Lecture 6 Krane Enge Cohen Williams
Properties of 2-nucleon potential
general central 4.4 3.6 9.9
spin dependence4.4 3.6 3.7 9.9
ℓ-s dependence 4.4 3.8 9.9
tensor 4.4 2.6 3.6 9.9
Meson Theory of Nuclear potential
4.5 3.11 3.9 9.10
I recommend Eisberg and Resnik notes as distributed

Problems, Lecture 6

1 Consider problem 15 in Krane Chap 4 (page 115). This requires a deal of thought.

2 Because both the neutron and the proton have magnetic dipole moments, there is a difference in the energy when they are aligned ↑↑ or ↑↑ .Estimate this difference when their centres are separated by r_d. It might be interesting to compare this value with the tensor force as plotted in Cohen fig 3.7ot Krane, Fig. 4.16.

3 Use energy conservation, in the light of the uncertainty principle, to estimate the relation between the range of the nuclear force and the mass of the π meason. Use this relation to estimate the pion mass if r' = 2 fm.

4 Because pions had not been discovered in 1936 when Yukawa proposed the meson theory of the nuclear force, it was suggested that the μ meson was responsible. What would the range of the nuclear force be if this were true?

Review Lecture 5

Nucleon-nucleon scattering

1 Study of deuteron tells us only about S=1 , ℓ=0 components of nuclear potential
Need to consider scattering of nucleons to study S=0 and ℓ>0 components.

2 Concept of nuclear cross section
the number scattered in thickness *t* is $N = N_o(1 - e^{-n\sigma t})$

3 Doing scattering experiments require accelerators to produce a beam of protons or neutrons of a controlled energy, and detectors to measure the number scattered at a particular angle.

4 Consider a plane wave incident on the target. The effect of the potential is to produce a phase change . Interference of the incident plane wave and the scattered wave, some distance from the nucleus explains the scattering probability.

5 The theory is most easily done by replacing the incident plane wave with a sum of spherical waves. $\varphi_{in} = e^{ikz} = e^{ikr\cos\theta} = \sum_{\ell=0}^{\ell=\infty} B_\ell(r) Y_{\ell,0}(\theta)$

We limit the scattering theory to ℓ=0.
You should be able to show that an incident nucleon with <10 MeV KE_{lab} has a classical AM similar to a wave with ℓ=1. Thus if KE is < 10 MeV or so, we can put B_ℓ= 1, and $Y_{\ell m}$➔ $(1/4\pi)^{1/2}$.

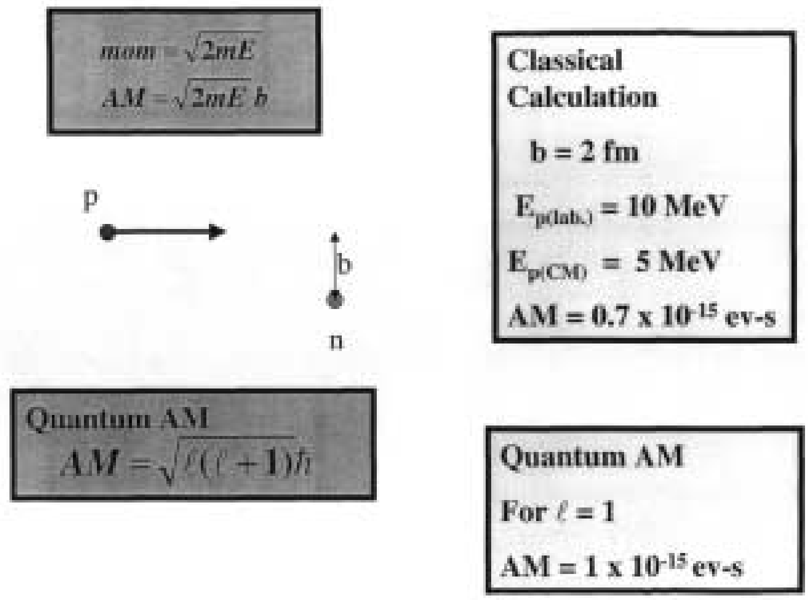
6 The calculated cross section is $\sigma = \frac{4\pi \sin^2\delta}{k^2}$ This has a value of 5 b, which is only 20% of the measured value. Reason is that we used nuclear potential parameters for from our solution of the deuteron S=1. As the neutron, with spin ½ approaches the proton in the target (with spin s = ½ , the probability of coupling to 1 is 3 times that of coupling to 0. Thus the measured cross section is $\boldsymbol{\sigma = \frac{3}{4}\sigma_3 + \frac{1}{4}\sigma_1}$.
So 20 b = ¾ (5 b) + ¼ σ_1 ¼ σ_1 = 20 - ~4 = 16 b So that σ_1 ~ 64 b. Confirms our conclusion that nuclear potential depends on relative spins of interacting nucleons.

