Lecture 16	Krane	Enge	Cohen	Williams
Reaction theories				
direct reactions	**11.11**	**13.11/12**	**ch 14**	
Admixed Wave functions				
residual interaction			**5.1-4**	

Problems Lecture 16

1 In the (d,p) reaction leading to states in ^{91}Zr (figs 11.23 and 11.24 Krane, and in the notes of this lecture) discuss the method of assignment of the I^{π}. Could this be done if the reaction proceeded via a CN process?

2 Estimate using semi-classical stripping theory, the angles at which the (d,p) cross section has its maximum for ℓ=0, 1, 2, 3. Use R= 5 fm and a deuteron energy of 10 MeV. The proton energy is 14 MeV .

3 Write down the approximate admixed wavefunctions for the following ℓ=3 states in ^{91}Zr.

The GS, 1.47 MeV state, the 2.044 MeV state. Use fig. 11.24 from Krane or the copy that is in the notes for today's lecture.

Review Lecture 15

1 The difference between a compound nucleus reaction mechanism and a direct one.

2 The CN lasts for $\sim 10^{-12}$ s. The Direct reaction occurs in the transit time of $\sim 10^{-21}$s.

3 The decay of the CN state is independent of its method of creation.

4 Energy of the incoming projectile is shared, and decay to the residual states is statistical.

5 The angular distribution of the products of a CN reaction is essentially isotropic, but if high AM present will have a minimum at 90 degrees.

6 How to calculate the possible total AM of the initial situation, and deduce the possible spins and parities of the possible final nucleus states.

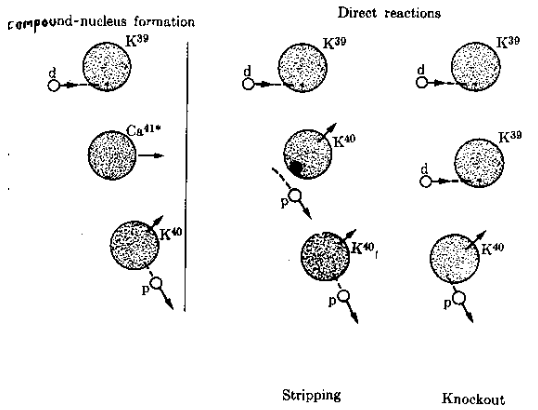
Lecture 16

The direct reaction mechanism

In a direct reaction mechanism, we postulate that the incident projectile interacts with (usually) one of the nucleons in the nucleus.

- It may be, as we showed for ^{41}Ca, a deuteron that lost its n in the interaction and came out as a proton : **Stripping**

- It may be that the deuteron impinged on a proton and knocked it out while the d remains in the nucleus **Knockout reaction**



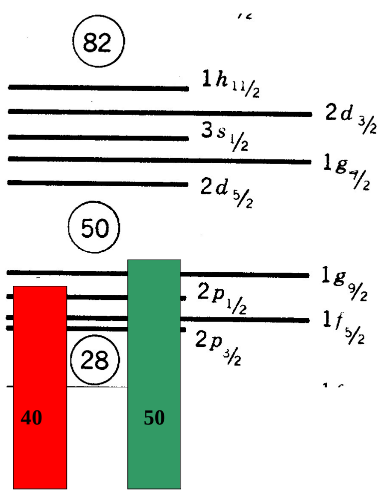
- It might be in another case that an incident proton interacts with a neutron which it picks up, and they form a deuteron. **Pickup Reaction.**

In any such case

- no CN is formed
- the interaction time is of order the transit time (10^{-21} sec)
- momentum and AM are conserved
- the angular distribution will be characteristic of the transferred AM

^{90}Zr Ground state

Z=40, N=50



Let's look at the implications and consequences of this model.

