Lecture 12	Krane	Enge	Cohen	Williams
a -decay				
Energetics	8.1	10.1		
Systematics	8.3	10.3		6.2
Coulomb effects	8.4	10.5	10.4	6.3
Angular mom.	8.5	10.6	10.5	6.4
g-decay				
Multipole radn.	10.2	9.5	12.1	
Decay constants	10.3	9.5	12.2	
Selection rules	10.4	9.6	12.4	

Problems

1 Calculate the Q and the energy of the emitted α, for α-decay from ^{242}Po, ^{208}Po, ^{242}Cm. Indicate the residual nucleus.

2 Use the uncertainty principle to estimate the minimum speed and energy of an α-particle inside a heavy nucleus.

3 In a semi-clasical picture an ℓ=0 α-particle is emitted along a line that passes through the centre of the nucleus..

(a) How far from the centre of the nucleus must an ℓ=2 particle be emitted? Assume Q = 6 MeV and A = 230.

(b) What would be the recoil rotational energy if all the recoil went into rotation of the daughter nucleus.

4 For the following γ-ray transitions , give all permitted multipoles, and indicate which would be the most likely.
$9/2^- \rightarrow 7/2^+$, $½^- \rightarrow 7/2^-$, $1^- \rightarrow 2^+$,

5 A decay process leads to final states in an even-even nucleus, and gives only three γ-rays of energies 100, 200 and 300 keV, which are found to be of E1, E2 and E3 multipolarity. Construct two different possible level schemes for tis nucleus and label the states with their most likely I^π assignments.

6 A nucleus has the following sequence of states beginning with the GS. $3/2^+$, $7/2^+$, $5/2^+$, $½^-$ and $3/2^-$. Draw a level scheme showing the intense γ-rays transitions and indicate their multipolarity.

Review Lecture 11

Isospin

1 The nuclear force is charge independent, hence the level structure of excited states in nuclei that have the same A, and the same configuration (but differing N and Z) will be the same. The example given was A=14. After allowing for the different coulomb energy

$\Delta E_c = 3/5 \dfrac{e^2(2Z-1)}{4\pi\varepsilon r_0 A^{1/3}}$, and the mass difference between a proton and neutron (0.78 MeV), the states in ^{14}C and ^{14}O (mirror nuclei) line up in energy, together with analogue states in ^{14}N.

2 The isospin Q No. of a configuration is the T_z value ($T_z = (N-Z)/2$) of the "most neutron-rich" form of the configuration that can be made consistent with Pauli principle. The low-lying states in ^{14}C and ^{14}O, and the analogue states in ^{14}N are all T=1, with projection T_z = -1, +1, and 0 respectively. The GS and most low-lying states in ^{14}N have T=0 (naturally T_z =0)

Beta Decay

The beta decay process

$^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}X + \nu$

The energy of a β^--particle emitted from a nucleus A,Z to form nucleus A, Z+1

$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}X + e^-$

$Q_{\beta^-} = \{[m(^{A}_{Z}X) - Zm_e] - [m(^{A}_{Z+1}X) - (Z+1)m_e] - m_e\}c^2$

$Q_{\beta^-} = [m(^{A}_{Z}X) - m(^{A}_{Z+1}X)]c^2$

Q_β represents the energy shared by the e^- and the antineutrino

$Q_{\beta^-} = T_e + E_{\bar{\nu}}$

And since when one is max the other is zero

$(T_{e^-})_{max} = (E_{\bar{\nu}})_{max} = Q_{\beta^-}$

For positron emission, a similar calculation gives that:

$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}X + e^+$

$Q_{\beta^+} = [m(^{A}_{Z}X) - m(^{A}_{Z-1}X) - 2m_e]c^2$

The **log ft** term and what it indicates.

The concept of forbidden-ness.

The importance of super-allowed transition, when the Nuclear wave-function overlap is exact.

Lecture 12

We discussed β-decay last lecture, and today I will discuss α and γ decay. We need to ask what determines whether decay of any kind will occur, and then look a little more closely as to what is the probability of certain decay.

Firstly let us remember we are discussing nuclei which lie outside the collection of 200 or so stable nuclei., and ultimately they will reach stability in order to attain a minimum energy of the nuclear system. These nuclei have either an excess of protons or neutrons, and in general they will attain stability by β-decay, changing a n➔ p or p ➔n. However for nuclei with A > 150 α-decay is the most likely possibility.

Ultimately the requirement for particle decay is that the final system is more stable than the original. One can show by considering the Q (energy difference between the final and initial system), that for light particle emission the only possibility is α.

