Lecture 8	**Krane**	**Enge**	**Cohen**	**Williams**
Shell Model	**Ch 5**	**6.2-**	**ch. 4**	**Ch 8**
excited states	**5.1**	**6.5**	**5.8**	**8.9**
magnetic moments	**5.1**	**6.6**	**7.4**	**8.7**
quadrupole moments	**5.1**		**7.5**	**8.8**

Questions Lecture 8

1 Why are the nuclear magic numbers different from the atomic ones?

2 How can the data on mag. Dipole moments of nuclei as plotted in Krane Fig. 5.9, be used to identify the orbital AM Q. No. ℓ of many nuclei when their nuclear AM Q. No. known?

3 Use the shell model to predict the ground state of ^{17}O: the spin, parity, ,sign of mag. Dipole moment, sign of electric quadrupole moment.

4 The ground state of a nucleus with an odd proton and an odd neutron is obtained by coupling the j of the unpaired nucleons. $I = j_p + j_n$. For ^{16}N (2^-) and ^{12}B (1^+) draw simplw diagrams illustrating these couplings. Now replace j_p and j_n with ℓ_n+s_n, and ℓ_p+s_p . From your diagrams deduce an empirical rule for the relative orientation of s_n and s_p in the ground state. Use this rule to determin I^π for the GS of ^{26}Na and ^{28}Na.

5 Calculate the mag. Dipole moments using the shell model, for the following nuclei, and compare them with the measured values.

nuclide	I^π	μ (exp)
^{75}**Ge**	$½^-$	-0.510
^{87}**Sr**	$9/2^+$	-1.093
^{91}**Zr**	$5/2^+$	-1.304
^{147}**Eu**	$11/2^-$	+6.06

Review Lecture 7

Shell Model

1 from the standpoint of any one nucleon, the forces acting on it by all the other nucleons in the nucleus can be represented to first approximation by an average potential—the shell theory potential

2 **Basic form** It must be based on the N-N potential (Vo) we have discussed previously. However since it involves interaction with all of the nucleons, the depth should reflect the density of nucleons. Our discussion in lecture 1 of the results of electrons scattering showed that

$\rho(r) = \frac{\rho_o}{1+\exp(r-R)/a}$ so the form of the potential is $V = -\frac{V_0}{1+\exp(r-R)/a}$

V_o is about 60 MeV R~1.3 $A^{1/3}$ fm a is the skin thickness ~0.65 fm

3 Coulomb effect for protons
Spin-Orbit interaction

4 Solve Schroedinger equ. for shell model $V + \frac{l(l+1)\hbar^2}{2mr^2}$ pot.
With centripetal potential to give allowed states with different ℓ.

5 Understand couplings of ℓ and s to give j

6 Understand ℓ-s coupling lowers energy of states with ℓ parallel to s

7 Note the grouping of levels and "magic numbers"

8 Filling of the states. In doing this must not violate Pauli principle. *J* has 2j + 1 projections (mj) onto the z axis. So there are, for any state specified by QN j, 2j+1 sub states with different QN m_j. So 2j+1 protons and 2j+1 neutrons can go into each state.

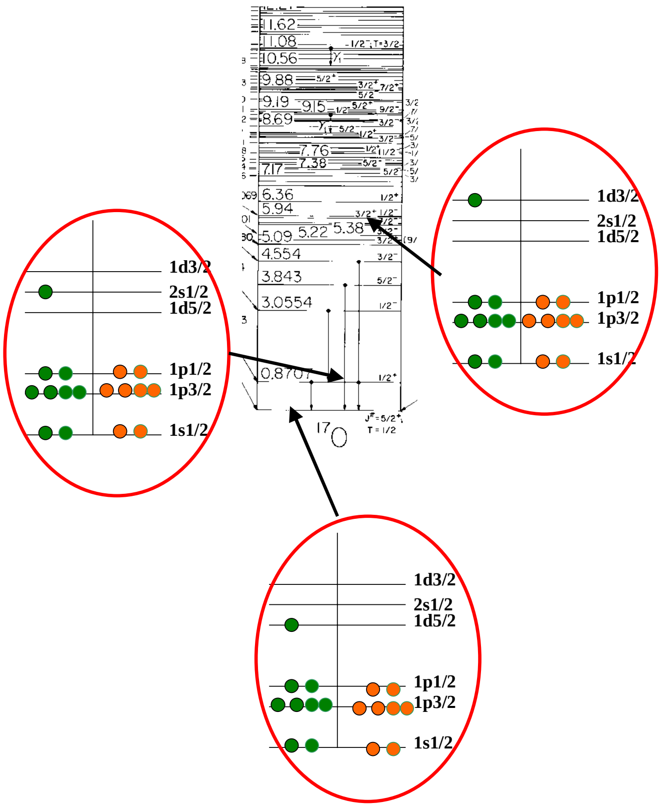
9 Be able to define the configuration of the GS of even-even nuclei, and nuclei with one unpaired nucleon.

Lecture 8

Excited states
What about excited state spins and parity?