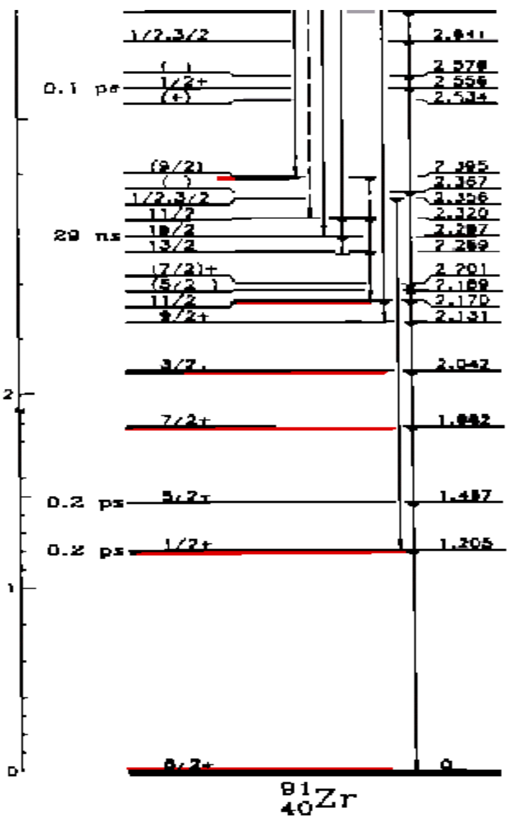
To show the importance of this, consider the example ^{90}Zr(d,p)^{91}Zr, a stripping reaction where **a neutron is stripped from the incident deuteron and placed into an available level in ^{90}Zr.**

^{90}Zr has 40 protons and 50 Neutrons. That is the neutron number closes one of the major shells. ^{91}Zr has one more neutron, and it will according to the shell model go into the 2d5/2 shell. This means that the GS spin and parity of ^{91}Zr should be $5/2^+$, which it is.

On the simple shell model, you might then expect that there would be other (excited) states corresponding to the proton configuration being unchanged, but the single neutron being sucked into one of the levels between (50) and (82). The states that should be formed would be “single neutron“ states. i.e form ^{91}Zr in states that correspond to a single neutron in a SM shell. e.g. I would expect to find a $7/2^+$ state, a $½^+$, a $3/2^+$ and an $11/2^-$ state. (even a $9/2^-$).

Certainly these states are found in the level structure of ^{90}Zr.

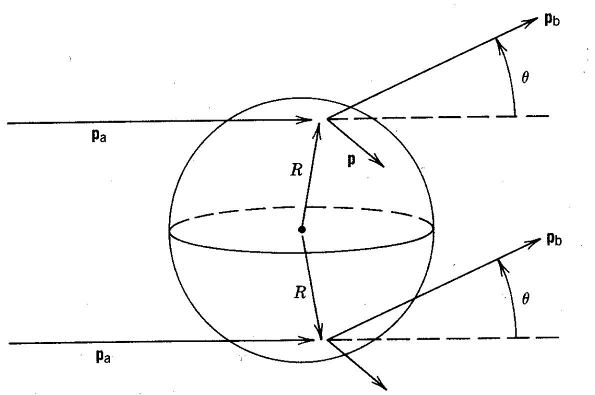
How can we tell which states are formed, and in particular how can we assign the spins and parities to these states?



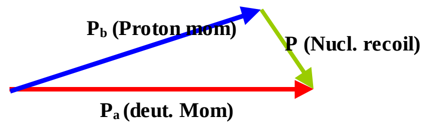
Naturally we will need to do an experiment, and we will come to this shortly. However let us now consider the AM conservation consequences.

Angular Momentum Considerations

$$p^2 = p_a^2 + p_b^2 - 2p_a p_b \cos\theta$$
$$= (p_a - p_b)^2 + 2p_a p_b (1 - \mathrm{co}$$



where p is the transferred momentum.



On this basis since we know the maximum value of the impact parameter to be R, we can estimate ℓ

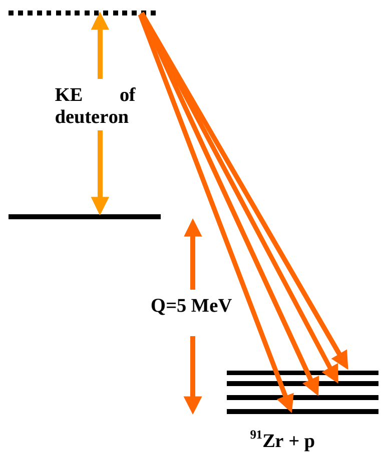
$$pR \sim \ell\hbar$$

and the equation above provides a direct connection between the transferred AM ℓ, and θ.

$$\boldsymbol{l}^2 = \frac{\boldsymbol{R}^2}{\hbar^2}[(\boldsymbol{p}_a - \boldsymbol{p}_b)^2 + 2\boldsymbol{p}_a\boldsymbol{p}_b(2\sin^2\frac{\boldsymbol{\theta}}{2})]$$

So measuring the angle at which the outgoing particle is emitted will lead us to the **value of ℓ that was given to the nucleus**. Note that the calculation above was a semi-classical one. In practice the distribution of emitted particles will be an interference pattern, but the 1st maximum will relate to the angle θ that we calculated above.

