Lecture 13	**Krane**	**Enge**	**Cohen**	**Williams**
γ-decay				
Decay constants	**10.3**	**9.5**	**12.2**	
Selection rules	**10.4**	**9.6**	**12.4**	
Nuclear Reactions	**Ch 11**	**Ch 13**	**Ch 13**	**7.1/2 7.5**

Problems Lecture 13

1 In a semi-clasical picture an ℓ=0 α-particle is emitted along a line that passes through the centre of the nucleus..

(a) How far from the centre of the nucleus must an ℓ=2 particle be emitted? Assume Q = 6 MeV and A = 230.

(b) What would be the recoil rotational energy if all the recoil went into rotation of the daughter nucleus.

Review Lecture 12

1 α-decay is the way heavy unstable nuclei (A>150) move towards the stability line.
$^{A}_{Z}x_N \rightarrow ^{A-4}_{Z-2}x'_{N-2} + ^{4}_{2}\alpha_2$ Energy considerations give $Q = (m_x - m_{x'} - m_\alpha)c^2$

2 For α-decay to be strong we must have:

- Q positive
- Penetration of the Coulomb barrier (~20+ MeV).
  Where (WF outside)^2/(WF inside)^2 = prob of α being outside nucleus
  $P \approx e^{-2\kappa a}$ where $\kappa = \frac{1}{\hbar}\sqrt{2m(V-Q}$ and a is the barrier width.
- the overlap of the initial and final WF must be large. $\theta_G = \int \psi^*{}_f . \psi^*{}_i$
- The AM (ℓ) required to be carried by the α to couple the GS spin of the parent to that of the residual state of the daughter should be 0 or small.

Electromagnetic decays

Two marks of a person literate in nuclear physics:

1. they say nu-clear, not nucu-lar

they know the difference between x-rays and γ-rays

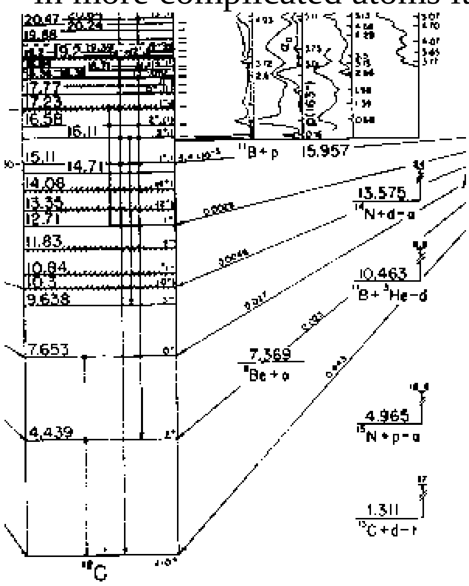
What are γ-rays?

Very-high-energy EM radiation
E=hf hc/λ
From nucleus (cf x-rays from atom)

As we said earlier. if a nucleus is in an excited (not GS) state, it will, if energetically possible decay by emission of an α particle, or by β emission.

We now want to look at the competition that γ-ray decay offers to α-decay. It turns out **that the decay probability for α-decay or any particle emission is much higher (half-life much shorter) than for γ-decay**, so will always happen if energetically possible. Otherwise γ-decay.

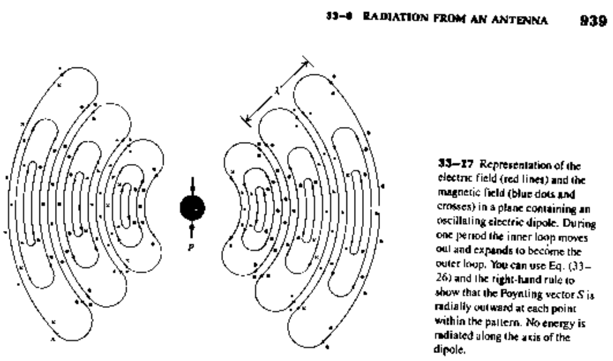
What can we learn from the study of γ-decay spectra?
You will recall that in atomic physics it was the study of the spectra (visible mainly but later IR and UV) that led to the unfolding and confirmation of the shell model of the atom. The spectrum of H is the supreme example. However in more complicated atoms it was necessary to identify the spectra with specific electron configurations. This involved measuring the energy (wavelength), the decay probability (intensity) and the related half-life of each excited state.



