Lecture 15	Krane	Enge	Cohen	Williams
Reaction theories				
compound nucleus	**11.10**	**13.7**	**13.1-3**	
direct reactions	**11.11**	**13.11/12**	**ch 14**	
Admixed Wave functions				
residual interaction			**5.1-4**	
Admixed Wave functions				
residual interaction			**5.1-4**	

Problems Lecture 15

1 The (d,p) reaction on ^{52}Cr leads to the $3/2^-$ GS of ^{53}Cr. How would the analysis of the AM transfer in this reaction differ between an analysis in terms of direct reactions and one in terms of compound-nucleus reactions?

2 Give the compound nucleus resulting from 6-MeV protons bombarding a target of ^{11}B, and indicate all the possible decay channels. The level diagram is available from my notes in the part 3 reading room or on the WEB.

3 In the (d,p) reaction leading to states in ^{91}Zr (figs 11.23 and 11.24 Krane, and in the notes of this lecture) discuss the method of assignment of the I^π. Could this be done if the reaction proceeded via a CN process?

4 Estimate using semi-classical stripping theory, the angles at which the (d,p) cross section has its maximum for ℓ=0, 1, 2, 3. Use R= 5 fm and a deuteron energy of 10 MeV. The proton energy is 14 MeV .

Review Lecture 14

1 The general form of a nuclear reaction is

X	+	a	→	Y	+	b
target		projectile		product		ejectile

or X(a,b)Y

2 General rules for nuclear Reactions

- Mass/energy is conserved

$m_Xc^2 + T_X + m_ac^2 + T_a = m_yc^2 + T_y + m_bc^2 + T_b$

$Q = [(m_X + m_a) - (m_Y + m_b)]\, c^2$

or $Q = T_{final} - T_{initial}$

if Q is negative the reaction is endoergic, Initial KE becomes mass or BE

if Q is positive it is exoergic. Binding energy released as KE of products

- Charge is conserved
- Baryon number is conserved. The number of p+n is conserved
- Linear momentum is conserved
- AM is conserved
- Parity is conserved

3 What is the object of performing nuclear reactions?

Ultimately it is to observe the quantum states of a nucleus, that is to define the AM, energy and parity of the quantum states that exist in a given nucleus. From this information one can use models to work backwards to better define the N-N potential, and particularly how these nucleons might be effected by being in close proximity with other nucleons.

4 Tools: Accelerators, detectors, Ref. Part3 Nuclear Lab.
The measurements will generally be made by bombarding the target nucleus with a projectile (say a proton, neutron, α or γ-ray) of known energy, and observing the various parameters of the ejectile (say its energy, and angular distribution).

5 What can be measured in a nuclear reaction, and what we might learn.

6 What are some of the types of nuclear reactions

Lecture 15

Reaction Mechanisms

Types of nuclear reactions		
1. **Elastic scattering**	$X + x \rightarrow X + x$	
no change in reacting particles or energy		
2. 2 **Inelastic scattering**	$X + x \rightarrow X^* + x + Q$ (Q is negative)	
3. **Radiative capture**	$X + x \rightarrow Y + \gamma + Q$	$X(x,\gamma)Y$
Q is positive and is the BE of x in Y		
4. **General Reaction**	$X + x \rightarrow Y + y + Q$	$X(x,y)Y$
Q is negative being the threshold energy		
5. **Photonuclear Reaction**	$X + \gamma \rightarrow Y + y + Q$	$X(\gamma,y)Y$

Now all of these reactions may occur when we throw x onto X. Their reaction probability depends on the energetics of the system, and on the spins and parities of the available states.

$$\sigma_T = \sigma_{x,x} + \sigma_{x,x'} + \sigma_{x,\gamma} + \sigma_{x,y} +$$
$$= \sigma_{inelastix} + \sigma_{reaction}$$

