like an elastic fluid, and the radius stretches at higher energy..

Lecture 10

Last lecture we looked at the interesting level structure that is seen in deformed nuclei, and how well this was reproduced by considering collective rotations of the nucleus. Today I want to talk about the application of the collective model to **spherical nuclei with even numbers of p and n.**

The shell model successfully gives the GS spin of these even-even nuclei as 0^+. When we look at the level structure of these nuclei, as for deformed even-even nuclei, there is always a 1st excited state with 2^+.

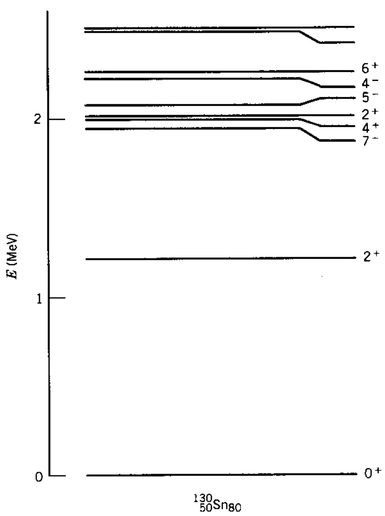
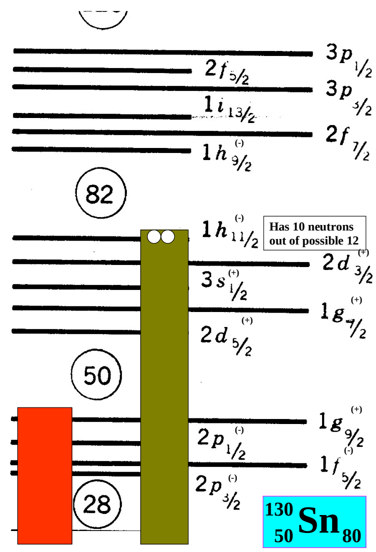


Figure 5.14 The low-lying energy levels of ^{130}Sn.



Well this is no problem for the SM to account for. Take ^{130}Sn (Z=50 N= 80 with 10 neutrons in the h11/2 state (out of a possible 12). We could uncouple the top pair of N or P and excite to the next level, and recoupling will always give even +ve parity states including 2^+ (remember `^{18}O). However the uncoupling and exciting is much greater in energy than the observed energy of the 2^+ states. This excited state in this mass energy region is always less than 2 MeV, and between A=150 and 200, a few 100's of keV. There are other ways of producing the +ve parity states. E.g. lift a pair of d3/2 neutrons to the holes in the h11/2 state and recouple the remaining neutrons in the d3/2, or excite an s1/2 neutron to the d3/2 and couple them to give even parity even spin states. Etc.

$$\begin{aligned}\Psi(2^+) &= a\Psi(\nu\, h11/2 \otimes \nu\, h11/2)\\ &+ b\Psi(\nu\, d3/2 \otimes \nu\, d3/2)\\ &+ c\Psi(\nu\, d3/2 \otimes \nu\, s1/2) + etc\end{aligned}$$

Discuss collective effects.

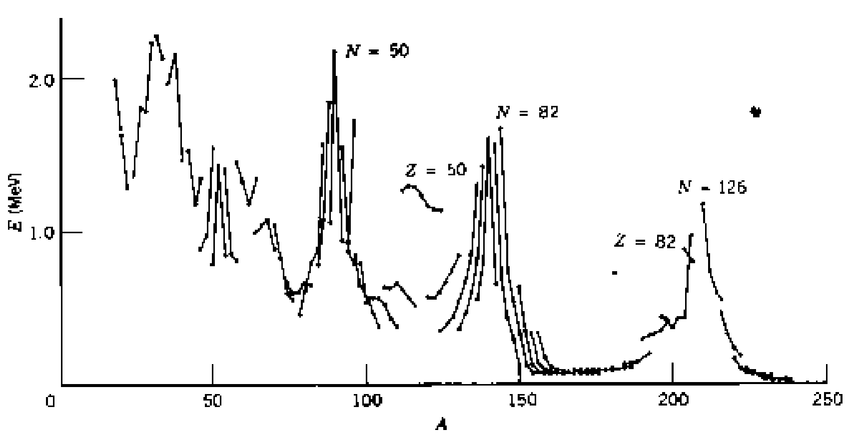


Figure 5.15a Energies of lowest 2^+ states of even-Z, even-N nuclei. The lines connect sequences of isotopes.

