Lecture 3	Krane	Enge	Cohen	Willaims
NUCLEAR PROPERTIES				
1 Binding energy and stability				
Semi-empirical mass formula	3.3	4.6	7.2	Ch 4
2 Nuclear Spin	3.4	1.5	1.6	8.6
3 Magnetic dipole moment	3.5	1.7	1.6	8.7
4 Shape				
Electric Quadrupole moment	3.5	1.8	1.6	3.9, 8.8

Problems

1 From the table of nuclear masses given in the text, calculate the binding energy B, and B/A for ^{116}Sn.

2 Calculate the various terms in the expression for the SEMF for ^{116}Sn. From them determine B, B/A, and the mass. Compare the results with the values you obtained in question 1.

3 Using the details in the attached sheet, which is copied from Krane pages 70-72, confirm that ^{125}Te is the most stable isobar with A=127. Also calculate the BE for both ^{127}Te and ^{127}I. , and thereby the energy of the β-decay. Is the β-decay β^+ or β^-?

4 Suppose the proton magnetic dipole moment were to be interpreted as due to the rotation of a positive uniform charge distribution of radius R, spinning about its axis with angular vel ω.
a. By integrating over the charge distribution show that $\mu = e\omega R^2/5$
b. Using the classical relation for AM and ω, show that $\omega R^2 = s/0.4\ m^2$
c. Show that $\mu = (e/2m)s$ (analogous to Krane equ 3.32 $\mu = (e/2m)\ell$

5. Calculate the electric quadrupole moment of a charge of magnitude Ze distributed over a ring of radius R with the axis along (a) the z-axis (b) the x-axis

Tutorial Tuesday lunchtime room 210 on podium

Lecture 2 Review

1. Nuclear Size

Electron scattering
Wavelength of 300 MeV electrons
Nucleus not a point particle
Form factor of nuclear charge. Distribution of charge

Charge distribution approx flat and constant for different A.
Skin thickness
Nuclear radius halfway down ski thickness

Shape of nucleon density distribution $$\rho(r)=\frac{\rho_o}{1+\exp(r-R_c)/t}$$

The number of nucleons per unit volume is essentially constant. $\dfrac{A}{4\pi R^3} \sim const$.

So that $R = R_0 A^{1/3}$
R_o = 1.2 fm from electron scattering studies.

Conclusions

Nuclear force short range
There is a repulsive core.

2 Stability of Nuclei

Half of all nuclei have even Z even N
Only 4 have odd Z and N
Conclusion: Nuclear force is stronger for nucleons with opposing spins and paired off

Binding energy of nucleons
Nucleon and nuclear masses
Atomic Mass Unit
Separation energy

Separation energies of Ca isotopes
Conclusion: Pairing energy is a about 3 MeV

Lecture 3_{2002}

Having discussed nuclear binding, we are in a position to look at the systematics of the stable nuclei, and use our limited knowledge to interpret it.
As we did for the case of ^{12}C, we can, from the mass of all the protons and all the neutrons compared with the mass of any nucleus, deduce the total binding energy. Naturally this will increase with A, and what is perhaps more tractable is the BE/A, the binding energy per nucleon. We already know this for ^{12}C to be 100/12 MeV/nucleon.

The semi-empirical Mass Formula
Shown in fig. 3.16 is a plot of BE/A for a selection of nuclei from A=1 to A=260 or so.

Comments

1. general shape; For the vast majority of nuclei, B/A is approximately constant, with a value of 8 MeV. Our observations last lecture that the nuclear force is short range is consistent with this. Each nucleon only interacts with those that can pack around it, so the contribution of each nucleon to the BE is the same, at least for nuclei where there are many nucleons. The exceptions are the very light nuclei.

2. maximum stability: The value of B/A has a maximum value for nuclei with A~60.

   For nuclei below this greater stability is obtained by aggregating. That is by fusing lighter nuclei together we increase B/A, and release energy. This is the principle of a fusion reactor or fusion bomb.

   For nuclei which have large A, the BE is increased of the nucleus of A nucleons splits into two separate nuclei. Again the increased BE leads to emission of energy. This of course is the principle of the fission reactor and the good old nuclear bomb.

What does this info tell us about nuclei and the nuclear force.