Here table 8.1

For α decay the process is

$$ {}^{A}_{Z}x_{N} \rightarrow {}^{A-4}_{Z-2}x'_{N-2} + {}^{4}_{2}\alpha_{2} $$

Energy considerations give

$$\textit{the Q is defined as}$$
$$Q = (m_x - m_{x'} - m_\alpha)c^2$$
$$\textit{This energy is shared between the}$$
$$\alpha \textit{ and the daughter nucleus}$$
$$\textit{so}$$
$$(m_x - m_{x'} - m_\alpha)c^2 = Q = T_{x'} + T_\alpha$$

So if Q is positive , in theory α-decay can occur. The emitted α will of course not have a KE (T) equal to Q, since the daughter nucleus will recoil (T'_x).

$$T_\alpha = \frac{Q}{1 + \frac{m_\alpha}{m_{x'}}}$$

$$\text{since } m_\alpha << m_{x'}$$

$$T_\alpha \approx \frac{Q}{1 + \frac{4}{A}}$$

However Q alone does not determine whether or not we will observe an α decay. Since the decaying nucleus has a large Z, the coulomb barrier acts to suppress decay. The probability of the α getting out depends very critically on the energy of the α. This was found out by Geiger and Nuttall in 1911 (the Geiger Nuttall law)

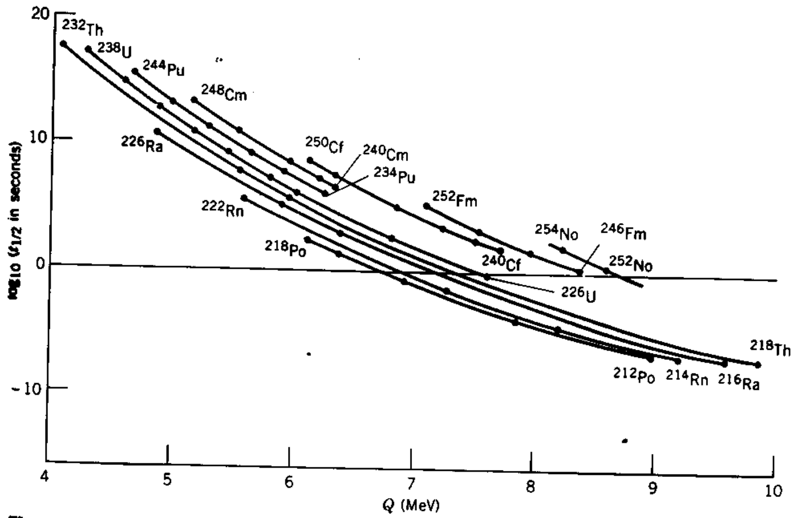
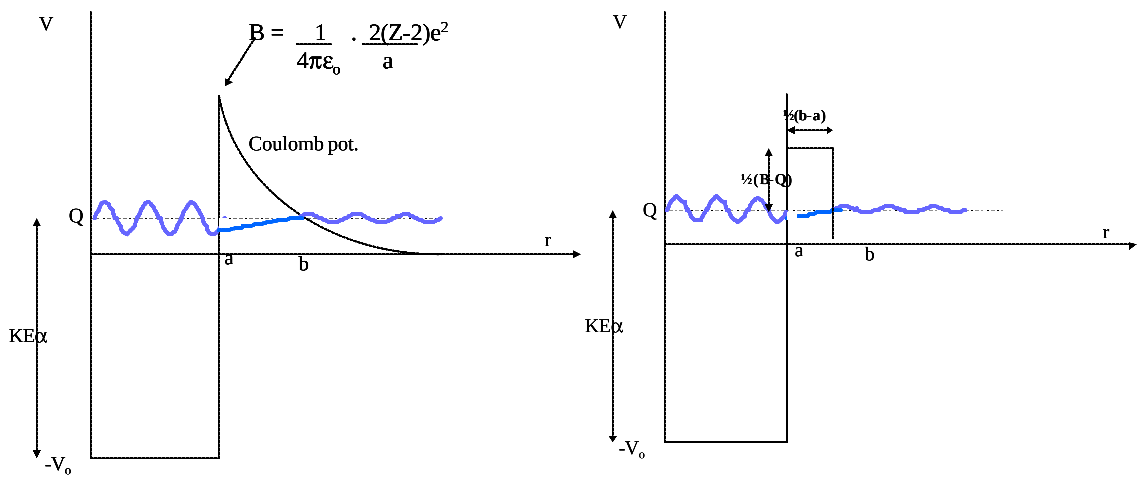


Figure 8.1 The inverse relationship between α-decay half-life and decay energy, called the Geiger-Nuttall rule. Only even-Z, even-N nuclei are shown. The solid lines connect the data points.