In the nucleus, γ-rays result from decay of a particular excited nuclear state to a lower one. The resulting spectrum, a succession of γ-rays leading to the GS and stability, can identify the energies of the excited states in the nucleus. The energies of these γ-rays can be determined using scintillation detectors and solid state detectors as most of you used in the part 3 lab.

The relative intensities of the γ-rays in the spectrum will tell us the decay probabilities of the decay (or half-life) of a particular state to a set of final states. This is relatable to the width of the state via the Heizenberg uncertainty relationship $\Delta E \Delta t \sim \hbar$.

Having collected data on the nature of the states it is then, as in the case of atomic physics, possible to try and find out the configuration of the state. In the case of nuclei near magic numbers, this is in part realised via the shell model, and in deformed nuclei rotational states can be identified; however in general it is very difficult.



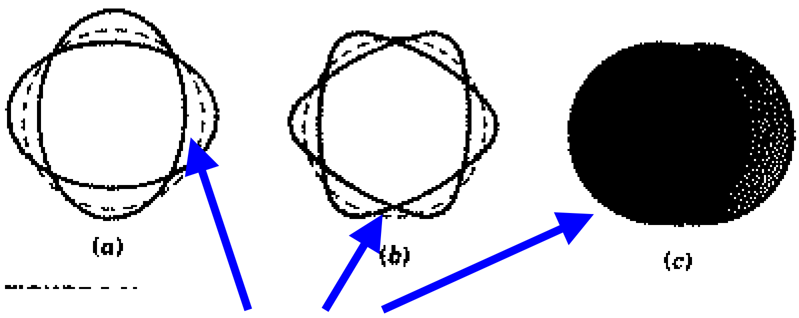
Classical Picture

EM radiation results from the acceleration of charges or the variation of currents. The simplest case is of the oscillation of an electric charge such as in a radio antenna.

This is called **electric** dipole radiation, **E1**, and it has a simple cosθ dependence.

The field set up by a changing current through a simple magnetic dipole has a similar behaviour, and is termed **M1** radiation.

More complicated charge (and current) distributions lead to higher order radiation modes, with more complicated distribution patterns. For example consider a quadrupole charge distribution.



This would lead to E2, E3, E1 radiation.

In general the radiation is specified as Eℓ or Mℓ. Where ℓ specifies the order of the multipole.

NOTE that a photon ALWAYS carries AM. And it is the amount of AM that must be carries away (or absorbed) that strongly influences the probability of its emission or absorption.

The power radiated by a particular charge distribution is given by:

$$P(E-\lambda) = \frac{2(\lambda+1)c}{\varepsilon\lambda[(2\lambda+1)!!]^2}\left(\frac{\omega}{c}\right)^{2\lambda+2} Q_{\lambda m}^2$$

$(2\lambda+1)!! = 1 \times 3 \times 5 \times 7 \ldots$

Q is the moment of the electric charge distribution, e.g. dipole, quadrupole etc.

- NB the strong dependence of P on frequency.

If we now consider a classical charge distribution, the size of a nucleus, due to Z protons confined in a radius R we can make some estimate of Q_ℓ.

$Q_1 \sim Ze\mathbf{d}$ where d is the amp of vibration

$Q_2 \sim Ze(3z^2-r^2) \sim$
$\cong 6.5ZeR^2(\Delta R/R)$
$< ZeR^2$

so to zero approximation $Q_\ell < ZeR^\ell$

Thus we could write

$$P(E-\ell) = \frac{2(\ell+1)c}{\varepsilon\ell[(2\ell+1)!!]^2}\left(\frac{\omega}{c}\right)^2\left(\frac{\omega}{c}R\right)^{2\ell} Z^2 e^2$$

so that the radiated power is approximately proportional to

$$\left(\frac{\omega R}{c}\right)^{2\ell} \quad or \quad \left(\frac{R}{\lambda}\right)^{2\ell}$$

so interestingly for medium size nucleus $R \sim 10^{-15}$ m, and emission of say, a 1-MeV γ-ray

$R/\lambda = 5 \times 10^{-3}$
E2/E1 power 2×10^{-5}

for atom $\lambda \sim 4000$ Angst
$R \sim 0.5$ angst.

