Lecture 7	**Krane**	**Enge**	**Cohen**	**Williams**
Shell Model	**Ch 5**	**6.2-**	**ch. 4**	**Ch 8**
complex nuclei	**5.1**	**6.4-7**	**ch 5**	**8.3/4/5**
Ang mom coupling	**2.5**	**6.3**	**5.13**	**8.3/4**
ground-state spins	**5.1**	**6.4**	**4.5**	**8.6**
excited states	**5.1**	**6.5**	**5.8**	**8.9**
magnetic moments	**5.1**	**6.6**	**7.4**	**8.7**
quadrupole moments	**5.1**		**7.5**	**8.8**

Questions Lecture 7

1 Give the expected shell-model spin and parity assignments for the ground states of ^{7}Li, ^{11}B, ^{13}C, ^{17}F, ^{31}P, ^{141}Pr.

2 Plot the shell-model potential for ℓ=0 protons, and neutrons in ^{208}Pb.

3 The low-lying states of ^{13}C are: GS ½$^-$, 3.09 MeV ½$^+$, 3.68 MeV, 3/2,.$^-$3.85 MeV, 5/2$^+$ The next states are above 7 MeV. Interpret the low-lying states in terms of the shell model.

4 Calculate the values of the mag. Dipole moments expected from the shell model, and compare with the exptal. values for ^{75}Ge, ^{87}Sr, ^{91}Zr, and ^{47}Sc.

5 Why are there no magic numbers that are odd?

6 ^{10}B has GS spin and parity 3$^+$. Show that the mass, charge, spin, and parity are consistent with a nucleus containing Z protons and A-Z neutrons. Which of these properties disagrees with the assumption that nuclei contain A protons and A-Z electrons?

Review Lecture 6

1 Evidence for meson exchange from forward peaking of n-p scattering at high energy.

2 Yukawa's OPEP (one pion exchange potential)

3 Creation and annihilation of virtual π mesons at surface on nucleons. allowed within the Heizenberg uncertainty limits. $\Delta E.\Delta t \sim \hbar$. Understand that for π mass of 140 MeV/c^2, the range is~1.4 10^{-15} m, 1.4 fm

4 Exchange of these field particles between nucleons communicates nuclear force.

5 a proton can have in its meson field either a π^+ or a π^o, but never a π^-. A neutron on the other hand can have in its meson field either a π^- or a π^o, but never a π^+.

6 Recall the charge distribution within π meson, and note why nucleon mag. Moments are so large.

7 Exchange of field particles leading to attractive force is a quantum effect (viz $_2H^+$ ion)

8 Note that the solution of Klein-Gordon equation for massed particles gives N-N potential (cf solution of Laplace equ gives electrostatic pot)

Lecture 7

The Shell Model

Today I want to introduce the **single particle** or **shell model** of the nucleus. It is one of the most successful, and as we shall see one of the most useful models available to us. It assumes that we can treat every nucleon in a nucleus as if it were confined by the average potential of all the other nucleons in the nucleus. It bears a great similarity to the shell model of the atom that you have known for several years, and have covered in some detail this year.

There is however something a bit strange if you think about it. In the case of the atom, there is a known central force that provides the potential for the “circulating” electrons. In either the semi-classical of quantum description of the atom the picture of the electrons forming shells around the central nucleus is there. This is somewhat harder to visualise in the case of the nucleus. Here the potential is provided by the nucleons that are internal to the nucleus.

Probably a more important question to raise, how can we have an “average” potential which must have a radius of the nucleus (~$A^{1/3}$), which is much bigger in general than the range of the nuclear force, which we have agreed is about 1.4 or so fm? And equally difficult to visualise is the idea of nucleons in the closely packed nucleus “orbiting round”? Surely they would collide so often that the concept of an orbit is far from the truth.

Well, if you wish to visualise them in orbits (and we will find this very useful) you should be reassured that in the quantum system the nucleons are all assigned unique energies. Although they might interact with other nucleons, there can be no change in energy, and we might well consider that the collisions did not occur.

As to the width of the **shell-model potential** we must consider that the nucleons, although limited to a **nucleon-nucleon potential** of only about 1 fm, do interact in their “orbits”, one nucleon at a time with all the nucleons in the nucleus. Thus the concept of an average nuclear potential is valid.

So the model is based on the premise that :

> **from the standpoint of any one nucleon, the forces acting on it by all the other nucleons in the nucleus can be represented to first approximation by an average potential—<u>the shell theory potential</u>**

What does this potential look like?