This anomalously low 2^+ state is not just a property of Sn130. The figure shows that this is common in the medium-mass nuclei. This reduction in anticipated energy of the 2^+ state is the result of collective motion of the nucleons within the nucleus, and as we saw last lecture, for the case of deformed nuclei, these collective effects can be modelled by motions of the nucleus as a whole.

A spherical nucleus cannot rotate (or at least QM-wise it can't), but it can oscillate.

Lord Rayleigh worked out the theory of oscillations of fluids in the 1800s. A fairly formal analysis for nuclei is given in Krane. The shapes of the vibrations can be expressed in terms of associated Legndre polynomials $P_{\lambda.m}$, and the standing wave patterns for λ = 1,2 3 are shown (krane 5.18) In essence λ can ve pictured as the number of wavelengths around the nucleus, and is the phonon quantum number.

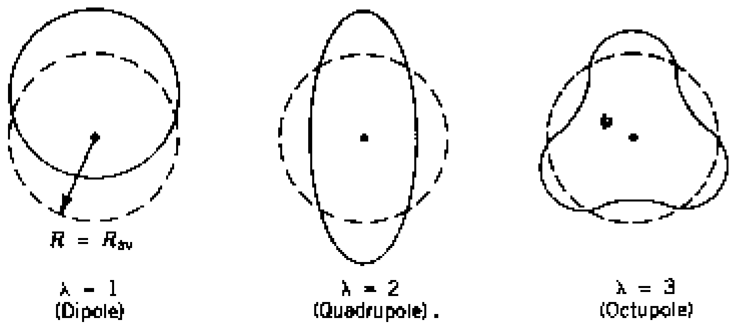


Figure 5.18 The lowest three vibrational modes of a nucleus. The drawir represent a slice through the midplane. The dashed lines show the spheri equilibrium shape and the solid lines show an instantaneous view of the vibrati surface.

The dipole mode involves a physical shifting of the nuclear CM and cannot occur from within the nucleus. λ = 2 is the quadrupole mode λ=3 is the octupole mode. The energy of the oscillations is quantised as phonons, and each phonon has a magnitude $\lambda\hbar$.

Thus if we set the spherical nucleus into the lowest realistic oscillation it will have one λ=2 phonon and a spin of + 2, since π= $(-1)^{\lambda}$. It is a 2^+ state. The next quadrupole oscillation state will have 2 λ=2 phonons, which can couple to give total AM of 0, 2, and 4, all with π positive.

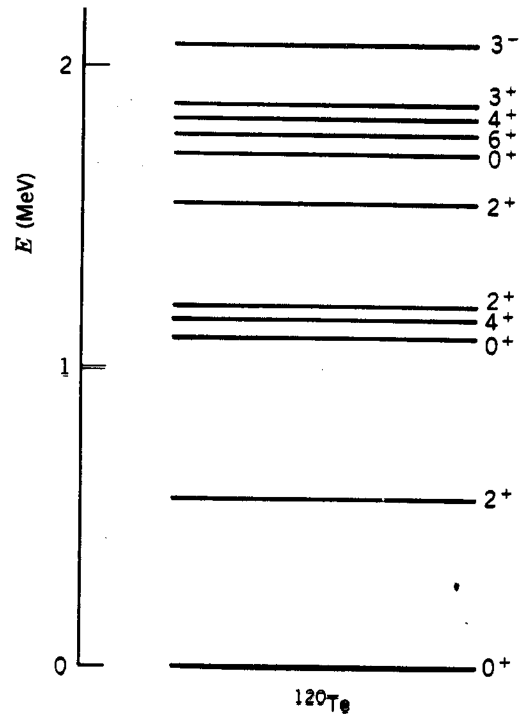
TABLE 5-5: VARIOUS m_λ CONFIGURATIONS FOR TWO-PHONON STATES