- The fact that B/A is essentially constant implies that each nucleon has the same number of neighbours. We implied this when we discussed the results of electrons scattering. The short range of the nuclear potential limits each nucleon's binding effect to a few others, and in essence each nucleon can contribute a certain amount to the stability or binding of the nucleus. Thus the contribution to the nuclear binding due to this **volume term is a_vA,** where a_v has a value of about 16 MeV.

- The nucleons on the surface do not have a full complement of neighbours, so the volume term a_vA, overestimates the binding. The number of nuclei on the surface is proportional to the surface area, $\propto r^2$, or $\propto A^{2/3}$. So we need **a term like $-a_sA^{2/3}$,** to allow for this.

- Although the protons bind with the same 16 MeV contribution as the neutrons via the nuclear force, they also destabilize because of their long-range coulomb repulsion against the (Z-1) other protons. Thus there is a destabilizing **coulomb term of the form $-a_cZ(Z-1)$**. On the assumption that the Z protons are in a uniform sphere of radius $R_{oA}^{-1/3}$, the constant a_c looks like

- Especially for light nuclei the preference is for N=Z. This becomes less evident as A increases. To allow for this symmetry a term of the form
  $a_{sym}(A-2Z)^2/A$ is inserted.

- We have already seen that more nuclei exist with N and Z even, only 4 with N and Z odd. Also we saw that there is a preference for nucleons to pair off. To allow for this pairing energy , a pairing term **$\delta = a_p A^{-3/4}$** is added. **δ is positive for even N and Z, negative for odd N and Z, and zero for odd A.**

$$B = a_v A - a_s A^{2/3} - a_c Z(Z+1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + \delta$$

$$B/A = a_v - a_s A^{-1/3} - a_c Z(Z+1)A^{-4/3} - a_{sym}\frac{(A-2Z)^2}{A^2} + \delta'$$

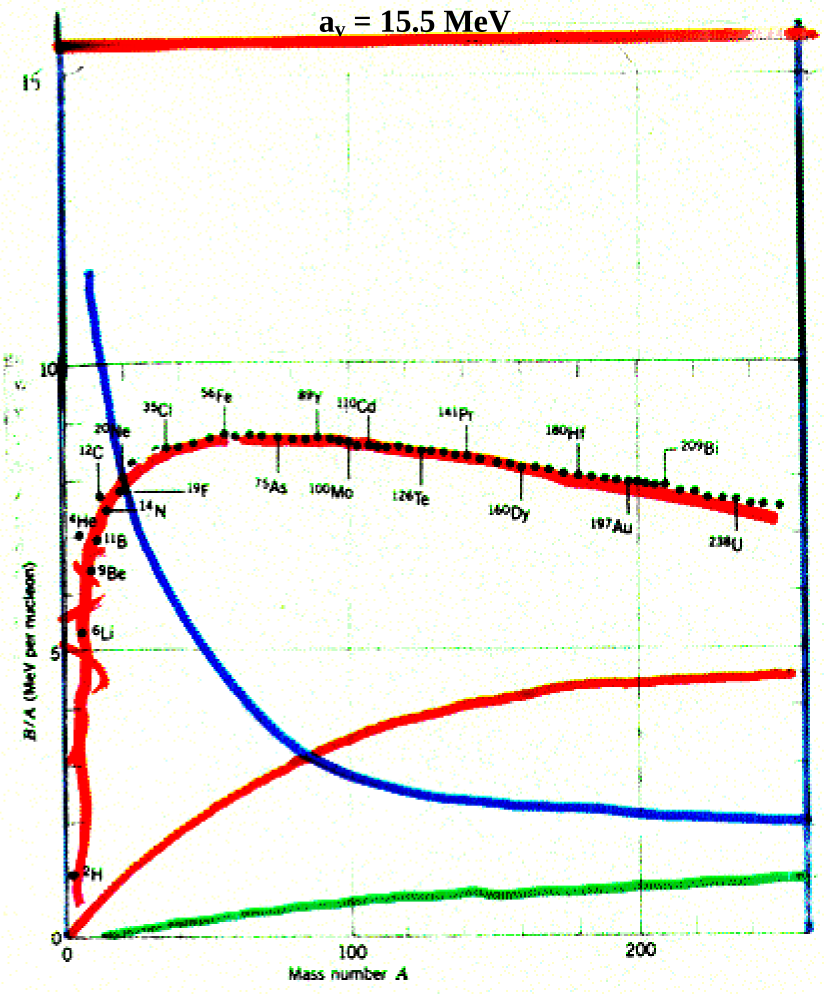


Figure 3.16 The binding energy per nucleon.