Note the Vertical scale is logarithmic, and ranges over some 25 orders of magnitude. Nuclei with the same Z are joined.

We are now in a position to estimate the rate of α-particle emission.



We model the system as an α and daughter nucleus, with a radius a. Within the radius r<a the α is free to move, but it cannot escape because of the coulomb barrier. Between a and b, classically the α cannot exist, since V>KE. Classically he particle will bounce back and forth in the region r=0 to a. However quantum mechanically there is a chance of leakage, or tunnelling through the barrier, and one can calculate the probability P of escape. The decay constant λ = fP per second. Where f is the frequency of presentation at the boundary.

As an example, for 238U the alpha takes ~10^{38} tries before it succeeds (~10^9 years.)

The QM problem is simplified by using a square pot barrier. The barrier height varies from (B-Q) above the particles energy at r=a, to zero at r=b. Lets take an average height ½(B-Q).

The width of the true coulomb barrier is b-a. Lets take the square one as ½(b-a).

(NOTE b is the value of r when the alpha is free of the coulomb potential. It is equivalent to the stopping distace from r=0 for in incoming alpha of energy Q. So conservation of energy gives

$$\boldsymbol{KE = PE}$$

$$Q = \frac{1}{4\pi\varepsilon_o}\frac{2(Z-2)e^2}{b}$$

$$b = \frac{1}{4\pi\varepsilon_o}\frac{2(Z-2)e^2}{Q}$$

P of α being outside nucleus is prop to (WF outside)^2/(WF inside)^2 = prob

After normalizing at boundary

$$P \approx e^{-2k(b-a)}$$

where

$$k = \frac{1}{\hbar}\sqrt{2m\tfrac{1}{2}(B-Q)}$$

Typical values
For Z=90 a~7.5 fm, B~ 34 MeV . With Q =6MeV, b~ 42 fm.

This gives P ~ 2 x 10^{-25}

if V (potential) ~ 35 MeV, and Q ~ 6 MeV, then

f = 5 * 10^{21} Hz.

So that remembering that λ = fP per second, $\lambda = 10^{-3}$ per second➔$t_{1/2}$ =700 s.

You can see the criticality of the dependence on E_α (~Q) since if we chose Q = 5 instead of 6, P➔ 10^{-30}, and $t_{1/2}$ ➔ 10^8 s. This is roughly consistent with the observations.

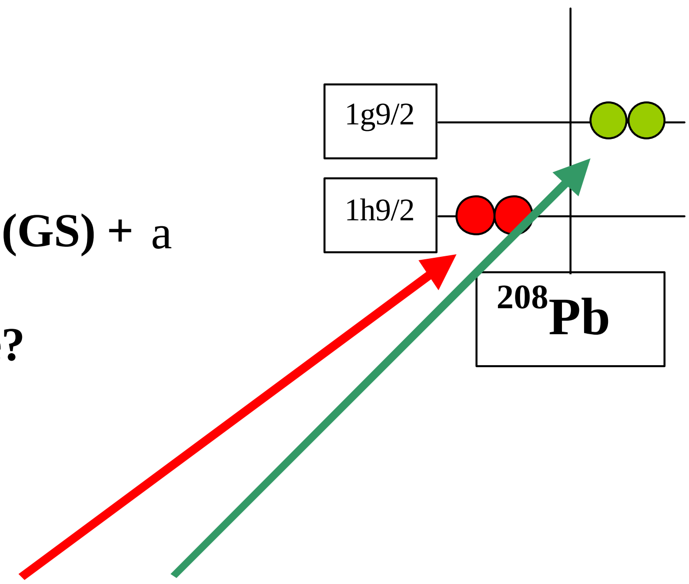
The Effect of WF

For a emission not only does the Q need to be quite high, but the overlap of the initial and final WF must be large.

$$q_G = \grave{o}_0\, y\ *_f \cdot y\ *_i$$

consider ^{212}Po ➔ ^{208}Pb(GS) + a

What does y_i look like?



$y_{I} \sim y_{(}$^{208}Pb + 2n + 2p)

these 2p and 2n in essence form an a particle since they are in a relative ℓ=0 state.

$$q_G = \grave{o}_0\, y_f \cdot y\ *_i$$

$$q_G = \grave{o}_0\, y\,(208Pb) \cdot y\ *(208Pb)(h9/2)^2(g9/2)^2 y\,(a)$$

$$= 1.\grave{o}_0\ y\,(a)(h9/2)^2(g9/2)^2$$

In fact these WF are to all intents and purposes the same. So the a decay should proceed strongly if Q is OK.