m_λ	m_λ *of the two phonons*														
+2	xx	x	x	x	x										
+1		x				xx	x	x	x						
0			x				x			xx	x	x			
−1				x				x			x		xx	x	
−2					x				x			x		x	xx
Σm_λ	+4	+3	+2	+1	0	+2	+1	0	−1	0	−1	−2	−2	−3	−4

m_I	+4	+3	+2	+1	0	−1	−2	−3	−4
No. of config.	1	1	2	2	3	2	2	1	1

Three λ=2 phonons can couple to 0,2,3,4 and 6 with π=+.
Each phonon of octopole oscillation has an AM of 3, and the parity for this is –ve.

Does it compare with reality

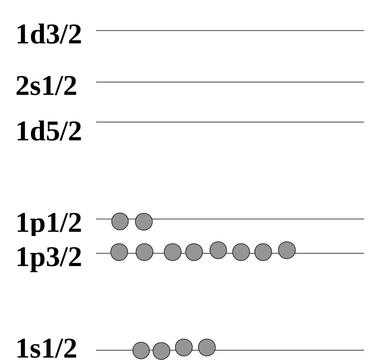


Note that to 1st order, the spacing between 2^+ and 4^+ and 6^+ is essentially equal: the spacing being the energy of one phonon.

Isospin
This seems a bit of an interloper here, but I find it a lovely example of the symmetry of nuclear structure. It reinforces the independence of the nuclear force on charge effects, and more importantly when we discuss beta decay it has a large role to play.

Look at this

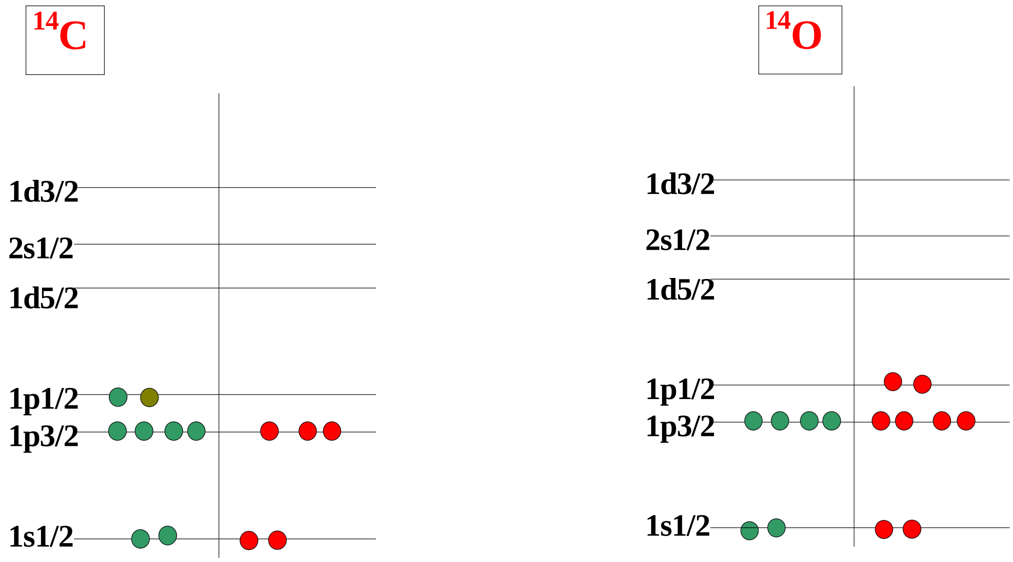
What nucleus is this?