So BE has the form

$$B = a_v A - a_s A^{2/3} - a_c Z(Z+1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + \delta$$

or to compare it with the BE/A curve

$$B/A = a_v - a_s A^{-1/3} - a_c Z(Z+1)A^{-4/3} - a_{sym}\frac{(A-2Z)^2}{A^2} + \delta/A$$

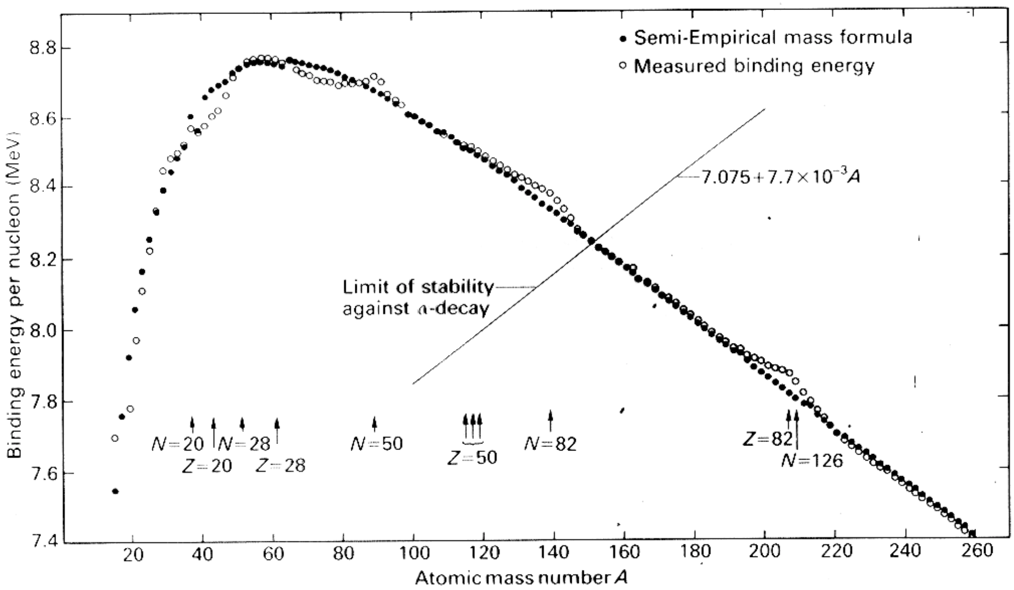
The fit shown has
a_v = 15.5 MeV
a_s =16.8 MeV
a_c = 0.72 MeV
a_{sym} = 23 MeV
a_p = 34 MeV where $\delta = +a_p A^{-3/4}$ for Z and N even
$\delta = -a_p A^{-3/4}$ for Z and N odd
δ = 0 for A odd

Comments

1 What we have done in the above is to model the nucleus in a number of different ways. We have assumed (correctly) that **the nuclear force is short-range**. It therefore exhibits saturation.

2 In the main we have considered the **nucleus to be like a liquid drop**. *The liquid drop model*, and to a good approximation this fits with reality. The liquid drop model is particularly appropriate for heavy nuclei, and the excited states of these nuclei are explained in terms of rotations and vibrations of the nuclear "fluid".

However some of the more specific effects, such as the pairing effects, invoke modelling the nucleus with orbiting nucleons, *the shell model.* We will come back to discuss this in far more detail later. The fact that we have not really modelled the nucleus correctly can be seen if we calculate the separation energy for the least bound neutron using the SEMF and **compare it with the measured values of S_n.**



The figure shows the predicted versus real BE/A. The significant deviations at N = 20, 28, 50, 82 and 126 indicate that the SEMF fails to account for the greater than expected stability. The data thus provide evidence for special stability of nuclei with these values of N. These are the so-called magic numbers that will be explained in the shell model of the nucleus.