Lets consider the situation of the reaction $^{39}K + d$

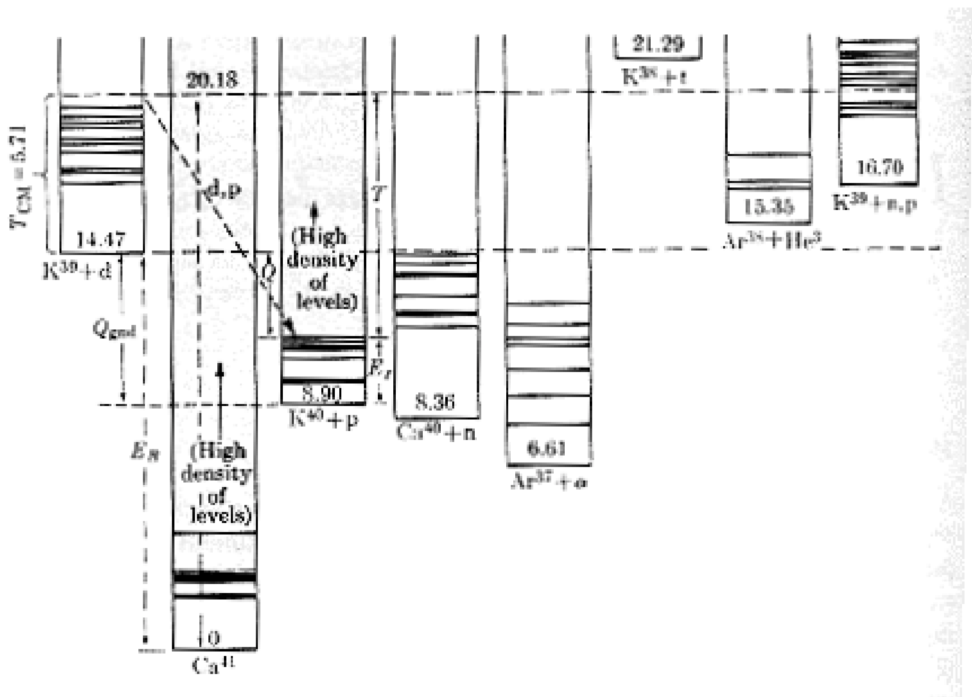


Fig. 13–1. Energy diagram of nucleus plus particle systems with 20 protons and 21 neutrons.

The system contains 39 + 2 nucleons. So the diagram shows the energy of all the likely combinations of these 41 nucleons. The most stable combination is the nucleus of ^{41}Ca . The deuteron is provided with 6 MeV by an accelerator, and we can indicate this on the energy diagram.

We need to be a bit careful, since the energy scale here is in the CM system, and of course the 6 MeV of the d is in the lab. system.

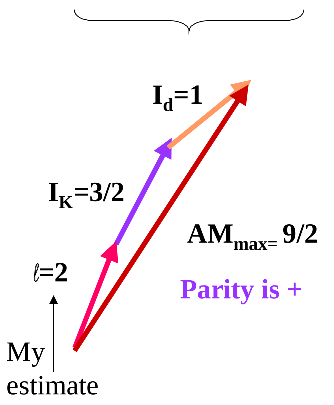
$$E_{cm} = \frac{E_{lab}}{1+\frac{m}{M}} = \qquad 5.71 \text{ MeV for } KE_{d(lab.)} = 6 \text{ MeV}$$

Go through figure discussing the possible and impossible reactions.

What determines which states might be populated?
The AM and the parity!   These must be conserved.

^{39}K   + ^{2}H   ==> ^{40}K * + ^{1}H

I^{π} 3/2^{+} 1^{+} ?   1/2



I_{K*}= ?

S_p=1/2

ℓ_p'= ?   Parity $(-1)^{\ell Æ}$

AM must be conserved

Parity must be conserved

So **s + ℓ + I_{K*} <= 9/2**

I_{K*} <= 9/2 - (s + ℓ)

I of ^{39}K is 3/2^{+}
I of d is 1^{+}

These can couple to give a total AM (called channel spin) ranging from ½ to 5/2.