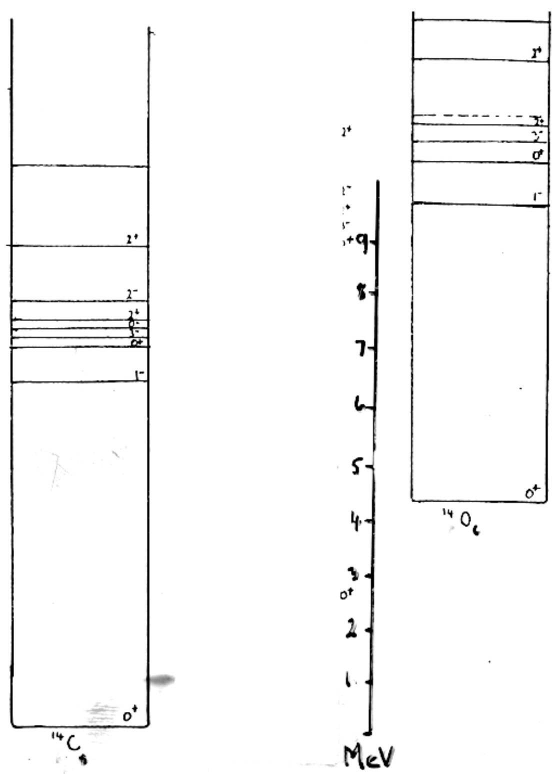
This nucleus has 14 nucleons. I know from the Pauli principle that at least 6 are protons, and 6 are neutrons. What about the 2 in p1/2 level?

They could be 2 n, 2 p, or p and n
Let’s look at the case where they are 2 n or 2 p.

(show 14C and 14O)



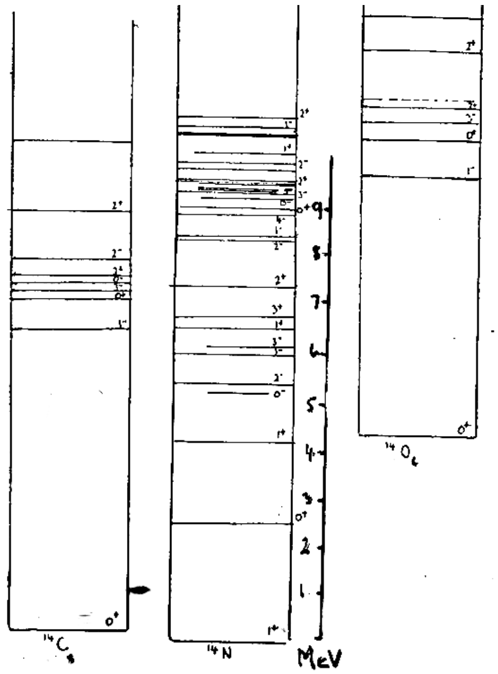
To all intents and purposes these are the same structure. The excited state energies and j^{π} look almost identical.



Conclusion. The nuclear wave function depends only on the N-N potential. There is an effect due to the Coulomb force that affects the absolute energy of the 14-nucleon system. ^{14}O has a larger coulomb repulsion and is less massive as a result. It also has 2 fewer neutrons, and is also less massive ($m_p > m_n$) because of this.

This pair of nuclei are called "mirror nuclei".

What about the other mass 14 nucleus ^{14}N?



It looks quite different at first glance. But look carefully.

(Indicate location of T=1) states.)

Let's make allowance for the energy differences due to the coulomb energy (which is different for each) and the different masses of p and n.