We can actually tell which states are formed in the reaction since from the equation above, the AD of the emerging proton is directly related to the AM ℓ transferred to the nucleus. That is, the AM ℓ of the orbit into which the neutron has been placed.



Energetics for ^{90}Zr (d,p)^{91}Zr

Let's determine the approximate angles of the first maxima for the reaction.

As an example if E_d=5 MeV, then because of the Q value, E_p will be 10 MeV or less depending on which states are populated. On this basis we make the approximation that $p_a \sim p_b \sim$ 140 MeV/c, the equation above gives

$$\ell \sim 8 \sin^2 \theta/2$$

➔	ℓ=0	θ=0°
	ℓ=1	θ=14°
	ℓ=2	θ=29°
	ℓ=3	θ=44°

These are the angles where the AD of the emitted proton should be a maximum if AM ℓ has been transferred to the nucleus. NB this transfer is carried by the neutron, and implies that the **neutron is in a shell-model level with the appropriate ℓ.**

Let's look at the experiment and its results.

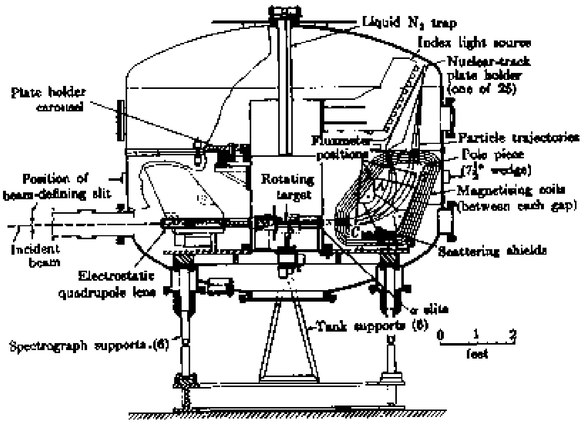


Fig. 13-8. Cross-sectional view of the Massachusetts Institute of Technology multiple-gap spectrograph. (From H. A. Enge and W. W. Buechner, op. cit.)

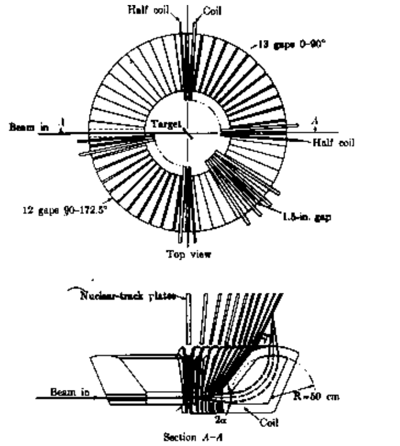
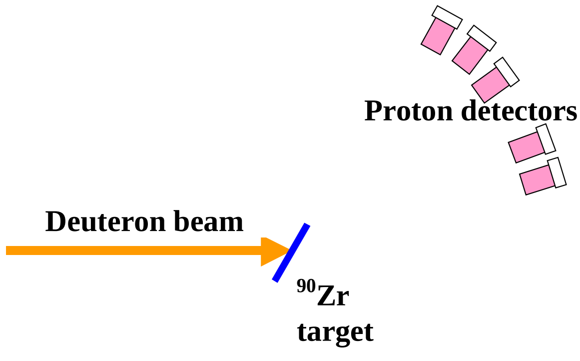


Fig. 13-7. Simplified drawing of the Massachusetts Institute of Technology multi-gap spectrograph. [From H. A. Enge and W. W. Buechner, *Rev. Sci. Instr.* 34, (1963).]