Note that using the formula for B, it is possible to estimate the stability of a set of Isobars.

$$M(Z,A) = Zm(^1H) + Nm_n - B(Z,A)/c^2$$

For a given value of A (for a set of isobars) the equation above is a quadratic in Z so that the binding energy for this value of A, and different Z can be found. For odd A there is generally only one stable isobar, the others decaying by β^- or β^+ emission. See fig 3.17 of Krane.

Mass of nucleus $^Z A$ is

$$\mathbf{M(Z,A) = Z\, m(^1H) + Nm_n - B(Z,A)/c^2} \qquad (1)$$

where

$$B = a_v A - a_s A^{2/3} - a_c Z(Z+1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + \delta$$

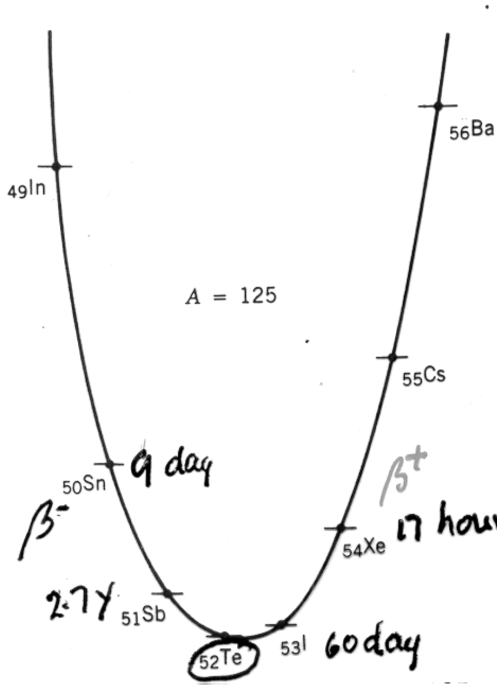
For a given A (a set of isobars) this becomes a parabola of M vs Z. The parabola will be centred about the value of Z where equ 1 is a minimum; i.e. dM/dZ =0

$$Z_{\min} = \frac{[m_n - m(^1H)] + a_c A^{-1/3} + 4a_{sym}}{2a_c A^{-1/3} + 8a_{sym} A^{-1}}$$

Note that it is the mass of the H atom, so that the 1st term on top is negligible with a_c=0.72 MeV, and a_{sym} = 23 MeV.

So $$Z_{\min} = \frac{A}{2}\,\frac{1}{1 + \frac{1}{4}A^{2/3} a_c / a_{sym}}$$

The text (Krane) shows the total BE of the mass A=125 set. You should notice that ^{125}Te is the stable isobar with Z= 52. The nuclear isobars successively change n➔p or p➔n by β^- or β^+ decay. I'm not going through this in detail now, but I suggest you do problem 15 in Krane Chap 4 to get a little practice.

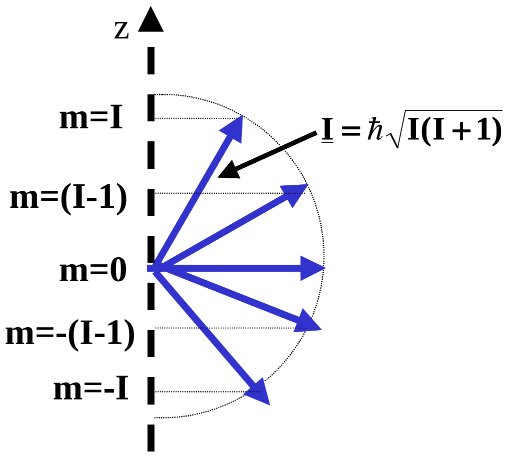


A few more nuclear properties

Nuclear Angular Momentum

The nucleus is a quantised system, and as such, if it has angular momentum, it will be quantised. The AM vector of a nucleus is specified by the quantum number, ***I***.