$R/\lambda = 10^{-4}$

E2/E1 power 10^{-8}

So although in general the higher the multipole the less power radiated, for atoms E2 radiation is 10^8 times less likely than E1 (cf 10^5 for nuclei). So in atomic spectra, all the observable transitions give rise to E1 radiation.

Quantization

The expression for P(E-ℓ) is the power radiated. Naturally in the classical situation this diminishes with time. That is the old problem of the Bohr orbits without quantization. The orbits collapsed.

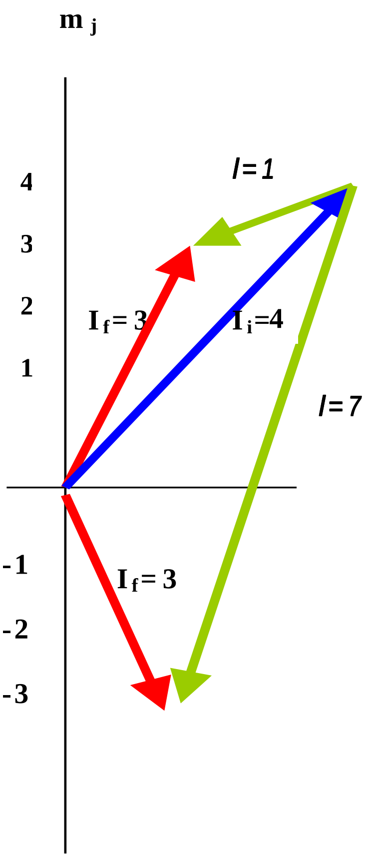
In the quantum situation the energy is emitted as a photon of energy hf or $\hbar\omega$, when the nucleus changes state from ψ_i to ψ_f. The transition probability is λ(E-ℓ) and is

P(E-ℓ)/ $\hbar\omega$ (energy/sec/energy).

$$\lambda(E-\ell) = \frac{2(\ell+1)}{\hbar\varepsilon\ell[(2\ell+1)!!]^2}\left(\frac{\omega}{c}\right)^{2\ell+1} Q_{\ell m}^2$$

For a quantum system $Q_{\ell m}$ is replaced by the overlap of the initial and final state charges as defined by the wavefunctions, and is known as the <u>multipole matrix element.</u>

$$Q_{\ell m} \Rightarrow \sum_{k=1}^{Z} \int \psi_f^* \, e r_k^{\ell} \, Y_{\ell m} \, \psi_i \, d\tau$$



Each photon of order ℓ, emitted in this transition, must carry with it an AM of $\hbar\sqrt{\ell(\ell+1)}$ with a z projection of $m\hbar$.

That is the vector difference between the AM of the initial state I_i and the final state I_f must be $\hbar\sqrt{\ell(\ell+1)}$ with the appropriate z component change Δm_j.

The consequence of this is that the allowed values of ℓ are given by

$$|\mathbf{I_i - I_f}| \leq \ell \leq \mathbf{I_i + I_f}$$

The transition probability for emission of a γ-ray between states is λ(E-ℓ) (energy/sec/energy) is:

multipole matrix element.

$$Q_{\ell m} \Rightarrow \sum_{k=1}^{Z} \int \psi_f^* e r_k^{\ell} Y_{\ell m} \psi_i \, d\tau$$

$$\lambda(E-\ell) = \boxed{\frac{2(\ell+1)}{\hbar \varepsilon \ell [(2\ell+1)!!]^2}} \boxed{\left(\frac{\omega}{c}\right)^{2\ell+1}} Q_{\ell m}^2$$

For a given energy (ω) 1st order calc. by Weisskopf gives relative transition probability at 1 MeV as
ℓ=1 λ~1
ℓ=2 λ~10^{-4}
ℓ=3 λ~10^{-8}
ℓ=4 λ~10^{-14}

For a given ℓ (ℓ=1) 1st order calcn. Gives the relative transition prob.,as a function of energy as:

100 keV λ~1
1 MeV λ~10^{3}
10 MeV λ~10^{6}

Each photon of ℓ, must carry with it an AM of $\hbar\sqrt{\ell(\ell+1)}$ with a z projection of $m\hbar$.