Lecture 6

In the last lecture, I indicated that components of the nuclear potential, other than S=1 (that we get from the deuteron BE), must be obtained by N-N scattering. Since in nthis case we can have the spins aligned to give S=1 or S=0. (hence see if there is a difference in the potential depending on spin alignment). In addition we can vary the energy of the projectiles and hence the AM l, and check any dependence on the rel AM of the nucleons.

One can get an indication of the AM of the nucleons pair from classical physics.

To see this imagine a neutron of momentum p approaching a proton at an impact parameter of b (about 1 fm. The range of the nuclear potential).



The Quantum AM is $\hbar\sqrt{\ell(\ell+1)}$. The classical AM = pb = b.$(2mE)^{1/2}$.

So if we assume b = the range of the nuclear potential we can find the approximate value of ℓ for a nuclein pair with a particular KE. Which if we put ℓ=1 gives about 10^{-15} ev-sec. At 10 MeV (lab) = 5 MeV (CM) the classical AM is ~ 0.6 x 10^{-15} ev-s. Much less than the case for ℓ=1. So at 10 MeV we can assume, on this semi-classical argument that the relative AM is ℓ=0.

In the notes from lect. 5, I gave some details of the calculation of p-n scattering for low-energy (ℓ=0) nucleons. The calulation gave a figure of about 5 barn (for KE<1 MeV)

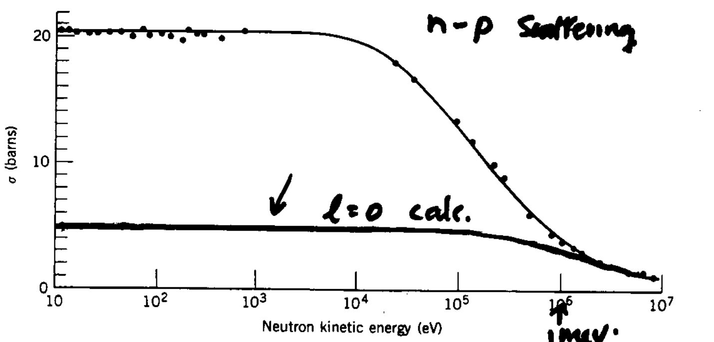


Figure 4.6 The neutron–proton scattering cross section at low energy. Data taken from a review by R. K. Adair, *Rev. Mod. Phys.* **22**, 249 (1950), with additional recent results from T. L. Houk, *Phys. Rev. C* **3**, 1886 (1970).

We see that the theoretical value is significantly below the measured value at low energies. Why is this? And what does it tell us?

We did the calculation using the parameters that gave the bound deuteron. This arrangement of n-p was parallel spins for n and p: S=1. However in scattering we are not limited to S=1, we also have S=0. As we noted before, the nuclear potential for S=0 is smaller than for S=1.Recall the S^1 state of the deuteron was unbound. If we assume that the large difference is due to singlet scattering we can determine the relative strengths of σ_3 and σ_1.

As the neutron, with spin ½ approaches the proton in the target (with spin
s = ½ , the probability of coupling to 1 is 3 times that of coupling to 0.

Thus the measured cross section is

$\boldsymbol{\sigma = \frac{3}{4}\sigma_3 + \frac{1}{4}\sigma_1}$.

We have calculated σ_3 to be about 5 b, since we used S=1 parameters.