The deuteron will also have an AM relative to the target. The value of ℓ will be from 0 upward**. There will be a limit, and this can be got from a classical consideration** by considering a d with KE incident at edge of nucleus.

$$AM = (2ME)^{1/2} R$$

(I've done the calc and get a value for ℓ_{max} ~2 for 6 MeV deuterons.. you check it)

On this basis the max AM in the entrance channel is 9/2. This must be conserved in the outward channel.

If the outgoing particle is a proton s= ½, and it can have ℓ so that the of the states that can be populated in ^{40}K can range so that **s +** ℓ **+ I_{final} <= 9/2**

Remember that parity is $(-1)^{\ell}$, so the parity of the available states can be determined.

$I_d=1^+$, $I_K=3/2^+$

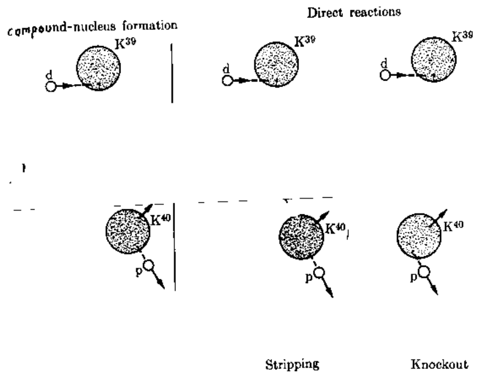
	Values of I_{total} for different l_D			Values of I_{final} for different l_p	
I_d+I_K	$l_D=0$	$l_D=1$	$l_D=2$	$l_p=0$	$l_p=1$
$½^+$	$½^+$	**$½^-$. $3/2^-$**		**$0^-,1^-,2^-$**	
$3/2^+$	**$3/2^+$**	**$½^-,3/2^-,5/2^-$**		**$0^-,1^-,2^-,3^-$**	
$5/2^+$	$5/2^+$	**$3/2^-,5/2^-,7/2^-$**	**$3/2^+,5/2^+,7/2^+,9/2^+$**	**$1^-,2^-,3^-,4^-$**	**$0^+,1^+,2^+,3^+,4^+,5^+$**

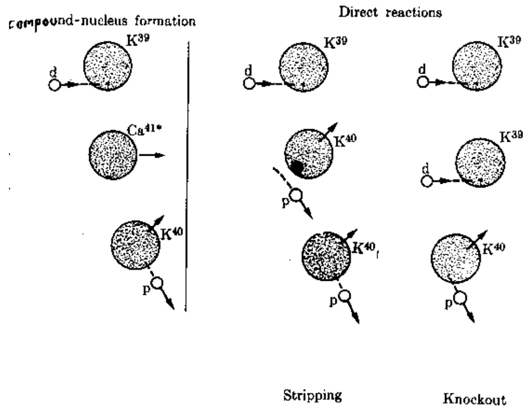
Reaction Models

The question needs to be asked "how can we predict or calculate the probability of any particular reaction occurring?"

Ideally we need to know the microscopic details of the target nucleus and do a detailed QM calculation. In practice this is impossible, since we can hardly do the calculation for a 3-body nucleus. **What is needed is a reaction model that might explain the processes involved between the initial situation (which we know) and the final situation (which we observe).**

Listed in the vugraph are 3 possible reaction mechanisms again illustrated with the reaction ^{19}K(d,p)





The results of the reaction are the same as shown in the left-hand picture, and they can be explained in a number of different ways, i.e. via a number of different reaction mechanisms, as revealed in the right-hand picture.

The first is the **compound nucleus** reaction. In this the deuteron enters the ^{39}K, and by a series of collisions, shares its energy with the nucleons in the target nucleus. This is a statistical process, and it will happen that at some stage one particular nucleon or nucleon group will gain sufficient energy to overcome the threshold (exceed the BE). This process takes time; the CN existing for a period of 10^{-15}- 10^{-13} sec. This is much longer than the transit time of the projectile across the nucleus (10^{-21} sec)

The life of the CN is so long (10^{-13} s) that its decay is independent of its mode of decay. Thus the CN reaction mechanism involves two steps:

FORMATION
DECAY

The decay mode is independent of the way it was created.

To fully understand this mechanism we need to consider the probability of creation, and then the probability of decay.