Effect of AM ℓ

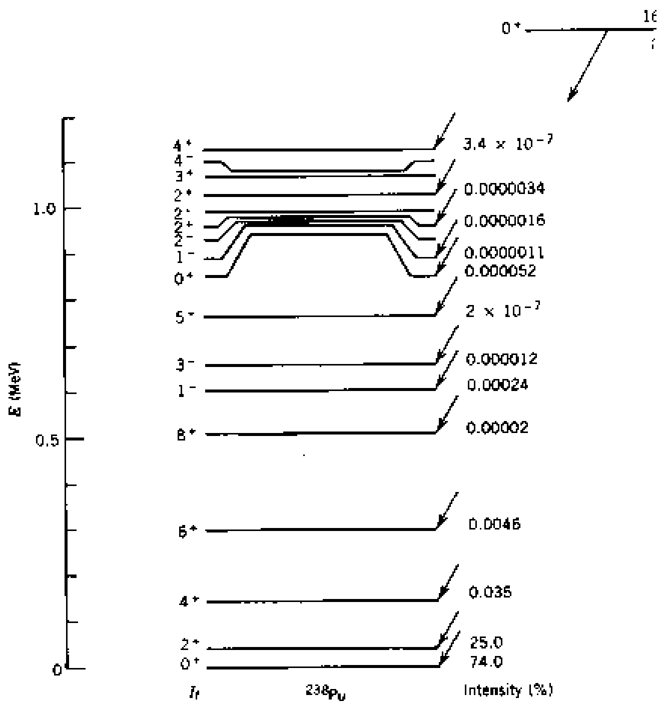


Figure 8.7 α decay of ^{242}Cm to different excited states of ^{238}Pu. T each α-decay branch is given to the right of the level.

^{242}Cm ➔ ^{238}Pu

Again the WF overlap is essentially 1 between the two GSs. However only 74% goes to GS. The rest goes to excited states. Note that Pu is a rotational nucleus. The first 5 states all have the same intrinsic WF. So overlap is same. The difference is that ^{242}Cm starts with AM = 0 and to get to 1st excited state the α must carry away 2 units of AM.
This means it must overcome a centrifugal barrier of

$$\frac{\ell(\ell+1)\hbar^2}{2mv^2}$$ (hence problem 3)

Electromagnetic decays

Two marks of a person literate in nuclear physics:

1. they say nu-clear, not nucu-lar

they know the difference between x-rays and γ-rays

What are g-rays?

Very-high-energy EM radiation
E=hf hc/λ
From nucleus (cf x-rays from atom)

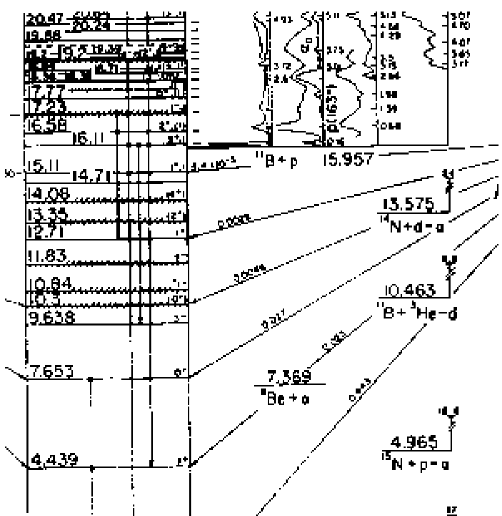
As we said earlier. if a nucleus is in an excited (not GS) state, it will, if energetically possible decay by emission of an α particle, or by β emission.

We now want to look at the competition that γ-ray decay offers to α-decay. It turns out that the decay probability for α-decay or any particle emission is much higher (half-life much shorter) than for γ-decay, so will always happen if energetically possible. Otherwise γ-decay.

What can we learn from the study of γ-decay spectra?

You will recall that in atomic physics it was the study of the spectra (visible mainly but later IR and UV) that led to the unfolding and confirmation of the shell model of the atom. The spectrum of H is the supreme example. However in more complicated atoms it was necessary to identify the spectra with specific electron configurations. This involved measuring the energy (wavelength), the decay probability (intensity) and the related half-life of each excited state.

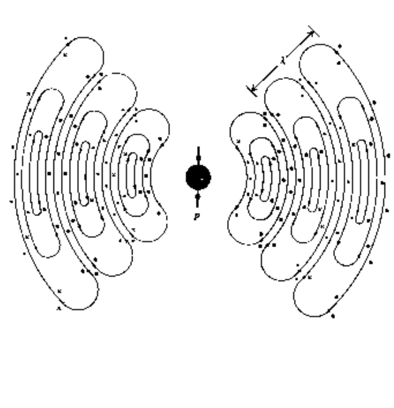
The γ-rays result from decay of a particular excited nuclear state to a lower one. The resulting spectrum, as a succession of γ-rays leading to the GS and stability, can identify the energies of the excited states in the nucleus. The energies of these γ-rays can be determined using scintillation detectors and solid state detectors as most of you used in the part 3 lab.