$\boldsymbol{I} = \hbar\,[I(I+1)]^{1/2}$.



$$\mathbf{I_z = m\hbar \qquad where\ m = +I........-I}$$

For odd A, I is half integral

For even A, I is integral (figure above)

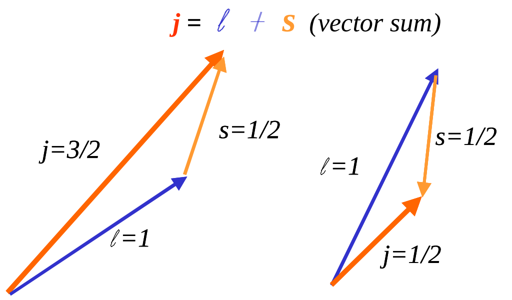
The vector **I** can have projections on to a spatially fixed axis, of $I_z = m\hbar$, where m = -I…..+I.

The nuclear Zeeman splitting that those of you that have done the Mossbauer experiment observed, is the result of this orientation of the spin in the strong magnetic field of the electrons in the atom of iron.

Where does this AM come from? It is the vector sum of the AM ***j***, of all the component nucleons . This spin **I**, is generally modelled as the result of the coupling of the angular momentum **j** of the individual nucleon that constitute the nucleus. This in turn is the result of the vector coupling of the orbital AM of each nucleon ℓ with its intrinsic spin AM ***s***.

Vector Coupling of Angular Momentum

ℓ and s to give j



In nuclear systems it is *j* that is good quantum number

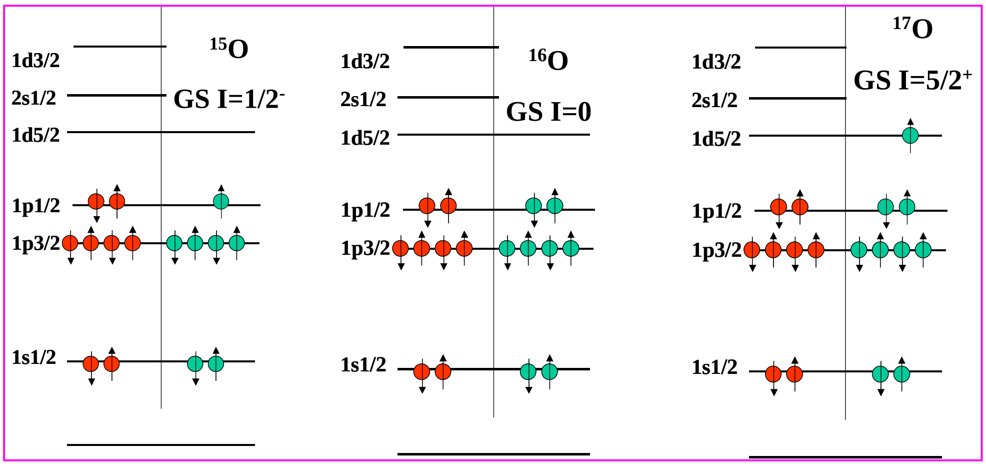
Because the value of the j quantum number is always half-integral,
if there is an even number of component AM j, then ***I*** will be integral,
If there is an odd number, then ***I*** will be half-integral.

Thus for odd A nuclei *I* is half integral
for even A nuclei *I* is integral.

In the same vein, a nucleus with even Z and even N will have ***I***=0. This is consistent with our observation about pairing effects. Nucleons like to pair off, and they in fact pair off with opposing j. (also can consider the pair as being in orbits with +ℓ and -ℓ).

Each nuclear state has a **parity** assigned to it. This indicates whether the wavefunction is even or odd on spatial reflection. This is the composite of the parity of the states of all the composite nucleons, and if we knew these, this could be determined. Except for fairly simple nuclei, this cannot be done, and the parity is determined as the result of conservation laws applied to nuclear reactions. The parity is indicated $\boldsymbol{I^{\pi}}$.

In odd Z or odd N nuclei it is possible to see what the spin and parity of the GS of the nucleus will be. (using the shell model to be discussed later)

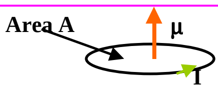


Magnetic dipole moment

The currents that exist within the nuclear substructure lead to the presence of magnetic fields. The simplest of these is the magnetic dipole moment.