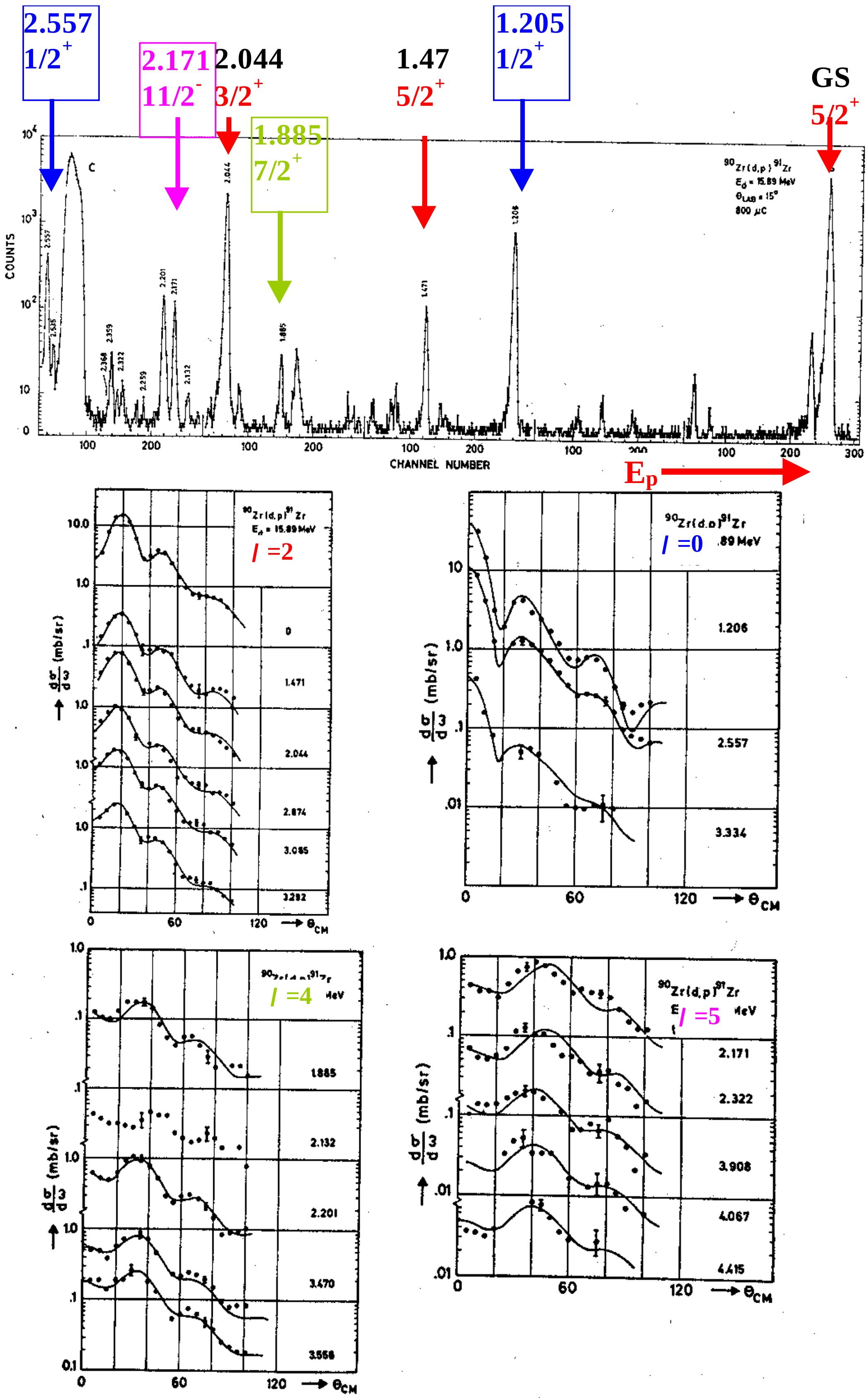
The measurement of the angle at which the proton comes off, and its energy is made with a multigap spectrometer. The beam of deuterons comes in and hits the Zr target at the centre of the spectrometer. At every angle we get an energy spectrum of the protons emitted at that angle.



Shown on the next page are the results of the experiment. Let me take you through them, and emphasise what is learnt.

Describe top section: This is the spectrum of protons at 15°. The higher E_p, the lower the state in ^{91}Zr. The labels indicate the state that has been populated. A spectrum like this was taken at many different angles, and the AD was plotted for protons leading to a particular state. Such an AD plot is similar to those shown in the lower section.

The lower sections have collected into one frame all the ADs that have similar behaviour. The frames correspond to proton ADs indicating a transferred AM ℓ of 1, 2, 3, 4. This means the neutron leading to the creation of the indicated states has ended up in an orbit with this value of ℓ. Note that as an extension of our simple semiclassical calculation of the angle at which nucleons of a given AM ℓ would appear, the quantum analysis results in a diffraction pattern, and the angle of the first maximum is the equivalent.