The relative intensities of the γ-rays in the spectrum will tell us the decay probabilities of the decay (or half-life) of a particular state to a set of final states. This is relatable to the width of the state via the Heizenberg uncertainty relationship $\Delta E \Delta t \sim \hbar$.

Having collected data on the nature of the states it is then, as in the case of atomic physics, possible to try and find out the configuration of the state. In the case of nuclei near magic numbers, this is in part realised via the shell model, and in deformed nuclei rotational states can be identified; however in general it is very difficult.

33-8 RADIATION FROM AN ANTENNA 939



33–17 Representation of the electric field (red lines) and the magnetic field (blue dots and crosses) in a plane containing an oscillating electric dipole. During one period the inner loop moves out and expands to become the outer loop. You can use Eq. (33–26) and the right-hand rule to show that the Poynting vector S is radially outward at each point within the pattern. No energy is radiated along the axis of the dipole.

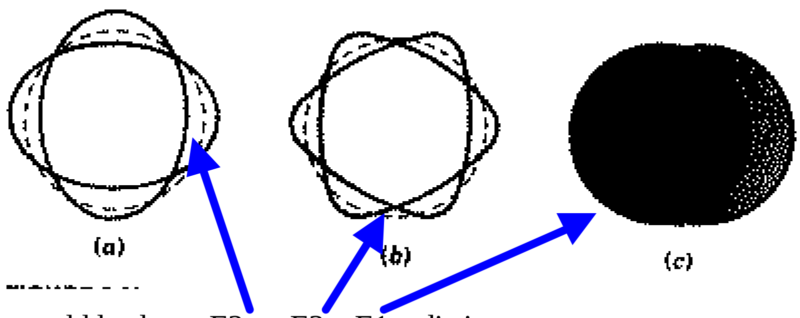
Classical Picture

EM radiation results from the acceleration of charges or the variation of currents. The simplest case is of the oscillation of an electric charge such as in a radio antenna.

This is called **electric** dipole radiation, **E1**, and it has a simple cosθ dependence.

The field set up by a changing current through a simple magnetic dipole has a similar behaviour, and is termed **M1** radiation.

More complicated charge (and current) distributions lead to higher order radiation modes, with more complicated distribution patterns. For example consider a quadrupole charge distribution.



This would lead to E2, E3, E1 radiation.

In general the radiation is specified as $E\ell$ or $M\ell$. Where ℓ specifies the order of the multipole.

NOTE that a photon ALWAYS carries AM. And it is the amount of AM that must be carries away (or absorbed) that strongly influences the probability of its emission or absorbtion.

The power radiated by a particular charge distribution is given by:

$$P(E-\ell)=\frac{2(\ell+1)c}{e\ell[(2\ell+1)!!]^2}\left(\frac{w}{c}\right)^{2\ell+2}Q_{\ell m}^2$$

$(2\ell+1)!! = 1 \times 3 \times 5 \times 7 \ldots$

Q is the moment of the electric charge distribution, e.g. dipole, quadrupole etc.

- NB the strong dependence of P on frequency.

If we now consider a classical charge distribution, the size of a nucleus, due to Z protons confined in a radius R we can make some estimate of Q_l.

$Q_1 \sim Ze\mathbf{d}$ where d is the amp of vibration

$Q_2 \sim Ze(3z^2-r^2) \sim$

@ $6.5ZeR^2(\Delta R/R)$

$< ZeR^2$

so to zero approximation $Q_l < ZeR^\ell$

Thus we could write

$$P(E-\ell)=\frac{2(\ell+1)c}{e\ell[(2\ell+1)!!]^2}(\frac{w}{c})^2(\frac{w}{c}R)^{2\ell}Z^2e^2$$

so that the radiated power is approximately proportional to

$$(\frac{w\ R}{c})^{2\ell} \quad or \ (\frac{R}{l})^{2\ell}$$

so interestingly

For medium size nucleus R~10^{-15} m
R/λ = 5 x 10^{-3}
E2/E1 power 2 x 10^{-5}

for atom
λ ~ 4000 Angst
R~0.5 angst.
R/λ = 10^{-4}
E2/E1 power 10^{-8}

So although in general the higher the multipole the less power radiated, for atoms E2 radiation is 10^8 times less likely than E1 (cf 10^5 for nuclei). So in atomic spectra, all the observable transitions give rise to E1 radiation.

Quantization

The expression for P(E-ℓ) is the power radiated. Naturally in the classical situation this diminishes with time. That is the old problem of the Bohr orbits without quantization. The orbits collapsed.