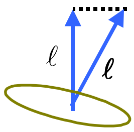
The magnetic moment of a current **I** circulating a circle of area A is
$\mu = iA$

The magnetic moment of a current i circulating a circle of area A is $\mu = iA$



If current due to charge e moving with speed v in radius r, i.e. with a period T= $2\pi r/v$, then

$$\mu = \frac{e}{(2\pi r/v)}\pi r^2 = \frac{evr}{2} = \frac{e}{2m}\ell$$



in classical physics ℓ is the AM mvr

in the quantum situation ℓ is the maximum projection of the AM $\ell_z = \ell\hbar$.

$$\boxed{\mu = \frac{e\hbar}{2m}\ell\,.}$$

The term $\frac{e\hbar}{2m}$ is called a magneton

For an electron in an atom, m is the electron mass and the term is called the Bohr magneton (μ_B)

$\mu_B = 5.7884 \times 10^{-5}$ ev/T.

For the heavier proton the nuclear magneton μ_N has a value

$\mu_N = 3.1525 \times 10^{-8}$ ev/T.

In terms of units of Nuclear Magnetons, the nuclear mag moment is written as

$$\mu = g\,\ell\,\mu_N \qquad \text{equ 1}$$

where g is the g factor!!

For protons g =1
for neutrons g =0, since they have no charge.

The circulating protons are not the only source of magnetic fields in the nucleus.
Nucleons have an intrinsic spin, and consequent spin magnetic moments.

Using the same form as equ. 1 above, the spin mag moment is

$$g_s\, s\, \mu_N$$

For a spin s = ½ point particle, such as an electron quantum electrodynamics gives a value of $g_s = 2$, and experiment confirms this.

For nucleons the value is far from the prediction for a point particle.

Proton $g_s = +5.5856912 \rightarrow \mu_s = +2.7928456\ \mu_N$

neutron $g_s = -3.8260837 \rightarrow \mu_s = -1.9130418\ \mu_N$

$\mu = ½$ gs in units of μN

Orbital Magnetic moment
If the current is due to a charge e moving with speed v in a circle of radius r, i.e. with a period T= 2πr/v, then

$$\mu = \frac{e}{(2\pi r/v)}\pi r^2 = \frac{evr}{2} = \frac{e}{2m}\ell$$

in classical physics ℓ is the AM, *mvr*
in the quantum situation ℓ is the maximum projection of the AM $\underline{\ell} = \ell\hbar$.

$$\mu = \frac{e\hbar}{2m}\ell\ .$$

The term $\frac{e\hbar}{2m}$ is called a magneton. For the case of a circulating electron, say in an atom, m is the mass of an **electron** and the term is called the **Bohr magneton** and has a value of

$\mu_B = 5.7884 \times 10^{-5}$ ev/T.

For the much heavier **proton** the **nuclear magneton** has a value

$\mu_N = 3.1525 \times 10^{-8}$ ev/T.

Thus in terms of units of Nuclear Magnetons, the nuclear mag moment is sometimes written as

$$\boxed{\mu = g\,\ell\,\mu_N \qquad \text{equ 1}}$$

where g is the g factor!!, and for protons g =1 , and for neutrons g =0, since they have no charge.

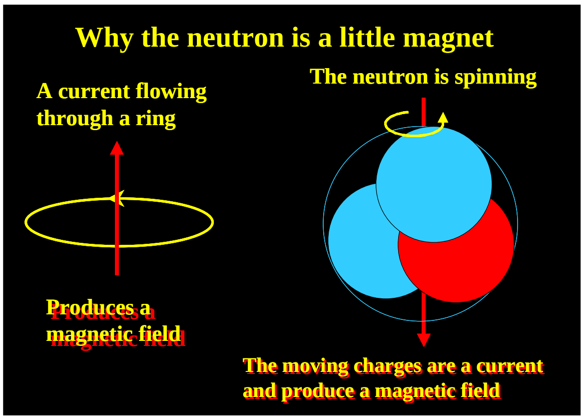
Spin Magnetic Moment
The circulating protons are not the only source of magnetic fields in the nucleus. Nucleons have an **intrinsic spin**, and consequent **spin magnetic moments**.

Using the same form as equ. 1 above, the spin mag moment is $\mu_s = g_s s \mu_N$

For a spin s = ½ point particle such as an electron quantum electrodynamics gives a value of $g_s = 2$, and experiment confirms this.