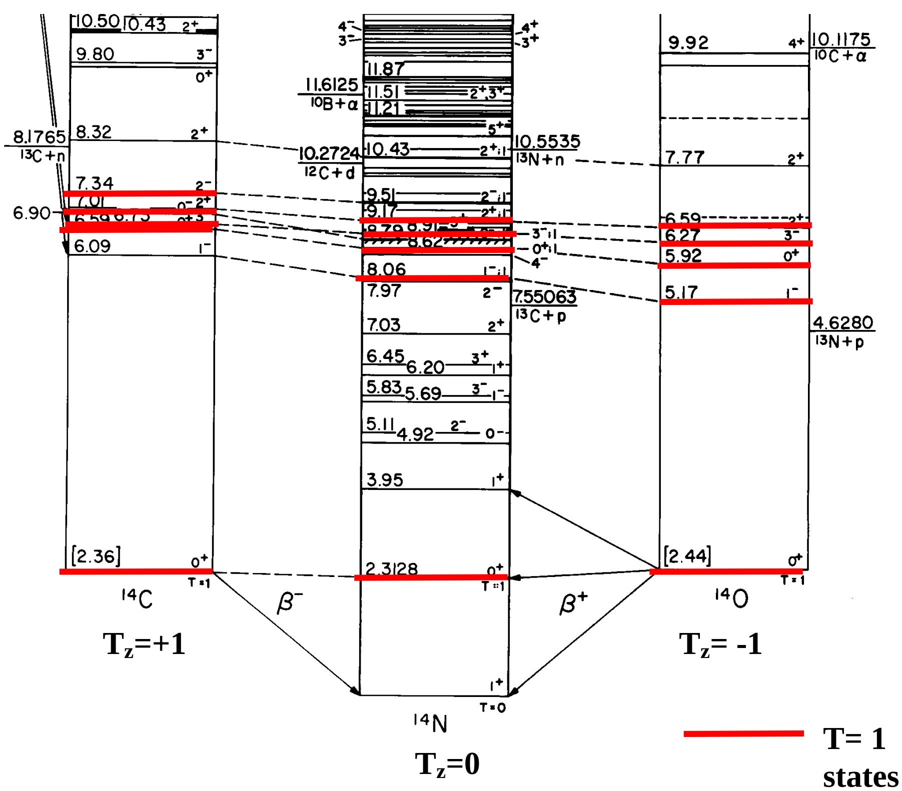
$$E_c = 3/5\frac{e^2 z^2}{4\pi\varepsilon R} = 3/5\frac{e^2 z^2}{4\pi\varepsilon r_0 A^{1/3}}$$

$$\Delta E_c = 3/5\frac{e^2(2Z-1)}{4\pi\varepsilon r_0 A^{1/3}}$$

The nucleus with Z protons is less stable by this amount rel to that with Z-1 protons.

However the nucleus with higher Z has one less neutron, and a neutron mass is greater than a proton mass by 0.78 MeV

So we can compensate for these mass-energy differences, and when we do so we see…



This is the result of ISOSPIN, T

All the states in ^{14}N that line up have T=1. The other states in ^{14}N have isospin T=0.
In words: If I did not consider isospin (if I painted all nucleons grey) all the T=1 states in ^{14}N would have the same configuration as states in the other mass 14 nuclei.
The T=0 states in ^{14}N have no equivalent states in the other nuclei. They are forbidden by the Pauli principle.

Let's formalise this.

We introduced the concept of isospin for the deuteron. We specify the isospin part of the nucleon wavefunction as !t,t_z> The vector t has projections t_z=+1/2 (n) and t_z=-1/2 (p). So
!n> = !1/2,+1/2> !p> = !1/2,-1/2>.

Since t_z determines the nature of the nucleon, the T_z of a nucleus is Σt_z of all A nucleons. For example for ^{14}N, T_z = 0. In fact we can see that T_z = (N-Z)/2. **T_z determines which element we have.**

Nucleon isospin	**Projections**	
t = ½	t_z = + ½ (neutron)	t_z = - ½ (proton)

Isospin part of wavefunction |t, t_z>

|p> |½, -½>

|n> |½, +½>

For nucleus A

$$T = \Sigma_{i=1}^{A} t$$ **Vector sum**

$$T_z = \Sigma_{i=1}^{A} t_z$$ **For a given T, there are (2T=1) projections T_z**

$$\boxed{T_z = \frac{(N-Z)}{2}}$$

The isospin part of the nuclear wavefunction is specified as

|T, T_z>

Well what about these states in mass 14 that line up. They all have the same isospin quantum number T (=1 in this case).

How do we find the value of T?

We take the configuration of the nucleons, and make the most "neutron rich" configuration of these A nucleons, that is consistent with the Pauli principle.

(illustrate with vugraphs)

For the configuration we have this most neutron-rich nucleus is ^{14}O, and for this nucleus the T is Σt for all 14 nucleons. Note this is a vector sum, **so the value is T = 1** (we could also say that T is the value of T_z for this most-neutron-rich configuration)

If T=1, it must have 3 projections T_z= 1, 0, -1.