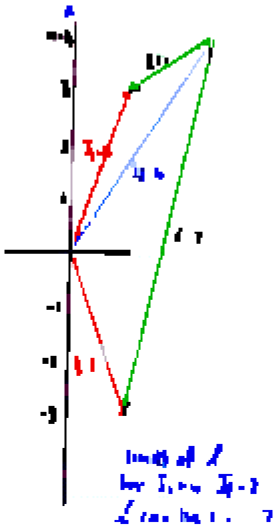
In the quantum situation the energy is emitted as a photon of energy hf or $\hbar\omega$, when the nucleus changes state from ψ_i to ψ_f. The transition probability is λ(E-ℓ) and is P(E-ℓ)/ $\hbar\omega$ (energy/sec/energy).

$$l(E-\ell)=\frac{2(\ell+1)}{\hbar e\ell[(2\ell+1)!!]^2}(\frac{w}{c})^{2\ell+1}Q^2_{\ell m}$$

For a quantum system Q_{lm} is replaced by the overlap of the initial and final state charges as defined by the wavefunctions, and is known as the <u>multipole matrix element.</u>

$$Q_{\ell m} \Rightarrow \sum_{k=1}^{Z} \int \psi_f^* \, e r_k^{\ell} \, Y_{\ell m} \, \psi_i \, d\tau$$

Each photon of order ℓ, emitted in this transition, must carry with it an AM of



$\hbar\sqrt{\ell(\ell+1)}$ with a z projection of m$\hbar$.

That is the vector difference between the AM of the initial state I_i and the final state I_f must be $\hbar\sqrt{\ell(\ell+1)}$ with the appropriate z component change Δm_j.

The consequence of this is that the allowed values of ℓ are given by

$$| I_i - I_f | \leq \ell \leq I_i + I_f$$

Note that we have not yet calculated the probability of emission of a particular ℓ-photon.

The transition probability for emission of a g-ray between states is l (E-ℓ) (energy/sec/energy) is:

multipole matrix element.

$$Q_{\ell m} \Rightarrow \sum_{k=1}^{Z} \int \psi_f^* \, e r_k^{\ell} \, Y_{\ell m} \, \psi_i \, d\tau$$

$$\lambda(E-\ell) = \frac{2(\ell+1)}{\hbar\epsilon\ell[(2\ell+1)!!]^2}\left(\frac{\omega}{c}\right)^{2\ell+1} Q_{\ell m}^2$$

For a given energy (w) 1st order calc. by Weisskopf gives relative transition probability at 1 MeV as

ℓ=1 l~1
ℓ=2 l~10^{-4}
ℓ=3 l~10^{-8}
ℓ=4 l~10^{-14}

For a given ℓ (ℓ=1) 1st order calcn. Gives the relative transition prob.,as a function of energy as:

100 keV l~1
1 MeV l~10^3
10 MeV l~10^6

Each photon of ℓ, must carry with it an AM of $\hbar\sqrt{\ell(\ell+1)}$ with a z projection of m .

Classification of g-ray transitions

For electric multipole transitions $p_i p_f = (-1)^{\ell}$.
For magnetic multipole transitions $p_i p_f = (-1)^{\ell+1}$.

Note that transition 0 → 0 cannot occur, since the photon must carry away AM.

Type	symbol	AM change	Parity change
Electric dipole	E1	1	Yes
Magnetic dipole	M1	1	No
Electric quadrupole	E2	2	No
Magnetic quadrupole	M2	2	Yes

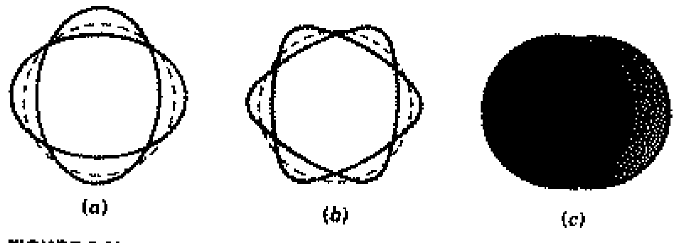
Magnetic transitions are ~10^{-2} times less likely than electric ones of the same multipolarity (ℓ)

Depending on the AM carried off by the photon there are requirements regarding the parity of the states involved.

For electric multipole transitions $\pi_i\pi_f = (-1)^{\ell}$.

For magnetic multipole transitions $\pi_i\pi_f = (-1)^{\ell+1}$.

The process of emitting a photon of quantized energy is not too difficult to imagine when we consider collective oscillations of the nucleus. The emission of a phonon of energy from an λ=1 dipole oscillation reduces the energy of the nucleus by that phonon. Similarly for quadrupole oscillations.