For nucleons the value is far from this prediction for a point particle.
proton $g_s = +5.5856912$
neutron $g_s = -3.8260837$
nb . $\mu = ½\, g_s$ in units of μ_N

Not only is the value for the charged proton much larger that for a point particle, but the uncharged neutron has a non-zero moment. This is reasonable evidence that the nucleons are not fundamental particles (as is the electron), but are composite particles. In nuclear parlance they each involve a core, plus a π-meson cloud. This seems even more reasonable when one notes that the value of g for the proton is larger than its expected value by about 3.6, and the value for the neutron is smaller than its expected zero by about the same amount. In the unified model they consist of 3 fractionally charged quarks.



Again, in terms of models, the MDM of nuclei might well be explained in terms of the WF (classically the motion) of individual nucleons. I mention only the deuteron and ^{17}O

Table 3.2 Sample Values of Nuclear Magnetic Dipole Moments

Nuclide	$\mu(\mu_N)$
n	-1.9130418
p	$+2.7928456$
^2H (D)	$+0.8574376$
^{17}O	-1.89379
^{57}Fe	$+0.09062293$
^{57}Co	$+4.733$
^{93}Nb	$+6.1705$

All values refer to the nuclear ground states; uncertainties are typically a few parts in the last digit. For a complete tabulation, see V. S. Shirley, in *Table of Isotopes* (Wiley: New York, 1978), Appendix VII.

Nuclear Shape

Electric Quadrupole Moment

If one is far enough away from a charge distribution, the electric potential can be considered to be due to a point charge. However you know from your EM lectures that in general

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{1}{R}\int \rho dV + \frac{1}{R^2}\int \rho z dV + \frac{1}{R^3}\int \rho(3z^2 - r^2)dV\right]$$

the 1st term is the total charge Q, and the potential is the monopole potential. The second leads to the electric dipole term in the potential, but for symmetry reasons this does not occur. The 3rd term reflects the non-uniform charge distribution about the Z axis of the nucleus. The electric quadrupole moment is then defined as

$Q = 1/e\int \rho(3z^2 - r^2)dV]$ and has units of area usually in barn (10^{-28} m^2).

If the charge distribution is prolate (stretched along the Z axis) Q is +ve. If squashed it is -ve.

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{1}{R}\int \rho dV + \frac{1}{R^2}\int \rho z dV + \frac{1}{R^3}\int \rho(3z^2 - r^2) dV\right]$$

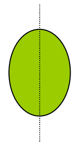
monopole **dipole** **quadrupole**

The electric quadrupole moment is defined as

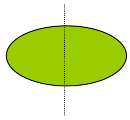
$$Q = 1/e\int \rho(3z^2 - r^2) dV]$$ **units of area**

usually barn (10^{-28} m^2).