Classification of γ-ray transitions

For electric multipole transitions $\pi_i\pi_f = (-1)^{\ell}$.
For magnetic multipole transitions $\pi_i\pi_f = (-1)^{\ell+1}$.

Note that transition 0 ➔0 cannot occur, since the photon must carry away AM.

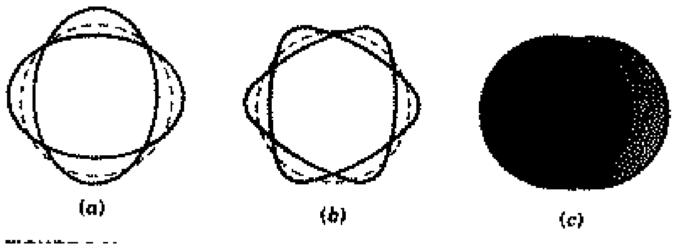
Type	symbol	AM change	Parity change
Electric dipole	E1	1	Yes
Magnetic dipole	M1	1	No
Electric quadrupole	E2	2	No
Magnetic quadrupole	M2	2	Yes

Magnetic transitions are ~10^{-2} times less likely than electric ones of the same multipolarity (ℓ)

Depending on the AM carried off by the photon there are requirements regarding the parity of the states involved.
For electric multipole transitions $\pi_i\pi_f = (-1)^{\ell}$.
For magnetic multipole transitions $\pi_i\pi_f = (-1)^{\ell+1}$.

Note that we have not yet calculated the probability of emission of a particular ℓ-photon.

The process of emitting a photon of quantized energy is not too difficult to imagine when we consider collective oscillations of the nucleus. The emission of a phonon of energy from an λ=1 dipole oscillation reduces the energy of the nucleus by that phonon. Similarly for quadrupole oscillations.



In a single particle model picture, we can imagine the initial and final states to involve a proton in one orbit jumping to another with the release of energy, in a similar way to the atomic shell model.
Indeed it is only possible to estimate the transition probabilities under some model assumption. A fair **order-of-magnitude estimate** is made on the assumption that the **initial wave function of the proton has AM ℓ and the final state is an s-state (ℓ=0).**

On this basis $Q_{\ell m} \Rightarrow \frac{e}{\sqrt{4\pi}} \frac{3R^{\ell}}{\ell + 3}$ and the graphs shown indicate values of λ for different electric multipole transitions for a range of nuclei

See Fig enge 9.13, 14for magnetic multipole transitions.
In this case the expression is identical except smaller by a factor of R^{2}.

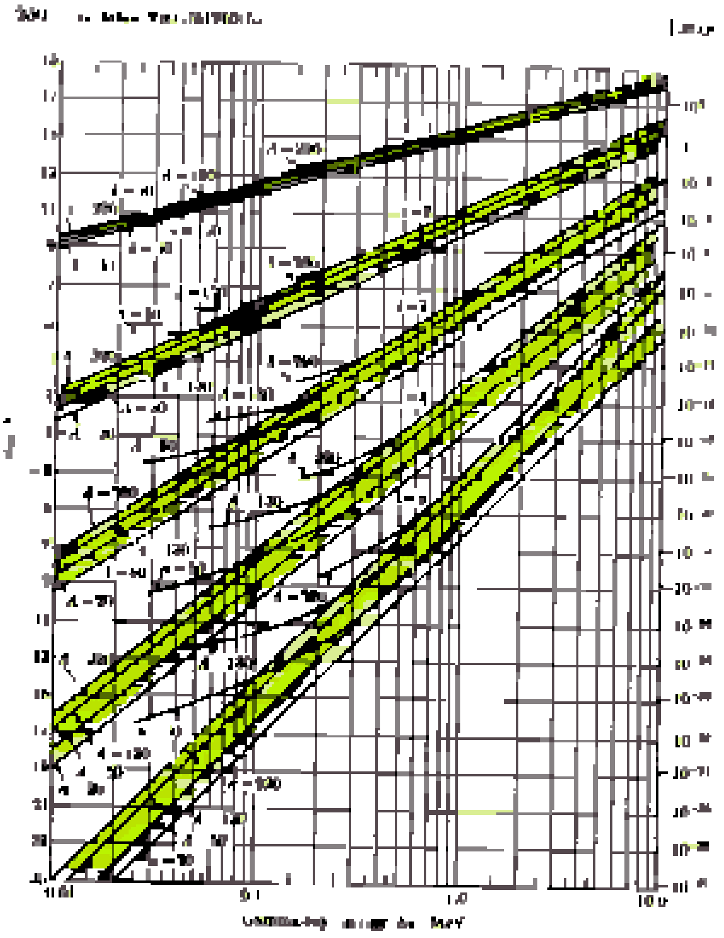
What do we observer from these theoretical estimates?