The other opposed mechanism is termed **direct**.

In one form, a **stripping reaction**, the deuteron passes close to the ^{39}K, and the neutron is stripped off by virtue of the nuclear potential. This leaves the proton to continue on as the observed ejectile.

The other direct reaction that is illustrated is the **knock-out** reaction. The deuteron collides with a proton near the surface of the target nucleus and ejects it. The deuteron remains in the nucleus.

For a direct reaction the interaction time is of the order of the transit time (10^{-21} sec)…no compound nucleus is formed.

Fortunately these reaction mechanisms can be distinguished by the magnitude of the cross section, but most importantly by the angular dependence of this cross section. The direct reaction ADs are particularly angle dependent.

Compound Nucleus Model

If an incident projectile x approaches a nucleus with an imact parameter less than the radius of the target nucleus X, it has a finite probability of entering the nucleus and interacting with a nucleon inside. This collision will impart energy and momentum to the hit nucleon, and it in turn will collide with other nucleons. In fact the process is statistical, and the energies of the nucleons in this compound nucleus will have a statistical distribution, like a boiling liquid.

$$X + x \rightarrow C^* \rightarrow Y + y$$

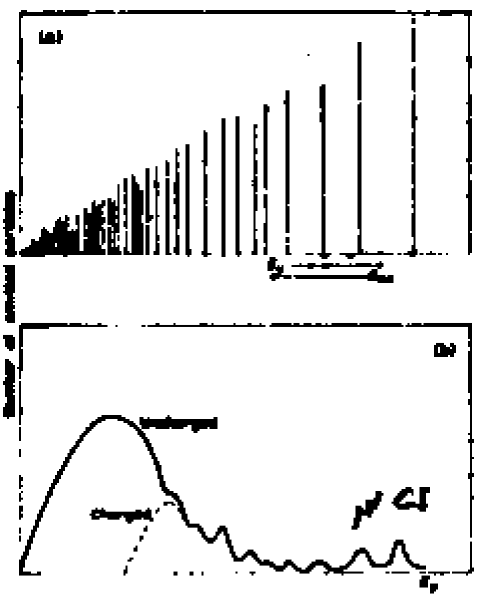
e.g. $\quad {}^{39}K + d \rightarrow {}^{41}Ca^* \rightarrow {}^{40}Ca + n$

Although in general any one nucleon will not have sufficient energy to escape, there is a probability that one will have an energy that exceeds the BE and be emitted.

When this happens the spectrum of emitted particles will reflect this.

Now discuss the compound nucleus of ${}^{41}Ca$. And its decay

(d,n)   discuss neutron spectrum



The spectrum of neutrons is consistent with a Maxwellian distribution. Most neutrons have little energy and are emitted to high-lying states. A few leave the residual nucleus in the GS and other low-lying states

(d,p)   discuss coulomb barrier effect on proton spectrum. The spectrum of protons has fewer low-energy protons emitted because of the additional coulomb barrier.

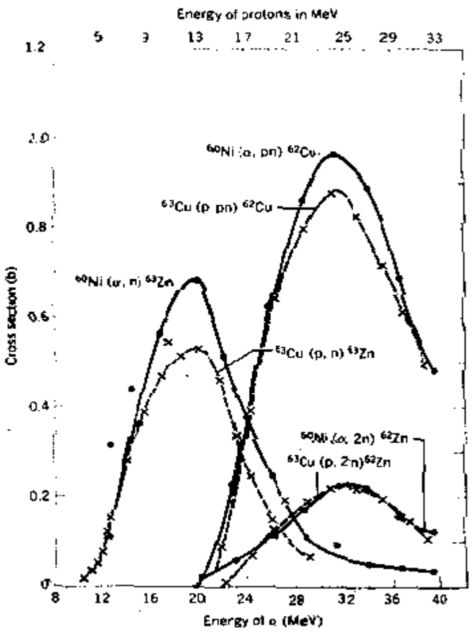


Figure 11.19 *Cross sections for different reactions leading to the compound nucleus ^{64}Zn show very similar characteristics, consistent with the basic assumptions of the compound nucleus model. From S. N. Goshal, Phys. Rev.* **80**, 939 (1950).