1 Basic form

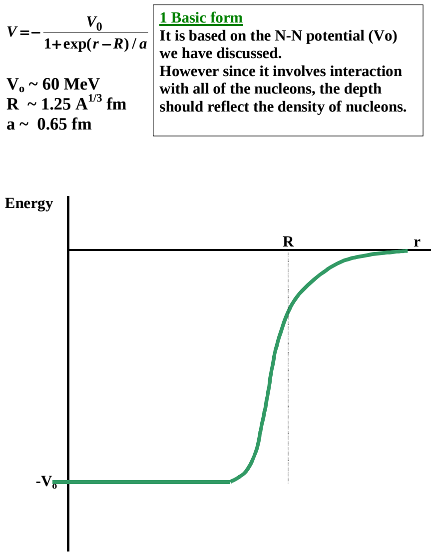
It must be based on the N-N potential (Vo) we have discussed previously. However since it involves interaction with all of the nucleons, the depth should reflect the density of nucleons. Our discussion in lecture 1 of the results of electrons scattering showed that

$$\rho(r) = \frac{\rho_0}{1 + \exp(r - R)/a}$$

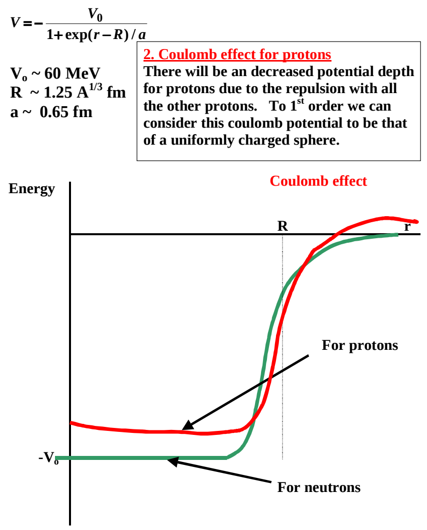
so the form of the potential is

$$V = -\frac{V_0}{1 + \exp(r - R)/a}$$

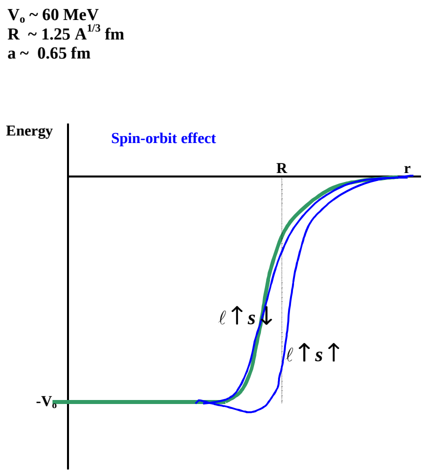
V_o is about 60 MeV
R~1.3 $A^{1/3}$ fm
a is the skin thickness ~0.65 fm



2 Coulomb effect for protons
There will be a **decrease in the depth of the potential for protons,** due to the interaction with all the other protons. To 1st order we can consider this coulomb potential to be that of a uniformly charged sphere.



3 Spin-Orbit interaction
Because we considered mainly the p-n system in an ℓ=0 AM state, I did not raise the dependence of the N-N potential on the relative alignments of ℓ and S. The fact is that the potential is deeper (nucleons more strongly attracted) if the ℓ and S are aligned parallel, and less deep if they are aligned opposing. This is in direct analogy with the alignment of the *s* for the nucleons in the deuteron. In the nucleus it is the ℓ relative to the nucleus that is involved.



Solution of Schroedinger's equation for a finite 3D square well
We can see that the potential is approximated by a square well, and it is this approximation that is made. So the allowed quantum states are obtained by solving the Schroedinger equation for a 3D square well, which you have studies, or know the solutions for.

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r\theta\varphi) = (E - V)\psi(r\theta\phi)$$

Where ***V*** is the shell model potential.

However the solutions will be for states with different AM ℓ, so the potential must include a centripetal term, so that the potential becomes:

$$V_o + \frac{l(l+1)\hbar^2}{2mr^2}$$

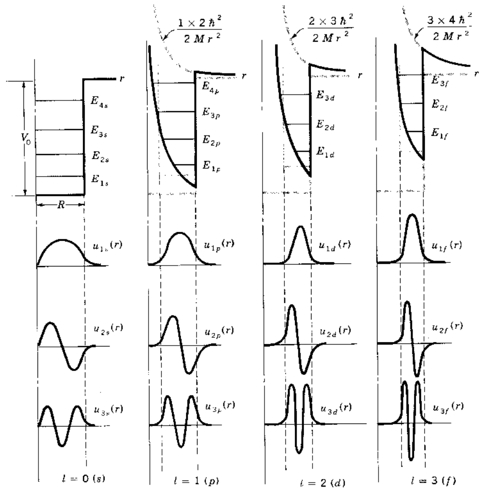
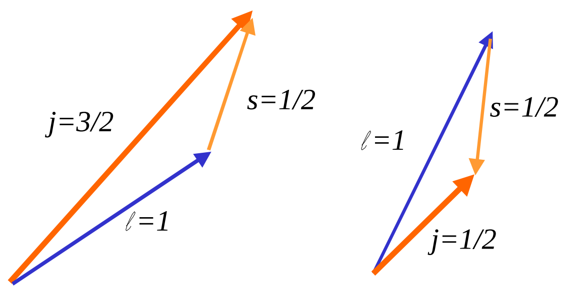


FIGURE 2-4 Energies and wave functions obtained from the solution of (2-18) for a square well. The four columns are the solutions for $l = 0, 1, 2$, and 3, respectively. The top row shows the two contributors to the effective potential, $V(r)$ and $V_{\ell f}$, as dashed lines and their sum, the effective potential, as solid lines. Square wells similar to these effective potentials are shown as dot-dash lines; they were used in drawing the wave functions which are plotted below. Note that the wave functions have n half wavelengths in the range of r covered by the potential well with exponential tails going rapidly to zero in the region outside the potential well. The energies of the states, shown in the top row, are moved upward as the well becomes shallower and narrower. The energies shown are not quantitatively correct for a square well.

Discuss the meaning of the quantum states, show the complement of states

Vector Coupling of Angular Momentum ℓ and s to give *j*

j = ℓ + ***s*** *(vector sum)*



In nuclear systems it is *j* that is good quantum number

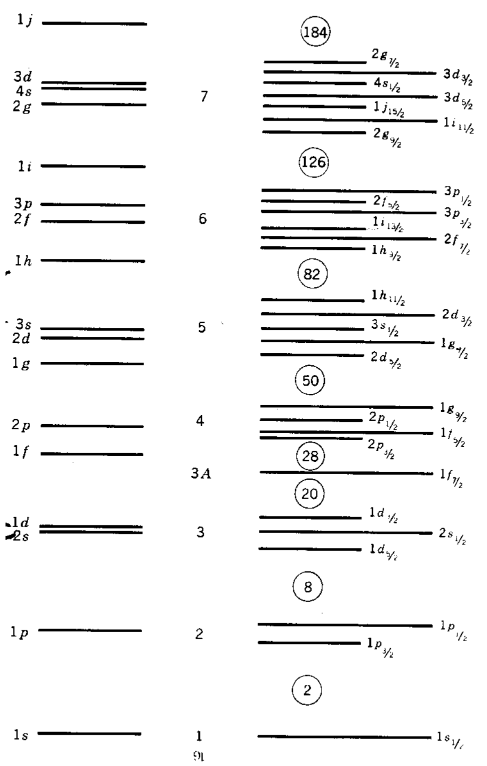


FIGURE 4-5 Energy levels in the shell-theory potential. The energy levels at the left are roughly what is expected without spin-orbit interaction. They are just the levels from Fig. 2-4 extended to larger potential wells. The right side shows the effect of the spin-orbit interaction in splitting each level (except $l = 0$) into two with $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$. The amount of this splitting is roughly proportional to l.

Discuss coupling of ℓ and s to give *j*.
Show effect of ℓ-s correction to potential well

Discuss grouping of levels and magic numbers.

The level sequence has been confirmed in a number of ways that we will discuss shortly.

The filling of the states

Let's just recap a few matters before moving on.

I want you to note that the reason the states with ℓ >0 split in energy is the result of the ℓ .s component of the N-N potential. For ℓ and s parallel the potential is deeper and for ℓ antiparallel to *s*, it is shallower.

Remember that $j = \ell + s$, so that the state with ℓ and s parallel has the larger *j*, and is **lower on the shell model level scheme**. Recall the deuteron s=1 (parallel) has deeper pot well than S=0 (antiparallel).

The other thing to remember is that because the ℓ .s coupling is so much stronger in the nuclear case compared to the atomic case, it is *j* that is the good quantum number in nuclear physics, not ℓ or *s*. So that when we work out the total AM of a nucleus with more than a single unpaired nucleon we will vector-sum the values of *j*.

The filling of the states

The level scheme we have seen indicates a set of quantum states available to nucleons as components of a nucleus. The object is, for a given nucleus specified by A, Z and N, to place nucleons in these states in order to produce the most stable (lowest energy) nucleus.