So $20\text{ b} = \frac{3}{4}(5\text{ b}) + \frac{1}{4}\boldsymbol{\sigma_1}$

$\frac{1}{4}\boldsymbol{\sigma_1} = 20 - {\sim}4 = 16\text{ b}$

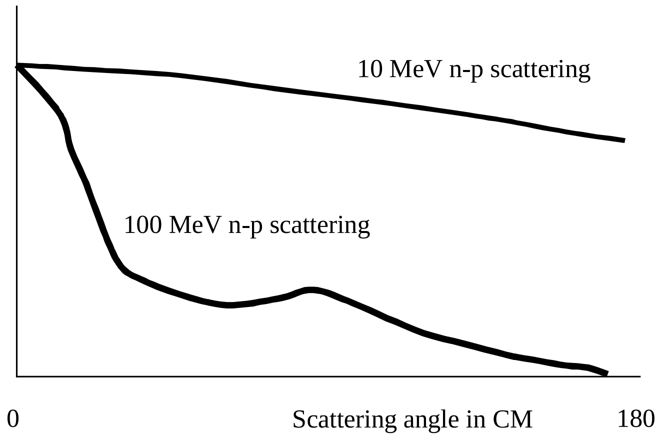
So that $\sigma_1 \sim 64$ b.

This is a huge difference, and an important indication of the difference in the singlet and triplet components of the nuclear potentials. This confirms our observation from the fact that the S^1 excited state was less bound than the S^3, that the nuclear force must be spin dependent.

In fact it is not possible to quantify this difference from these scattering experiments. It was resolved by scattering VERY low-energy neutrons (0.01 eV) from hydrogen MOLECULES, which come in 2 forms Para- (proton spins S=0) and Ortho- (Proton spins S=1). These neutrons have deBroglie wavelengths of 0.05 nm (much larger than the separation of the nuclei in the H mol. So that the neutron interacts coherently with BOTH nuclei, and scatters coherently. The resulting interference patter (scattering cross-section) allows the S=1 and S=0 potentials to be quantified, and the S=0 one is 40% weaker.

The reason scattering of nucleons with energies <a few MeV is not much help, is that their deBroglie wavelength is much bigger than the nucleon size: the Angular distribution is isotropic. (In other words the angle of the first diffraction minimum given by sinθ =λ/d is at an angle greater than π (in the CM system)). As we use higher energy probes, we would expect that the scattering cross section would begin to show forward peaking.

[The classical reason form this expectaion is that the impuse given to the nucleon as it gets into the range of the nuclear potential is small id it is moving quickly (the SPEED bus-



effect).

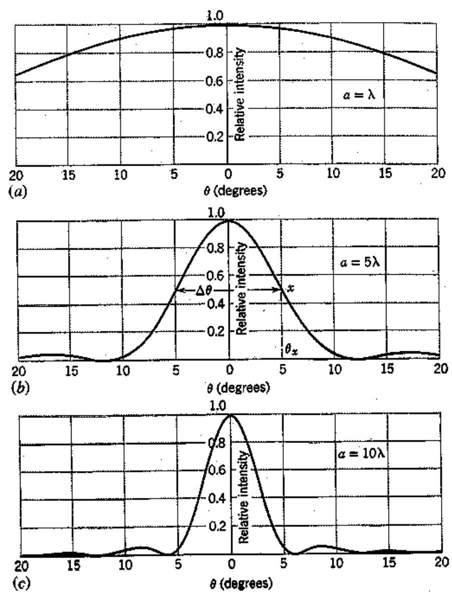
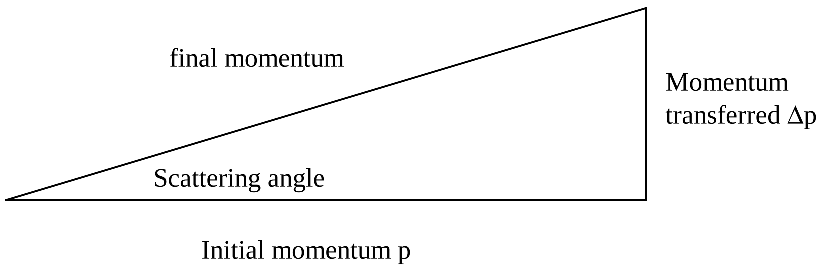


Figure 11 The relative intensity in single-slit diffraction for three different values of the ratio a/λ. The wider the slit, the narrower is the central diffraction peak.