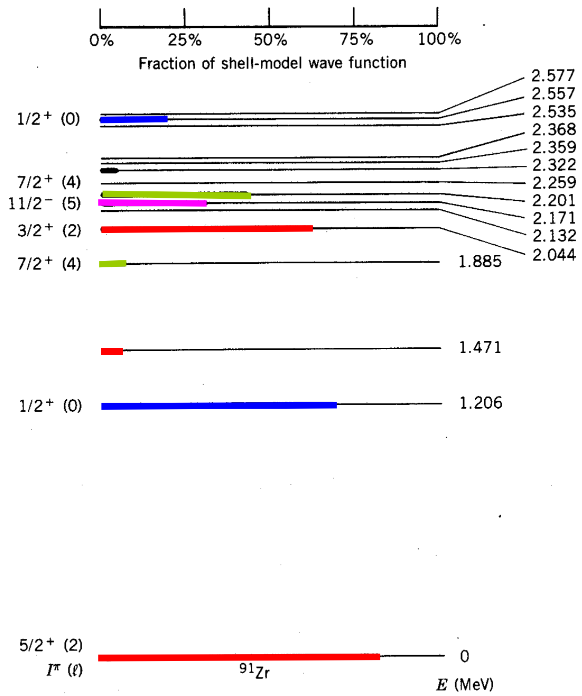
Now let's see if these results agree with our simple "single neutron" wave function.

The GS (5/2$^+$) is populated when the neutron brings in ℓ=2 units of AM, It is identified as a neutron going in to the d5/2 shell ………we agree

The 1st excited state at 1.206 MeV (1/2$^+$) is populated when a neutron brings in ℓ=0 units of AM. That is it puts the neutron into the 3s orbit…….we agree.

The state at 2.044 MeV (3/2$^+$) is populated when a neutron brings in ℓ=2 units of AM. The neutron goes into the d3/2 shell….we agree.

The question that this data answers is: What is the wavefunction of these states? If the state in ^{91}Zr that is populated is clearly populated with transfer of a particular ℓ, transferred in by the neutron, it means that we can assign the neutron in the SM picture to a particular orbit. We can say that the WF for **<u>this</u>** state in ^{91}Zr looks like the WF for ^{90}Zr plus a neutron in the appropriate shell model orbit.



So we see that the GS WF of ^{91}Zr is clearly a d5/2 neutron tacked on to ^{90}Zr. Similarly the 1st excited state is a 2s1/2 neutron tacked on to ^{90}Zr.

You will notice that there are several states in ^{91}Zr that correspond to a neutron being placed in a particular orbit of AM ℓ. One of these is dominant, e.g the GS is the dominant ℓ=2 (I=5/2$^+$). However there are several other I=5/2 + states where the AD indicates that the neutron has gone into a d5/2 orbit, but the cross section for populating these is smaller In figure 11.24 are plotted the fraction of all the cross sections of a particular ℓ, that is seen in a particular state.

e.g. the GS has about 70% of the ℓ=2 I=5/2 strength, the state at 1.47 MeV has another 5% and other states make contributions. Similarly the $1/2^+$ at 1.206 MeV has 60% of the ℓ=0 strength, and the $½^+$ state at 2.557 has another 25%.

What does this mean? It means that 70% of the GS WF looks like ^{90}Zr plus a d5/2 neutron. It means that ~5% of the WF for the 1.47 MeV state looks like ^{90}Zr plus a d5/2 neutron. Etc.

But wait a minute! I thought that the SM picture of the GS of ^{91}Zr was that it was a d5/2 n coupled to ^{90}Zr. In other words 100% of it should be a d5/2 neutron coupled to closed proton shells. Well sorry, the real world is not as simple as any model. The model gives a good first approximation, but there are complications.