In a single particle model picture, we can imagine the initial and final states to involve a proton in one orbit jumping to another with the release of energy, in a similar way to the atomic shell model.

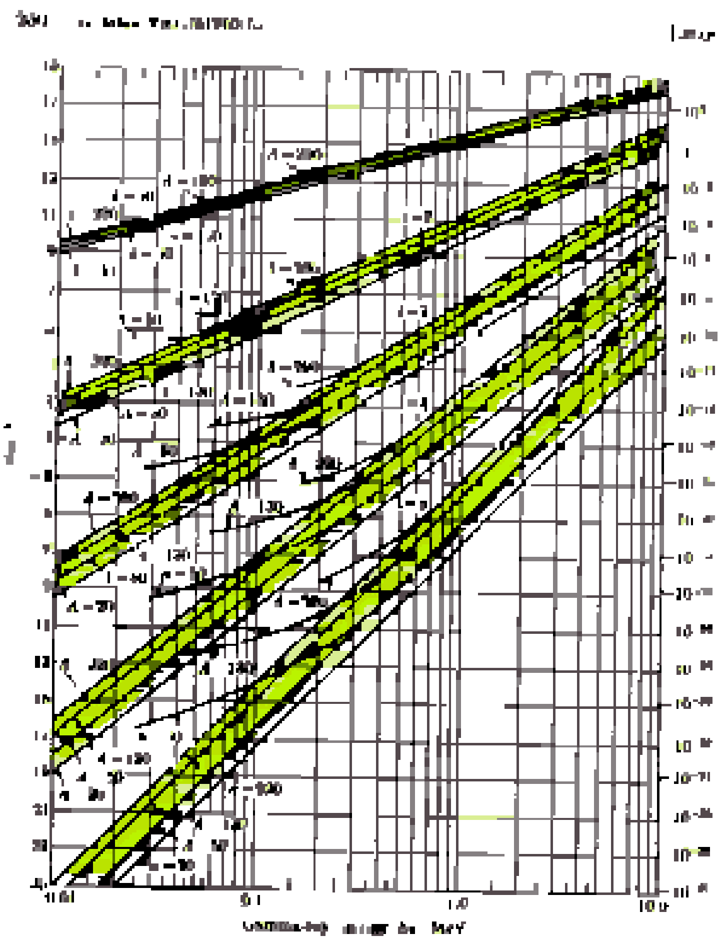


Fig. 9-13. Probability for electric multipole transition based on Weisskopf and Moszkowski's single-proton estimates. The curved lines indicate the total transition probabilities with internal conversion included. (From E. U. Condon and H. Odishaw, Handbook of Physics, New York: McGraw-Hill Book Company, 1958, p. 9-109.

Indeed it is only possible to estimate the transition probabilities under some model assumption. A fair order-of-magnitude estimate is made on the assumption that the initial wave function of the proton has AM ℓ and the final state is an s-state (ℓ=0).

On this basis

$$Q_{\ell m} \Rightarrow \frac{e}{\sqrt{4p}} \frac{3R^{\ell}}{\ell + 3}$$

and the graphs shown indicate values of λ for different electric multipole transitions for a range of nuclei

See Fig enge 9.13, 14for magnetic multipole transitions. In this case the expression is identical except smaller by a factor of R^{2}.

What do we observer from these theoretical estimates?

- First note that $T_{1/2} = \ln 2/\lambda$
  so that a high transition rate ➔ short life time

- the width of the state (how well its energy is defined) is inversely proportional to λ. (Heisenberg)

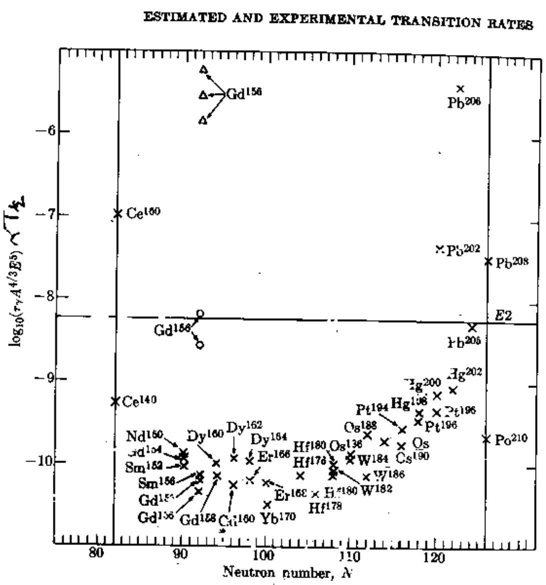
- The transition rates vary dramatically with γ-ray energy. E.g. 12 orders of magnitude for λ=5 transitions between 100 kev and 10 MeV, for E1 it is about 6 orders.