$\Delta p/p \sim F\Delta t/p$

$\sim F(r/v)/mv$ r = width of potential

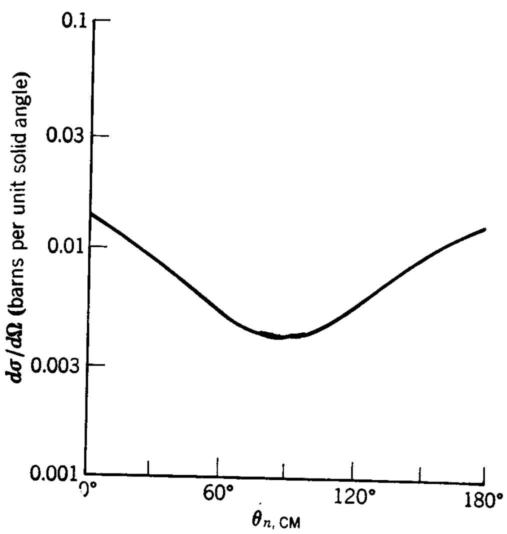
$\sim V_0/mv^2$ $(F = -dV/dr)$

$\sim V_0/K$

If $K > V_0$ angle of scattering is small

When they did n-p scattering at 90 MeV they got the following:

Note we are in the n-p CM ref. Frame.



Scattering of neutrons by protons at 90 MeV.
Results plotted in the CM reference frame.

Not what was expected! There is an equal number of neutrons (or protons) scattered through large angles as there are through small.

What was expected is shown in the top diagram, and the explanation in the lower one.

The interpretation of the data was that in about half of the scatterings,whilst in the region of the N-N

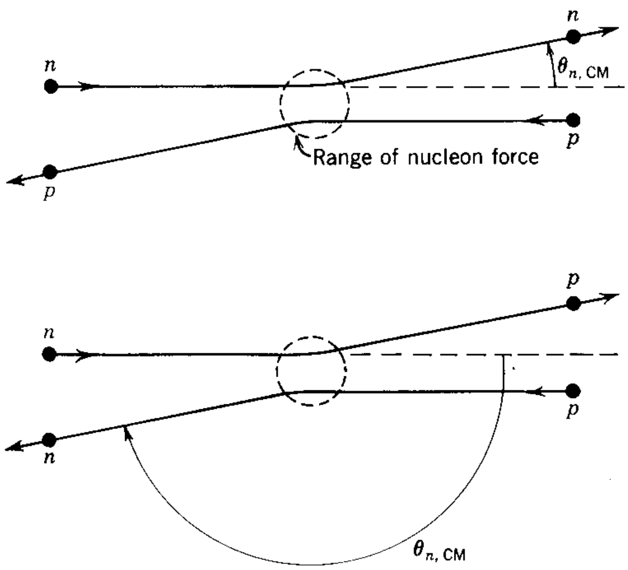


Figure 17-6 *Top:* Neutron-proton scattering as seen in a frame of reference in which the center of mass of the system is stationary. If the kinetic energies of the nucleons are large compared to the depth of the nucleon potential, the momentum transfers are small and the neutron and proton scattering angles are small as well. *Bottom:* The same, for a scattering in which the neutron changes into a proton and vice versa when they interact. Although the momentum transfers are still small, because of the exchange the scattering angles are large.

potential, a p changed to a n, and vice verca. This exchange effect is one of the terms in the N-N potential (called the exchange term)

$$V \approx \frac{V(r) + V(r)P}{2}$$

Scattering occurs via

$$\Psi_f^* V \Psi_i$$

so

$$V\Psi_i = \left[\frac{V(r) + V(r)P}{2}\right]\Psi_i$$

$$= \frac{V(r)}{2}\Psi_i + \frac{V(r)}{2}P\Psi_i$$

replacing i with ℓ one can write this as

$$\approx \frac{V(r)}{2}\Psi_\ell + \frac{V(r)}{2}P\Psi_\ell$$

since interchanging a P with a N is equivalent to spatial exchange, it is also equivalent to the parity operation. So

$$P\Psi_\ell = (-1)^\ell \Psi_\ell$$

so that

$$V\Psi_\ell = \frac{V(r)}{2}\Psi_\ell + \frac{V(r)}{2}P\Psi_\ell$$

$$= \frac{[1 + (-1)^\ell]}{2}V(r)$$

Thus if ℓ is odd V is near zero!

i.e. Nuclear potential depends strongly on ℓ.

It would be unfair of me to suggest that we have really covered N-N scattering. Most of the information about the nature of the N-N force was obtained from more complicated scattering than ℓ=0. I do not intend to cover that. You will find it in Enge Chap. 3 and Krane Chap. 4.