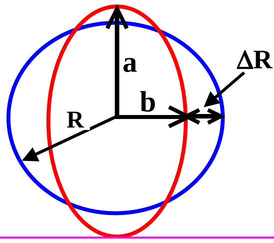
For sphere Q=0



prolate **Q +ve**



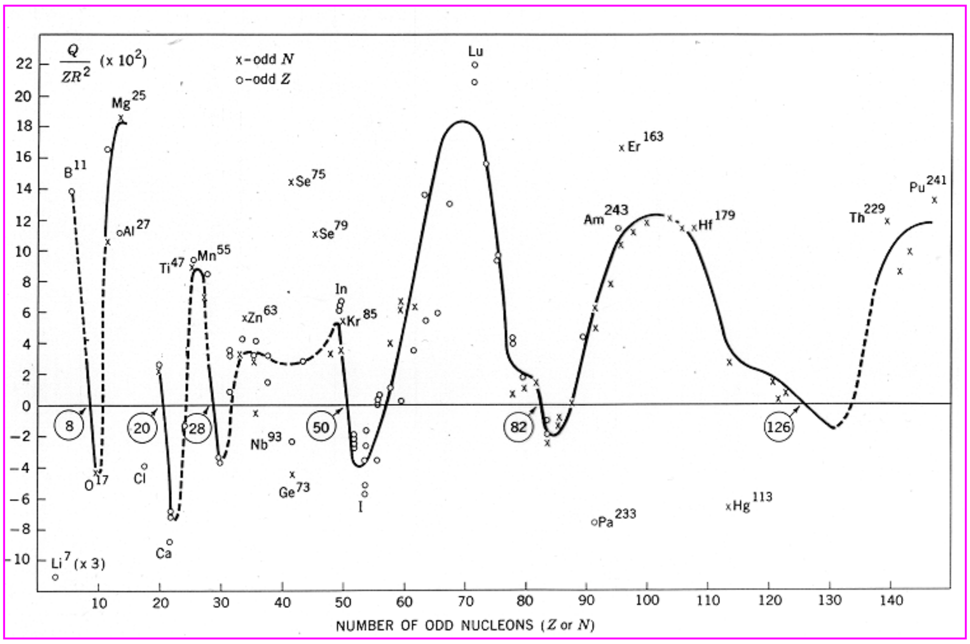
oblate **Q -ve**



$Q = 2/5\, Z(a^2 - b^2)$
$\cong 6.5ZR^2(\Delta R/R)$

If the spin of the nucleus is 0 (or in fact ½) it is not possible to measure the value of Q (usually done from studies of hyperfine splitting in spectra) since there is no spatial reference to define the Z axis.

The figure to the left shows the known Qs for a series of nuclei. Note that there are some nuclei with extreme deformations. Again note that the nuclei that are close to sperical are those near the “magic” numbers.

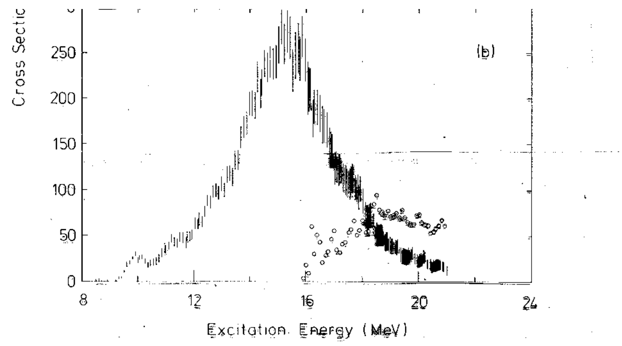


Now by way of a diversion, I want to mention some of the work that both Roger and I did. As I mentioned our research area is photonuclear reactions. This means that we examine the nucleus using high-energy photons. Currently we work in the energy region of 100 MeV or so, and are particularly interested in the short-range interaction of nucleons within the nucleus.

However some 10 years or so ago, we had in this School a 35-MeV betatron, which provided us with photons up to 35 MeV. At this energy, and at that time this allowed us to study the macroscopic properties of nuclei.

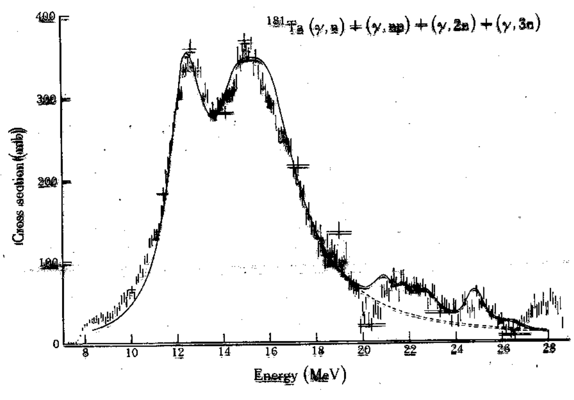
As we have just discussed in considering the Semi-empirical mass formula, the nucleus, especially heavy nuclei, can be considered to be modelled as a liquid drop. Thus we might consider that the EM field of a high-energy photon might interact with the total charge of the nucleus (essentially a fluid of protons) and force this fluid into sympathetic oscillations at the frequency of the EM field. In fact it happens that the fundamental resonance of a nucleus is about 20 MeV, so it is quite easy to set the nucleus into oscillation.

The oscillation is a simple dipole collective mode. If the nucleus is like a spherical drop, the resonance has a well defined energy and width. The nucleus of Sn, has Z=50, and is thus at a closed shell or magic number for protons. It is in reality a good spherical nucleus.



If however the nucleus is deformed, as is Ta, the cross section splits into two resonances with different energies.

(a) The ^{121}Sb(γ, sn) reaction cross section and (b) the cross sections for ^{121}Sb(γ, n) and ^{121}Sl (circles).



You should be able to guess whether the Q of Ta is +ve or -ve. Is Ta stretched out or squashed?