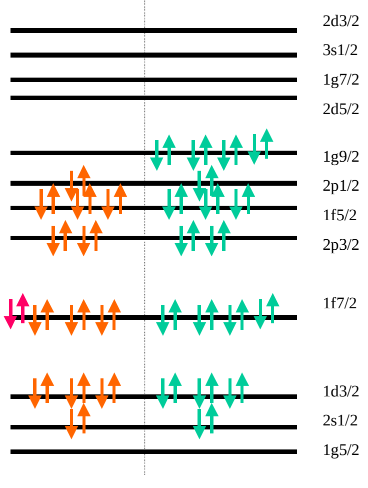
You should recall that to a good approximation the WF for the deuteron was a p and n with spins coupled to S=1, in an orbit with rel AM ℓ=0. However you should recall that this did not explain the Mag moment. We had to believe that the WF was not all ℓ=0 but there had to be an ℓ=2 component, coupled to S=1 to give a GS with I=1 π=+.

$$\boldsymbol{\psi_{deut} = a_s\psi_s + a_d\psi_d}$$

where the a_i quantify the amount of each term in the expression of the WF.

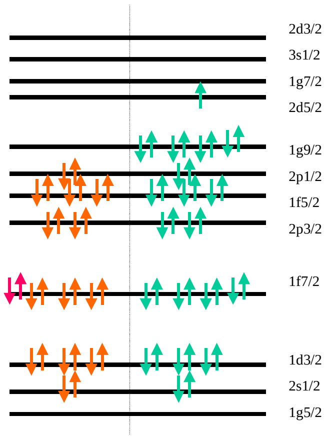
ψ_s and ψ_d are simple SM TERMS in the expression of the total WF. (sum of squares = 1).

Similarly for any real nuclear WF, it is never a pure SM, and **importantly this type of stripping experiment tells you how much it does not conform to the model.** I want to try and clarify the results of this experiment and hopefully at the same time clarify your picture of the true shell model.



The major term for ^{90}Zr ground state wave function

Let's think a little more carefully about the nature of these 1-neutron states that are populated in ^{91}Zr by capturing a neutron from the deuteron. The ^{90}Zr is **in its GS as the deuteron approaches**. If it looks like (show GS term) I can understand that the n could



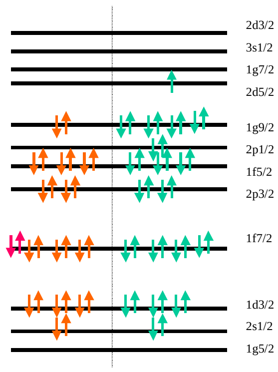
The major d5/2 term for ^{91}Zr ground state wave function

be captured to form a 5/2+ , 7/2+ etc states.

For example I can see the GS of ^{91}Zr as a neutron in the d5/2 shell coupled to a closed proton configuration.

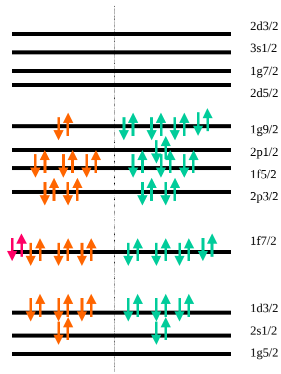
However I find it hard to understand how >5% of the time it creates ^{91}Zr in the 2nd excited state (5/2+), which looks like this

show 2nd ex state WF



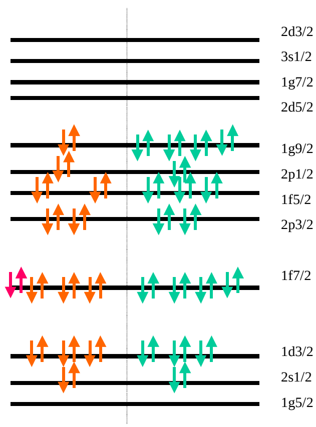
The major d5/2 term for ^{91}Zr 2nd excited state wave function

Since we know the ^{90}Zr is in its GS, we can only form these different states in ^{91}Zr with a neutron in the d5/2 orbit, if the GS of ^{90}Zr was not purely (GS term) but sometimes looked like this etc