- First note that $T_{1/2} = \ln 2/\lambda$
  so that a high transition rate ➔ short life time

- the width of the state (how well its energy is defined) is inversely proportional to λ. (Heisenberg)

- The transition rates vary dramatically with γ-ray energy. E.g. 12 orders of magnitude for λ=5 transitions between 100 kev and 10 MeV, for E1 it is about 6 orders.

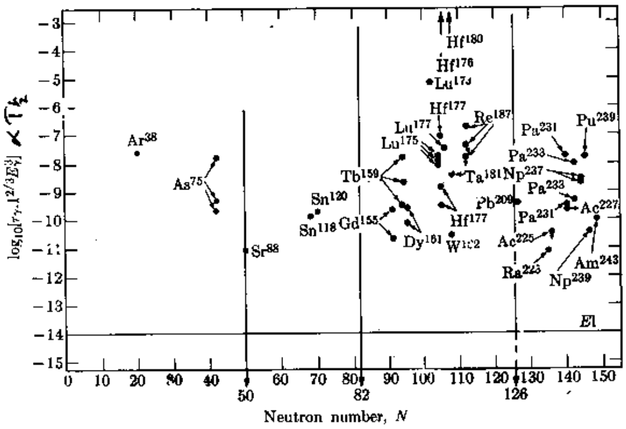
- at low energies the effect of ℓ is dramatic. 6 orders of magnitude between E1 and E2 at 100 keV. Only 2 orders at 10 MeV.



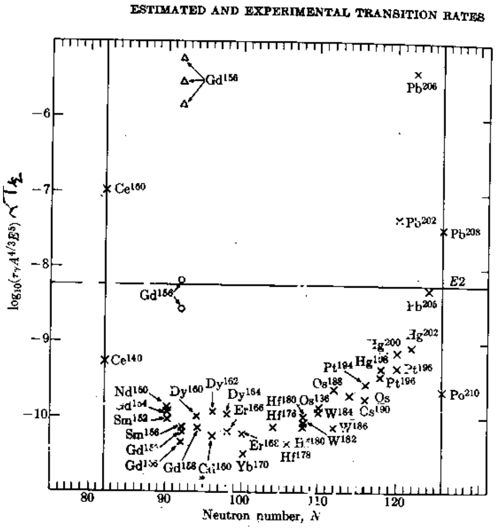
- the magnetic transitions show the same trends, but the rates are lower by about 2 orders of magnitude.

How realistic are these numbers?

Real states live longer



In view of the approximations made it would be surprising if the predictions were correct. In general (not for E2) the calculated transition rates are significantly larger than the measured ones. (real states live longer than predicted) This is not surprising since it has been assumed that protons only are involved in the transition. In half the cases it is the neutron that moves orbit. The calculations also assume pure simple single-particle wavefunctions. This is seldom likely to be the case.



E2 transitions for deformed nuclei decay more rapidly.

Going against the trend E2 transitions for deformed nuclei have larger transition rates than estimated. This is likely to be the case since the quadrupole moments for these nuclei is much larger than assumed.