In doing this we must not violate the **Pauli exclusion principle**. We discussed this and indicated that this meant that the total wave function for any individual indistinguishable nucleon must be anti-symmetric. In practical terms this means that each nucleon must have a unique set of quantum numbers.

What quantum numbers do we have available.

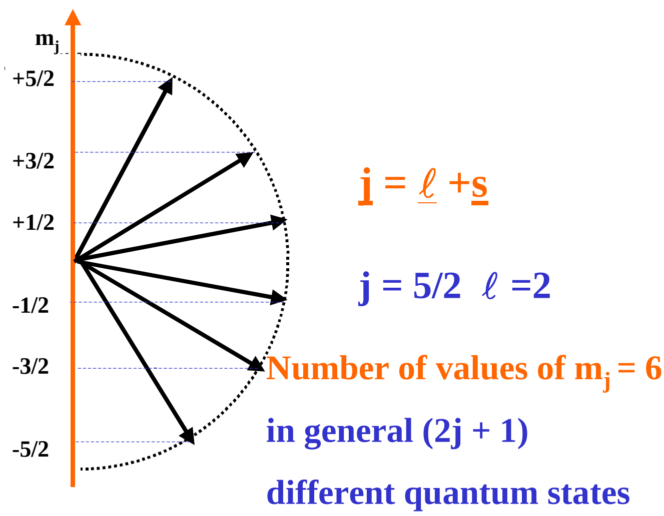
ℓ $\quad$ $\ell = 0$ (s), $\ell = 1$ (p), $\ell = 2$ (d) etc

s $\quad$ ½

t $\quad$ ½ $\quad$ $t_z = +$ ½ (neutron) $\quad$ $t_z = -$ ½ (proton)

However in the nucleus ℓ and s combine to give *j*, where $\boldsymbol{j} = \boldsymbol{\ell} + \mathbf{s}$ (see figure previous page), and it is QN *j* that determines the state.

***j* has 2j + 1 projections of m_j** on to the the z axis. So there are for any state specified by QN j, 2j + 1 sub states with different QN m_j. The vugraph shows how this might be envisaged in terms of precessing AM vectors. This may or may not be helpful for you. But this is why I will often draw in the nucleons in a given j-shell as ⇑⇓. The nucleons DO pair off ($+m_j$, $-m_j$) as they are added to a nucleus in order to minimise the



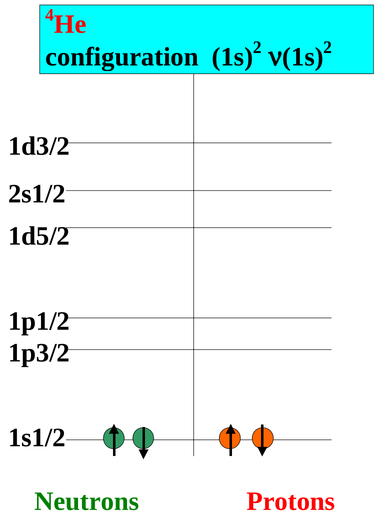
energy of the system. We saw in an earlier lecture that the S_n for even N or Z nuclei was greater by several MeV than the S_n for odd nuclei.

Thus in a sub-shell with j = 3/2 we can put 2 x 3/2 + 1 = 4 protons, each having a unique assigned m_j.

We can also put in 4 neutrons even though a neutron and a proton will have the same value of m_j . This is because there is another QN associated with the nucleon itself: its isospin. So each n has an isospin QN t_z of +1/2, and each p a t_z = -1/2. So we can have **up to 2j +1 protons and 2j+ 1 neutrons** in each subshell.
So we can see why the magic numbers occur at the values they do.

As an example take ^{4}He. A=4, N=2, Z=2. The lowest available unoccupied shell is the 1s shell. Here *j* = ½.

	n_1	n_2	p_1	p_2
m_j	**+1/2**	**-1/2**	**+1/2**	**-1/2**
t_z	**+1/2**	**+1/2**	**-1/2**	**-1/2**



The next **major** shell closes at n = 8
2 p and 2 n in s1/2→ 4 ^{4}He
2 p and 2 n in p1/2 → 4 ^{16}O