- at low energies the effect of ℓ is dramatic. 6 orders of magnitude between E1 and E2 at 100 keV. Only 2 orders at 10 MeV.

- the magnetic transitions show the same trends, but the rates are lower by about 2 orders of magnitude.

How realistic are these numbers?

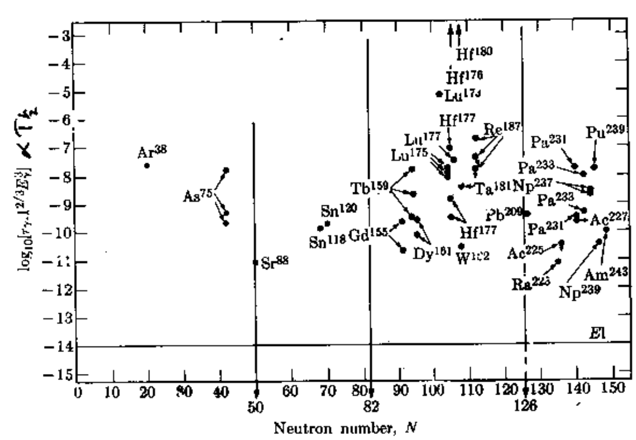
Real states live longer



In view of the approximations made it would be surprising if the predictions were correct. In general (not for E2) the calculated transition rates are significantly larger than the measured ones. (real states live longer than predicted) This is not surprising since it has been assumed that protons only are involved in the transition. In half the cases it is the neutron that moves orbit. The calculations also assume pure simple single-particle wavefunctions. This is seldom likely to be the case.

E2 transitions for deformed nuclei decay more rapidly.

Going against the trend E2 transitions for deformed nuclei have larger transition rates than estimated. This is likely to be the case since the quadrupole moments for these nuclei is much larger than assumed.



Internal Conversion

These is an interesting effect on these transition-rate graphs at low γ- energies for heavy nuclei. Note that the transition rate increases markedly about 100 keV. So that, **particularly for heavy nuclei** the rate increases by several orders of magnitude for low energies. This is the result of internal conversion.

What is it?

In this instance the energy difference between the initial and final states of the nucleus is transferred to an orbital (most likely a k-shell) electron, and this is emitted with a kinetic energy of E -BE. Thus one sees emission of electrons with very discrete sharp energies that can be related directly to energy differences of states within the nucleus.

How does this come about?

The whole process of emission of a γ-ray is the result of the interaction of charge distribution with the all-pervading EM field. So that the Proton in the higher state drops back to a lower state within this field (provided by the electric moment of the nucleus), and the energy difference is emitted as a γ-ray. However this EM field also pervades the atomic electrons even though they are much further away (4 or so orders of magnitude). So it is possible that the energy difference could be given to an atomic nucleus.

This is more likely if the orbital electrons are close to the nucleus as they are for heavy atoms (high Z and r_{atom} is prop 1/Z).

It is also more likely if the γ-decay has a small decay constant (the state has a long life), since the EM interaction probability goes up. Thus when a γ-ray transition involves a large AM transfer, the electron conversion rate increases. In fact in some instances (e.g. a 0 ➔ 0 transition) when γ-decay is forbidden electron conversion is the only decay mechanism.

The picture below shows the energy spectrum of electrons from ^{212}Pb. The smooth curves are due to β particles, and the sharp resonances are internal-conversion electrons. (Enge 9.19)

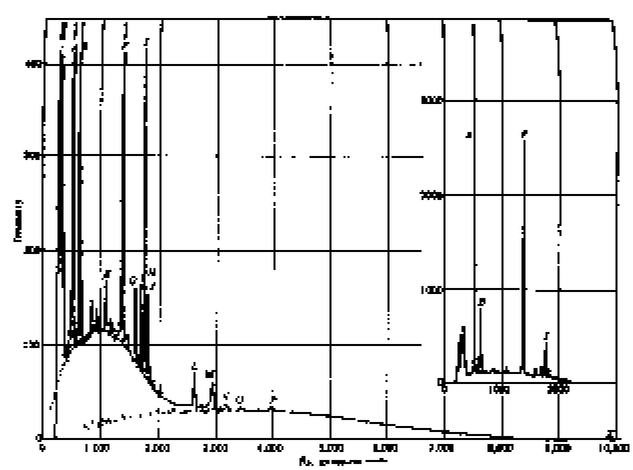


Fig. 9-19. Electron spectrum from Pb^{212} and daughter products. [From A. Flammersfeld, Z. Physik 114, 227 (1939).]