As the incident energy increases in a reaction such as (α,n) (see Krane ch 11.10) the higher in α energy, ther higher the Average energy of the nucleons in the CN, and hence the chance of multiple neutron emission.
Discuss multi nucleon emission

In such a model, the time to set up this equilibrium condition is relatively long (10^{-15}- 10^{-13} sec). So as mentioned earlier the decay of the compound nucleus is essentially independent of how it was formed. A good example of this is shown in Krane for the compound nucleus of ^{64}Zn.

The figure shows that whether the CN of ^{64}Zn is made by firing α on ^{60}Ni, or protons on to ^{63}Cu, the cross section for decay by emission of p + n to ^{62}Cu, or 2n to ^{62}Zn, or n to ^{63}Zn are essentially the same.

Expected AD

If a nuclear reaction involves the formation of a CN, one might imagine that the emitted particles would have an equal probability of leaving in any direction. That is the AD of the cross section would be isotropic.

To first order this is so. However there is an angular dependence, which becomes evident if large AM is brought in was when a heavy ion is the projectile. It derives from the fact that the incoming particle x, brings with it an amount of AM depending on its value of ℓ. This of course has to be conserved, so the CN will begin to spin. The axis of spin will be different for each incident particle, however it will always be perpendicular to the direction of incidence of the projectile. The presence of a centrifugal force will mean that it is easier for particles to be emitted in a direction perpendicular to the spin axis. Since the spin axis is perpendicular to the direction of incidence, fewer particles are emitted in a direction 90^{o} to the beam. Thus the AD has a slight **minimum** at 90^{o}.

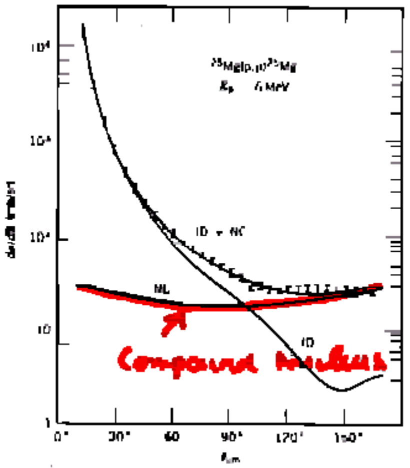


Figure 11.20 The curve marked NC shows the contribution from compound nucleus formation to the cross section of the reaction $^{25}Mg(p, p)^{25}Mg$. The curve marked ID shows the contribution from direct reactions. Note that the direct part has a strong angular dependence, while the compound nucleus part shows little angular dependence. From A. Gallmann et al., *Nucl. Phys.* **88**, 654 (1966)

Already we see in this figure that other reactions modes exist, in particular a direct reaction process (en francais Interaction Direct). Let's see when and why this becomes important.

For a nucleon, say a neutron of energy < 8 MeV, the deBroglie wavelength is of the order of 5 fm; importantly about the size of a medium size nucleus. It is thus not surprising that for reactions using incident particles of this energy, the particle interacts with the whole nucleus (simplistically the wave envelops the nucleus), and a CN is the result.

At higher energies, the wavelength becomes smaller; of the size of a nucleon, and the probability increases that the projectile will **interact with a nucleon**, or nucleon bundle, **at the surface** of the nucleus. This reaction type is called **Direct.**

The direct reaction mechanism

In a direct reaction mechanism, we postulate that the incident projectile interacts with (usually) one of the nucleons in the nucleus.

- It may be, as we showed for ^{41}Ca, a deuteron that lost its n in the interaction and came out as a proton : **Stripping**

- It may be that the deuteron impinged on a proton and knocked it out while the d remains in the nucleus **Knockout reaction**

- It might be in another case that an incident proton interacts with a neutron which it picks up, and they form a deuteron. **Pickup Reaction.**

In any such case

- no CN is formed
- the interaction time is of order the transit time (10^{-21} sec)
- momentum and AM are conserved
- the angular distribution will be characteristic of the transferred AM