At higher values of ℓ the interference pattern becomes more complex, and the analysis more difficult (and more subjective).

Suffice it to say that at higher energies the ℓ-s components can be found, and as the energy gets higher the nucleon probes closer to the repulsive core, and this can be resolved.

To summarise some of the dependencies of the nuclear potential:

The central potential (a function of r only)
attractive short-range central potential with a repulsive core. Modelled as a square well for nucleons with $\ell=0$ and parallel spins

c = 0.4 fm (core width)
V_o = 73 MeV
width b = 1.337 fm

Orbital AM of the nucleons
However there are several components of the central potential that have different depences on r, depending on the relative AM ℓ of the nucleons.

Spin dependence
Depth significantly less for the p and n aligned anti-parallel. In general the potential depends on whether the two nucleons are in a relative singlet (S=0) or triplet (S=1) state.

We observed this from 2 facts.
- The S=0 state of the deuteron is unbound,
- The n-p scattering cross section is very different depending on whether the nucleons couple to S=1 or S=2.

Tensor potential

We have already noted that this is necessary to explain the QM of the deuteron. We stated that it must depend on S.r.

***l*.s dependence (spin-orbit)**
As well as the so-called static components to the nuclear potential, scattering experiments show that it also depends on dynamic variables such as the relative nucleon momentum (in practice the AM) .The simplest form of this is a term such as

$$\mathbf{V_{SO}(r)}\ \boldsymbol{l}.\mathbf{s}$$

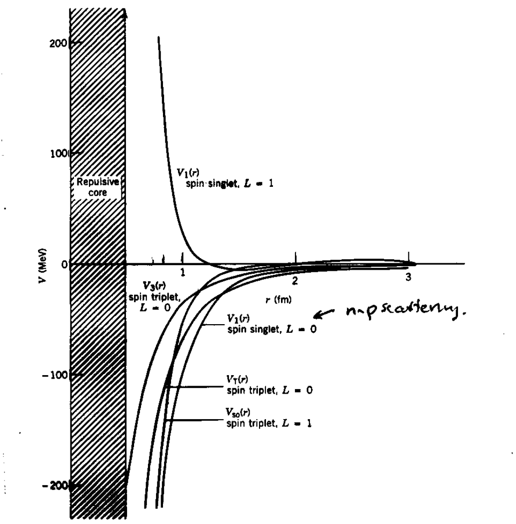
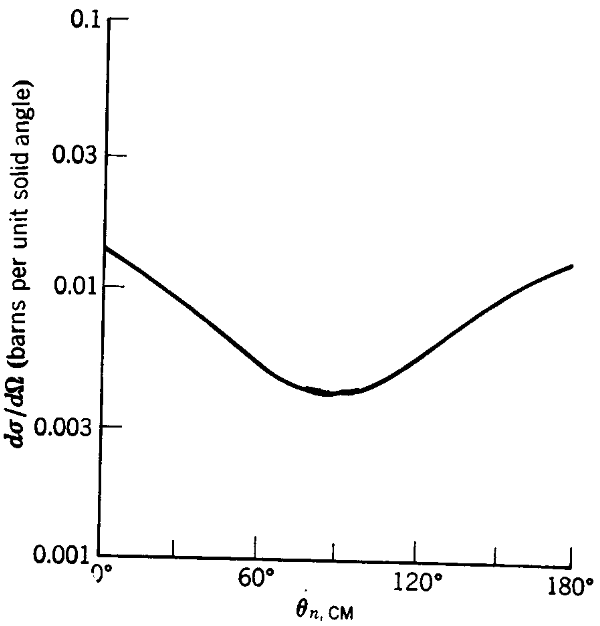