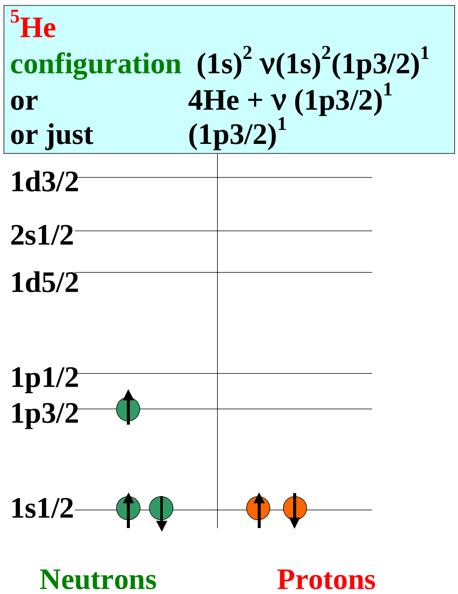
and so on to n = 20 → ^{40}Ca

Ground state spins of odd N or Z nuclei

The spin I of a nucleus is the vector sum of the AM ***j*** of all the nucleons in it. As mentioned protons will pair off, and neutrons will pair off so that a **nucleus with Z even and N even will have a spin I = 0, and even parity,** eg ^{4}He, ^{12}C ,^{16}O.

In a nucleus with a **single unpaired nucleon** the nuclear spin is that of the unpaired nucleon. i.e. I = j.

eg	^{5}He,	n in 1p3/2 shell	I = 3/2
	^{5}Li	p in 1p3/2 shell	I = 3/2
	^{13}C	n in 1p1/2 shell	I = ½
	^{17}O	n in d5/2 shell	I = 5/2
	^{17}F	p in d5/2 shell	I = 5/2



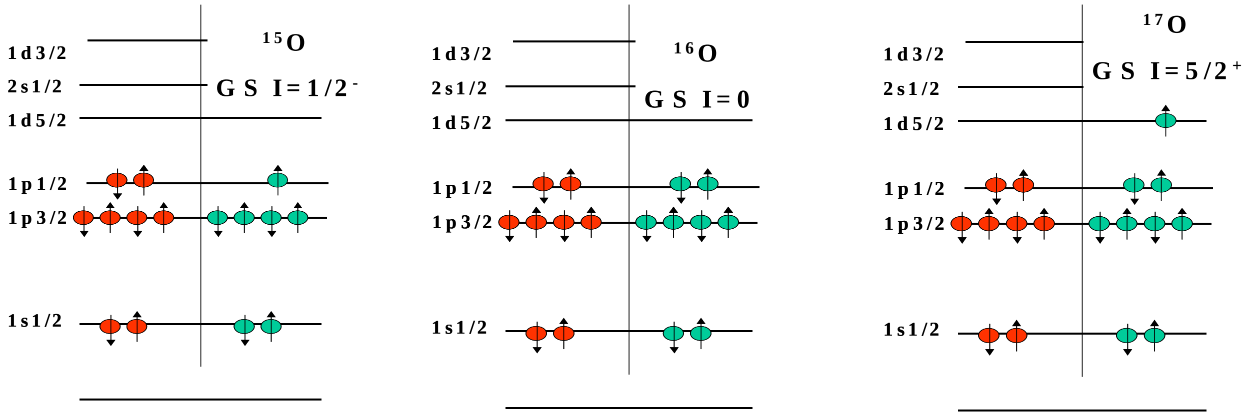


Table 6–2. Shell-model terms and observed ground-state angular momenta

Nuclide	Z	N	Shell-model terms	Observed J^π
O^{17}	8		$(1s_{1/2})^2\ (1p_{3/2})^4\ (1p_{1/2})^2$	$5/2^+$
		9	$(1s_{1/2})(1p_{3/2})^4\ (1p_{1/2})^2\ (1d_{5/2})^1$	
Al^{27}	13		$(1s_{1/2})^2\ (1p_{3/2})^4\ (1p_{1/2})^2\ (1d_{5/2})^5$	$5/2^+$
		14	$(1s_{1/2})^2\ (1p_{3/2})^4\ (1p_{1/2})^2\ (1d_{5/2})^6$	
K^{39}	19		$(1s_{1/2})^2\ (1p_{3/2})^4\ (1p_{1/2})^2\ (1d_{5/2})^6\ (2s_{1/2})^2\ (1d_{3/2})^3$	$3/2^+$
		20	$(1s_{1/2})^2\ (1p_{3/2})^4\ (1p_{1/2})^2\ (1d_{5/2})^6\ (2s_{1/2})^2\ (1d_{3/2})^4$	
Zn^{67}	30		(28 protons) $(2p_{3/2})^2$	$5/2^-$
		37	(28 neutrons) $(2p_{3/2})^4\ (1f_{5/2})^5$	
Mo^{95}	42		(28 protons) $(2p_{3/2})^4\ (1f_{5/2})^6\ (2p_{1/2})^2\ (1g_{9/2})^2$	$5/2^+$
		53	(50 neutrons) $(2d_{5/2})^3$	