Another term for ^{90}Zr ground-state wave function

Or this



Another term for ^{90}Zr ground-state wave function

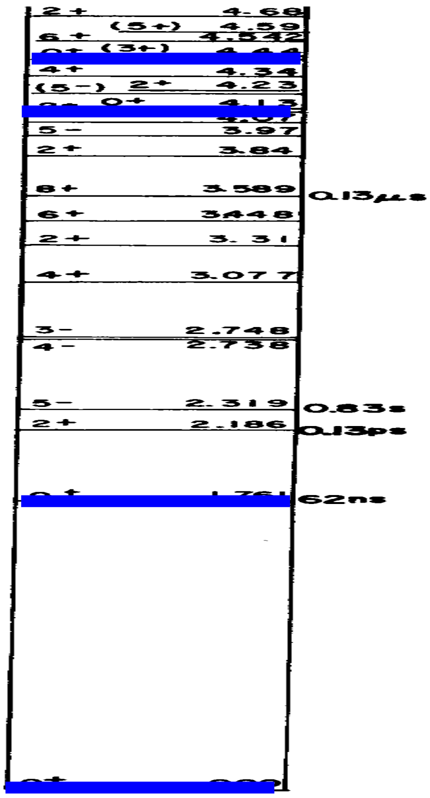
Then when the deuteron comes in there is a certain probability of finding the GS WF looking like one of these configurations.

In fact the GS of ^{90}Zr is

$$\boldsymbol{\psi_{GS} = a_1(\text{closed at p1/2}) + a_2(2p1/2)^{-2}_p(1g9/2)^2_p}$$
$$\boldsymbol{+ a_3(1f5/2)^{-2}_p(1g9/2)^2_p + a_3(2p3/2)^{-2}_p(1g9/2)^2_p}$$

Now when the neutron is captured it has a chance of seeing the ^{90}Zr looking like term 1, in which case it forms ^{91}Zr in the GS. (70 % of the time). Some 5% of the time it finds ^{90}Zr looking like the 2nd term, and forms the 2nd excited state of ^{91}Zr .

Now the terms in this WF of the ground state of ^{90}Zr are all the SM configurations with I=0 and π +ve. They correspond to the GS SM WF of the GS, the next 0+ state, and the next



How come the GS of ^{90}Zr can contain these terms?

Consider the 40 protons in ^{90}Zr. They are all paired off. With 2j + 1 protons in each shell of spin j. In the 2p½ there are 2 one with m_j = ½ and the other with m_j = - ½ . That is they are in the same orbit and moving in opposite directions. In the 1f5/2 there are 6: 2 with m_j = +/-5/2, 2 with m_j = +/-3/2, 2 with m_j = +/-1/2. Each proton moving in opposite direction to its mate.

These pairs of protons can collide. But remember we are dealing with a quantum system and energy is quantised, so to first order the collisions cannot occur since by definition a collision requires a change in energy and this cannot happen. But wait we have forgotten the Heizenberg uncertainty principle. If there is another quantum state separated by an energy ΔE then the pair may scatter into that state for a time $\Delta t \Delta E \sim \hbar$. As it happens there is an orbit, the 1g9/2 very close to the 2p1/2. So there is a good chance of the WF of the GS including a term with 2 proton holes in the 2p1/2 orbit and 2 protons in the g9/2 orbit. The proton pair in the f5/2 can also be scattered up to the g9/2, but for a shorter period since ΔE is bigger. Similarly the 2p3/2 pair.

So now we see that the true WF of each state is in fact a sum of terms which are each pure SM configurations. The relative importance of them depends on the spacing for the lowest available SM orbit.

Similarly the GS WF of ^{91}Zr should be written as

$$\boldsymbol{\psi_{GS} = a_1(1d5/2)^1_\nu + a_2(2p1/2)^{-2}_p(1g9/2)^2_p(1d5/2)^1_\nu}$$
$$\boldsymbol{+ a_3(1f5/2)^{-2}_p(1g9/2)^2_p(1d5/2)^1_\nu + a_3(2p3/2)^{-2}_p(1g9/2)^2_p(1d5/2)^1_\nu)}$$

Now the terms in this WF of the ground state are all SM configurations with I=5/2 and π +ve. We know that $a_1 > a_2 > a_3$ from the experiment

You might well consider the WF of the next $5/2^+$ state in ^{91}Zr. The 2[nd] excited state at 2.044 MeV. In this case we have given energy to a pair of protons and excited them to the g9/2 level. However again scattering of pairs of protons in nearby orbits may create holes and fill the now vacated p1/2 holes. In this case the terms in the WF will be the same ones, however the largest one will now be a_2

This scattering of pairs is called the “residual interaction”. When the SM levels were calculated we used an average potential for the total nucleus. This was OK, however it ignores the interaction of nucleons that come close to each other as they do in coupled pairs. Such periodic and coherent motion leads to these WF admixtures.