Internal Conversion

These is an interesting effect on these transition-rate graphs at low γ- energies for heavy nuclei. Note that the transition rate increases markedly about 100 keV. So that, **particularly for heavy nuclei** the rate increases by several orders of magnitude for low energies. This is the result of internal conversion.

What is it?

In this instance the energy difference between the initial and final states of the nucleus is transferred to an orbital (most likely a k-shell) electron, and this is emitted with a kinetic energy of E -BE. Thus one sees emission of electrons with very discrete sharp energies that can be related directly to energy differences of states within the nucleus.

How does this come about?
The whole process of emission of a γ-ray is the result of the interaction of charge distribution with the all-pervading EM field. So that the Proton in the higher state drops back to a lower state within this field (provided by the electric moment of the nucleus), and the energy difference is emitted as a γ-ray. However this EM field also pervades the atomic electrons even though they are much further away (4 or so orders of magnitude). So it is possible that the energy difference could be given to an atomic nucleus.

This is more likely if the orbital electrons are close to the nucleus as they are for heavy atoms (high Z and r_{atom} is prop 1/Z).

It is also more likely if the γ-decay has a small decay constant (the state has a long life), since the EM interaction probability goes up. Thus when a γ-ray transition involves a large AM transfer, the electron conversion rate increases. In fact in some instances (e.g. a 0 ➔ 0 transition) when γ-decay is forbidden electron conversion is the only decay mechanism.

The picture below shows the energy spectrum of electrons from ^{212}Pb. The smooth curves are due to β particles, and the sharp resonances are internal-conversion electrons. (Enge 9.19)

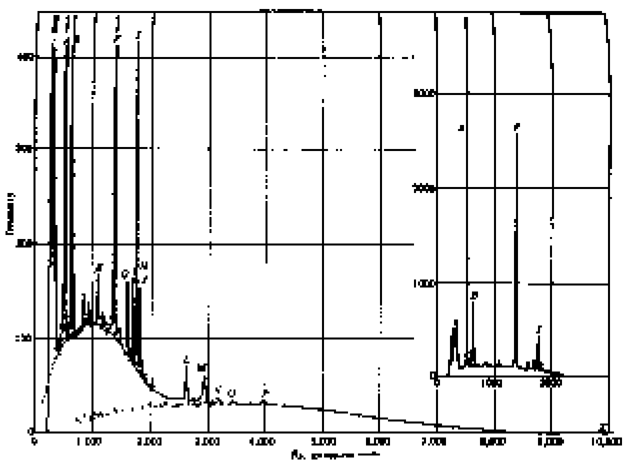


Fig. 9-19. Electron spectrum from Pb^{212} and daughter products. [From A. Flammersfeld, Z. Physik 114, 227 (1939).]

Nuclear Reactions

In the mediaeval times, the alchemists had hoped to turn lead into gold. The chemists have never succeeded. However we nuclear physicists can do that. 23% of lead is $^{205}Pb_{82}$ and gold is $^{197}Au_{79}$.

	Z	N
Pb	82	123
Au	79	118

All we need do is remove 3 protons and 5 neutrons. So a reaction like

$^{205}Pb_{82} + {^1p_1} \rightarrow {^{197}Au_{79}} + {^4\alpha_2} + {^4\alpha_2} + {^1n}$ would do it (not very well).

However in nuclear reaction transformations of the innate nature of the nucleus occur.

<u>Nuclear reactions are an essential part of the study of the nucleus, and are designed to reveal the spins, parities, energies of the nuclear states. Knowing these one can try to model the nucleus and hence refine our understanding of the basic N-N force between the nucleons.</u>

The general form of a nuclear reaction is

$$\underset{\text{target}}{X} + \underset{\text{projectile}}{a} \rightarrow \underset{\text{product}}{Y} + \underset{\text{ejectile}}{b}$$

or X(a,b)Y

The projectile may be a particle (p, n, α), a photon (photonuclear reaction) or even another nucleus (heavy ion reaction).

The **ejectile** may be a particle, a photon, or again a nucleus.