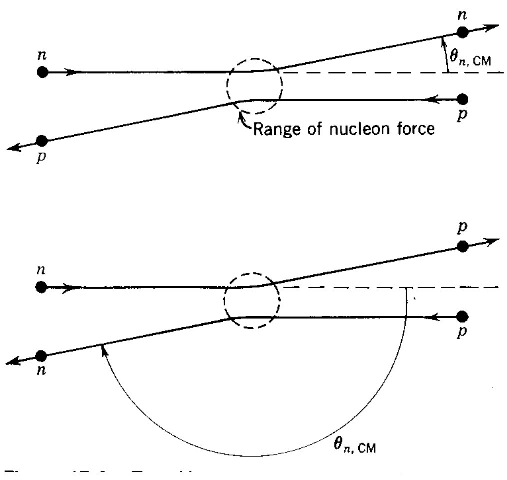
Figure 4.16 Some representative nucleon–nucleon potentials. Those shown include the attractive singlet and triplet terms that contribute to s-wave scattering, the repulsive term that gives one type of p-wave ($L = 1$) scattering, and the attractive tensor and spin-orbit terms. All potentials have a repulsive core at $r = 0.49$ fm. These curves are based on an early set of functional forms proposed by T. Hamada and I. D. Johnston, *Nucl. Phys.* **34**, 382 (1962); other relatively similar forms are in current use.

The figure to the left shows the forms of the various components of the nuclear potential

In essence one of the questions we need to ask, and to resolve at least qualitatively *is "how is this force promulgated*?"

A clue to the answer to this question can be got from the (n,p) scattering at higher energies data that I mentioned above.





This was unexpected, and particularly interesting relative to the microscopic source of the nucleon potential.

If we look at the reaction from the CM frame where the p and n approach with equal mom and energy we can see the reason.

What do we detect …neutrons! Where should they appear ? At forward angles since KE is greater than V_o, the higher KE the smaller the transverse impulse .
There are as many neutrons scattered through large angles as small. How come?

In about half the interactions, the neutron has been transformed into a proton! Viewed in the CM system, the exchanged particle scattering angle is small as it should be since KE is much greater than V_o, however the interpretation following the exchange results in an increase in large angle scattering.

This observation leads on to the last section of the lectures dealing with the N-N potential. I want to discuss the meson exchange theory of this potential.

π-meson theory of nuclear potential
We now look at the source of the binding potential between the nucleons. The so-called one pion exchange potential (OPEP). It was proposed way back in 1935, by Yukawa, that the **nucleons were surrounded by a field of massive particles**. These virtual particles now called π-mesons or pions, were emitted from the nucleon, and existed for a short time. At the end of their lifetime they were re-absorbed back into the nucleon. However if within their lifetime another nucleon came within the range of this field, the pion could be absorbed by this nucleon. The momentum would thus be transferred from one nucleon to the other, and transfer of impulse is akin to a force .

This exchange of mesons was postulated as the source of the nucleon attractive potential. The picture is a **direct analogy of the source of the electromagnetic force**. Each charge is pictured as surrounded with a field of virtual photons, and it is the exchange of these photons that produces the EM potential. Because the **photons are massless, they travel at the speed of light**, and can have an infinte range. The pions on the other hand have a **mass of about 140 MeV/c^2, and its range is consequently finite.** More on this in a moment.

There are several questions that should be raised:

1. **How come the existence of these virtual particles does not violate energy conservation?**
2. **How does exchange of particles lead to an attractive potential?**
3. **Does the model fit the bill?**

1 The mass of the pions
The pion comes in 3 kinds: π^+, π^- and π^o. The masses are

$\pi^+$and π^-	140 MeV/c^2
π^o	135 MeVc^2

This small difference has some implications that we will look at later.

You should recall that we said the nucleon was a particle with an isospin of ½, with two projections t_z = + ½ being a neutron, and t_z = - ½ being a proton. In the same way the pion is a particle that has an isospin t = 1, and thus can have 3 projections: t_z= +1 is a π^-, t_z = 0 is a π^o, and t_z = -1 is a π^+. It is an isospin triplet.

Now how come these pions can be created without violating conservation of energy?
In classical mechanics this is vorboten, however in quantum mechanics violations are allowed within the Heizenberg uncertainty limits.

$$\Delta E.\Delta t \sim \hbar$$

in this case the energy violation is 140 MeV/c^2, so we see that the pion can exist for a time

$$\Delta t \sim \hbar/\Delta E$$
$$\sim 6.5\ 10^{-16}\ (eV\text{-}s)/140\ 10^{6}\ (eV)$$
$$\sim 4.6\ 10^{-24}\ sec$$

not very long, however if we assume that the pion has a velocity close to that of light (an over estimate) it can travel a distance

~ 4.6 10-24 x 3 108 m.
~1.4 10-15 m,   1.4 fm

The approximate range of the nucleon potential.

Interestingly, the π meson although predicted in 1937 was not discovered until 1947 in cosmic ray studies. The pion is in fact unstable and decays to a mu meson.