T_z= +1 means a nucleus with 2 nett neutrons out of 14 nucleons…. ^{14}C

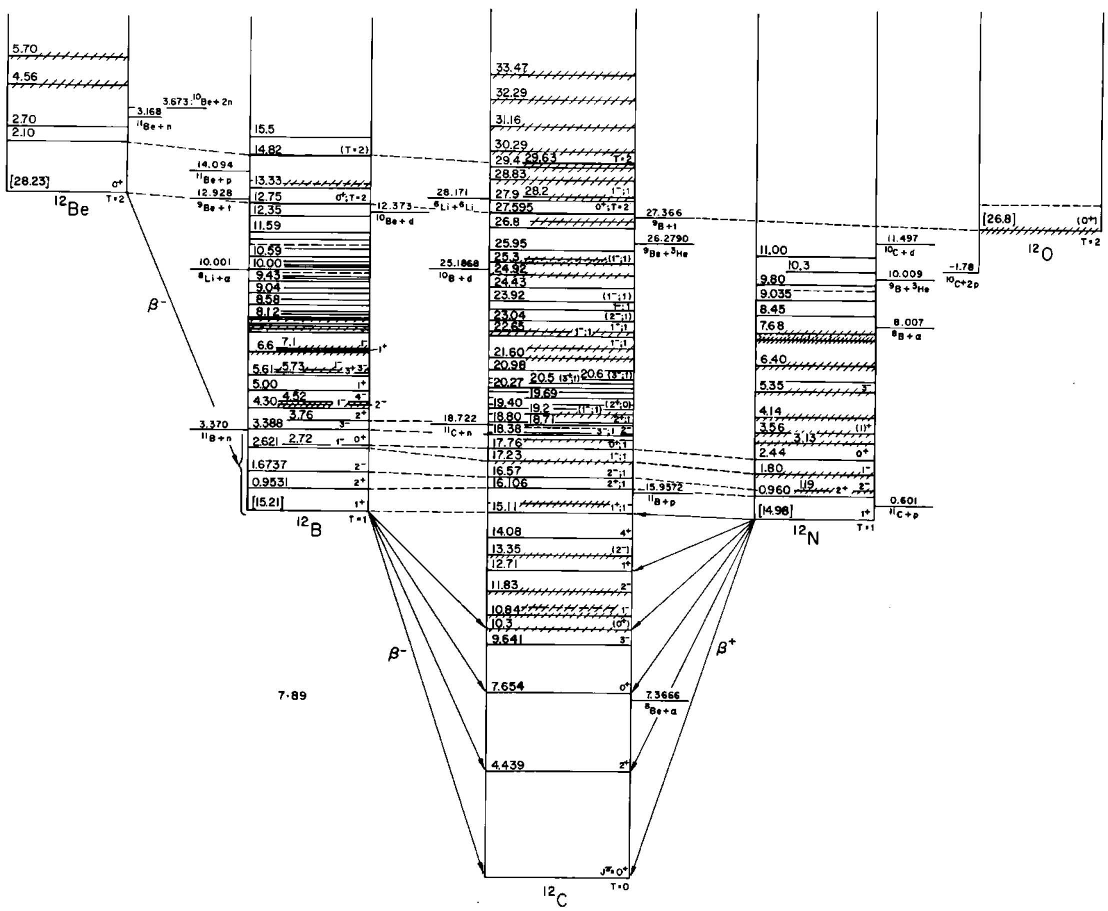
$T_z = 0$ means Z=N………………..^{14}N
T_z= -1 means 2 nett protons……………^{14}O

What is the T of the GS of ^{14}N?

GS has I=1 . ie p and n are parallel, and there is no way I can change a p to an n without violating Pauli principle.

So T is the value of T_z for the most neutron-rich form of this configuration. **This is it!!** The GS and the remaining unmatched states all have T=0. There is only one projection of this T_z = 0.

Look at ^{12}C as a further example .



This illustrates the situation for those students who have done the ^{11}B(p,α)^{12}C experiment in the part 3 lab.

Lead on to beta decay