A real meson can be made from the cloud of virtual mesons surrounding the nucleon by providing sufficient energy to "pay back the energy bank". This is usually done by bombarding a nucleon with another nucleon with at least 140 MeV of kinetic energy.

2 Why an attractive potential?
First let's look at what we might expect to explain the N-N potential.

In terms of the 90 MeV n-p scattering data we discussed before, where the AD was symmetric about 90 degrees. We said that this was explained in terms of a p → n. The mechanisms possible are:

$$n \rightarrow p + \pi^- \quad \text{and} \quad p \rightarrow n + \pi^+$$

That is a neutron emits a virtual π^- into its field and becomes a p, or conversely a p emits a virtual π^+ into its field and becomes a n. The interacting p or n absorbs the pion and changes into the opposite nucleon.

Interestingly in that scattering experiment, the p → n and n → p exchange accounted for about half of the events. The non-charge exchange is accounted for by the creation of a virtual π^0 meson.

i.e. $p \rightarrow p + \pi^o$ **and** $n \rightarrow n + \pi^o$.

p-p and n-n interactions can only come about by exchange of a neutral pion.

Note that these possibilities dictate that a proton can have in its meson field either a π^+ or a π^o, but never a π^-. A neutron on the other hand can have in its meson field either a π^- or a π^o, but never a π^+. Thus the charge distributions of these two nucleons will be markedly different.

High energy electron scattering, similar to that which we mentioned in lecture 1 confirms this

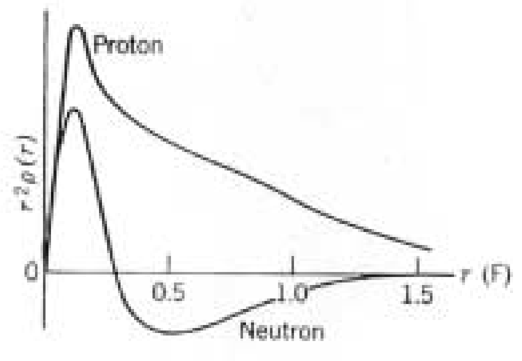


Figure 17-1
charge dens

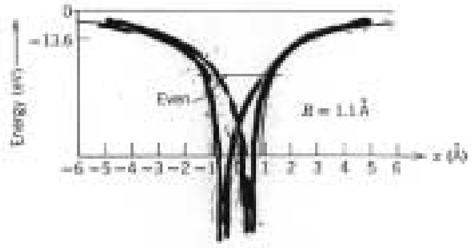
The proton charge distribution is everywhere positive, way out to the range of the nuclear potential. For the neutron it is positive at small r, where the p from n → p + π^- would be, it is positive, but where the π^- would be at larger r it is negative.

You no doubt have realised why the p and n have intrinsic magnetic dipole moments that are larger than for the electron, and in particular why the neutron has one at all!

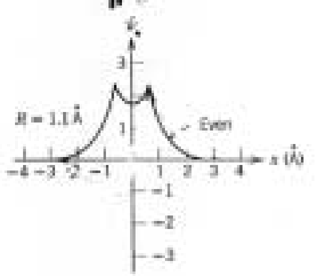
Exchange leading to attractive potential
Now the explanation as to how the exchange of a meson leads to an attractive force is not simple to understand. In any classical illustration, such as that used in 1st year texts of scatters exchanging pillows, leads to a repulsive force (I have actually thought of exchanging boomerangs, but this also leads to repulsion**). <u>There is no classical example</u>.**
The exchange force is a purely quantum mechanical phenomenon.

The example I have used, and is used by Eisberg in the notes I have given you, is the $_2H^+$ ion.

In this situation we have a stable combination of two +ve protons, which must repel each other, remaining stable by virtue of the exchange of the single electron between the two protons.



Here is the electrostatic potential of the electron as it approaches the two protons in the ion. The protons are separated by ~1.1 angstrom (10^{-10} m). You can see that this is the PE diagram for two protons. Note the energy scale. -13.6 eV is the BE of a single p-e system.

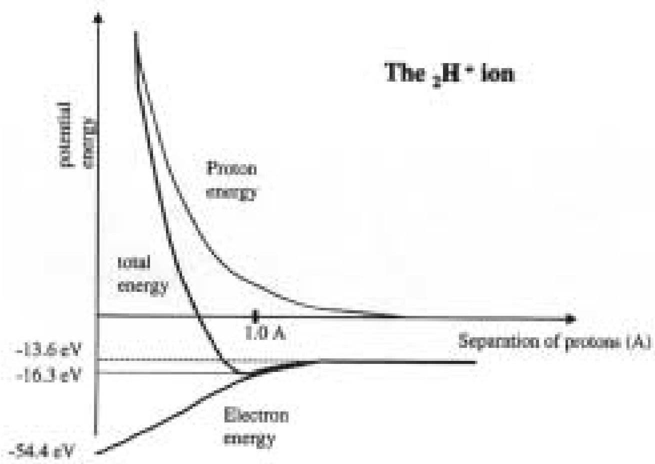


Similarly is shown the WF of the electron. Note again that it looks like a H s-state WF for R large. The important thing to note is that the value of

$|\psi|^2$ shows that the single electron spends most of its time BETWEEN the two protons.

Now we want to know if the p-p system will form a stable system with the energy of the system being a minimum for a particular separation of the protons. This is illustrated in the next figure.

Here the **energy of the total system** (e + 2p) is plotted as a function of **Proton separation.**



When the p's are well separated, the electron sees itself as belonging to one of them. Hence it has a potential energy of -13.6.

As the separation decreases the energy of the electron decreases. It begins to feel the attraction of the other proton. Remember it spends most of its time between the protons. The limit of this attractive potential on the electron would be when R=0, and we have Z=2 and the electron would bind as for the He^+ with BE 2^2 x 13.6 ev = -54.4 eV. On the other hand the repulsion between the protons can increase essentially to infinity, so it dominates, and we get the PE diagram shown.

Note that there is a minimum, and hence a stable separation for the ion. This comes about only because of the motion of the single electron which is exchanging between the protons.

This is an extremely good analogy for the OPEP situation.

3 Does it do the job?

Well it certainly gives a potential that has the basic shape to fit the experiments. I don't want to spend a lot of time on it, but you should know that the meson field that surrounds the nucleon is analogous to the photon field around a charge.

The electrostatic potential is obtained from the relativistic energy of a photon:

$$E^2 = p^2c^2$$

or $$-p^2 + E^2/c^2 = 0$$

replacing p and E with the appropriate operators:

$$E = i\hbar \frac{d}{dt}, \;\; p = -i\hbar\Delta$$

gives

$$(\hbar^2 \Delta^2 - \frac{\hbar^2}{c^2}\frac{d^2}{dt^2})V = 0$$

$$\Delta^2 V - \frac{1}{c^2}\frac{d^2 V}{dt^2} = 0$$

Laplace's equation

$$V = -\frac{e}{4\pi\varepsilon r}$$

for the meson with mass m the energy equation becomes

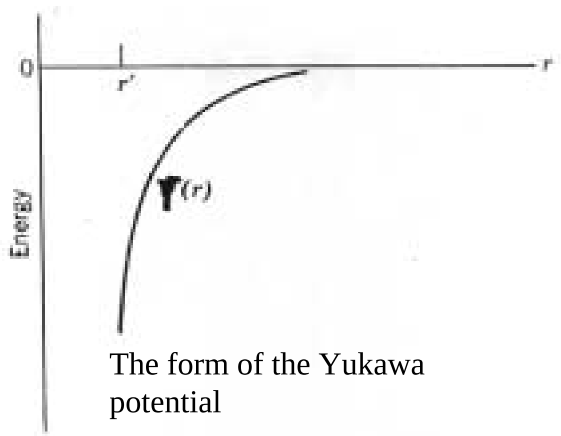
$E^2 = p^2c^2 + m_o^2c^4$ which leads to an equation

$$\Delta^2\varphi - \frac{1}{c^2}\frac{d^2\psi}{dt^2} = \frac{m_o c^2}{\hbar^2}\psi$$

Klein – Gordonequation

solutionis

$$\psi = g\frac{e^{-\mu r}}{r} \mathit{where} \mu = \frac{mc}{\hbar}$$



The form of the Yukawa potential

This is probably as far as I want to go with discussion of the N-N potential. But in moving on I should say that although we are now going to look at more complicated nuclei, it is a fact that all we can do for these is to provide models. These models have assumptions, and are always only approximations.