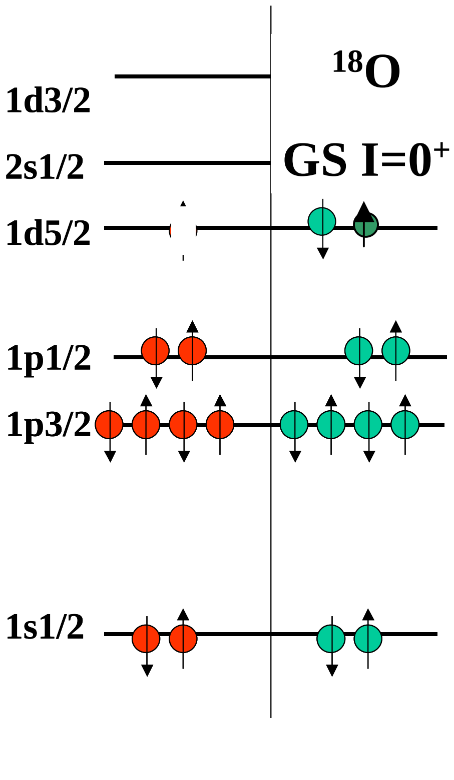
In simple cases there is some joy

Some simple excited state configurations in ^{17}O



What about when there is more than 1 uncoupled nucleon

Consider case of ^{18}O.



Discuss coupling of AM
Here there are two d5/2 neutrons and they can give states in ^{18}O, by coupling the AM vectors consistent with the Pauli princple.

Show coupling of 2x 5/2 neutrons.

TABLE 5-3: VARIOUS WAYS OF PUTTING TWO IDENTICAL NUCLEONS IN $d_{5/2}$ ORBITS

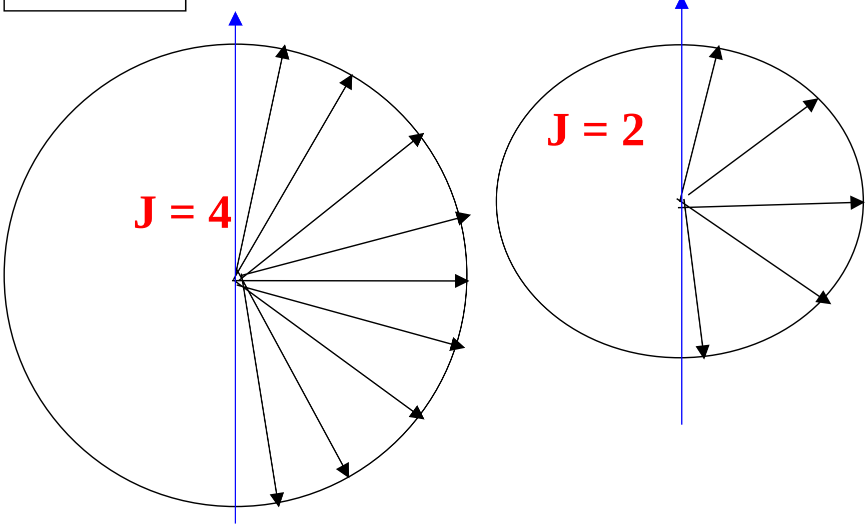
m	Occupied orbits														
$+5/2$	x	x	x	x	x										
$+3/2$	x					x	x	x	x						
$+1/2$		x				x				x	x	x			
$-1/2$			x				x			x			x	x	
$-3/2$				x				x			x		x		x
$-5/2$					x				x			x		x	x
Σm	+4	+3	+2	+1	0	+2	+1	0	−1	0	−1	−2	−2	−3	−4

J: 4(2) 3(2) 2(4) 1(4) 0(2)

$m_j=4$ (1)
$m_j=3$ (1)
$m_j=2$ (2)
$m_j=1$ (2)
$m_j=0$ (3)
$m_j=-1$ (2)
$m_j=-2$ (2)
$m_j=--3$ (1)
$m_j=-4$ (1)

Some Results from Angular-momentum Coupling

j = 4 (9 projections of m_j)
j = 2 (5 projections of m_j)
j = 0 (1 projections of m_j



Note this says that these two neutrons can couple to 3 states with j = 0, 2, 4.. We can also get this level structure by considering rotation of the nucleus as a whole, and these states would then be quantum states of rotation.

We will come back to this nucleus later. As we can also get this level structure by considering rotation of the nucleus as a whole, and these states would then be quantum states of rotation.

Magnetic dipole moments for single particle shell-model nuclei

As well as predicting the GS spins of single particle nuclei, the shell model also predicts (though not so well) the magnetic dipole moments.

You might refer back to lecture 3 when we discussed the natural consequence of charged protons and spinning charges setting up magnetic fields.

For a proton with AM QN ℓ we found that expressed in units of the nuclear magneton the observed dipole moment due to its motion was

$\mu_\ell = g_\ell \, \ell$ units of NM

$g_\ell = 1$ for a proton
` $g_\ell = 0$ for a neutron

We also found that the spinning proton, and strangely the spinning neutron also produced a mag dipole moment

$\mu_s = g_s \, s$ units of NM

$g_s = 5.585$ for a proton → $\mu_s = 2.79$ NM
$g_s = -3.826$ for a neutron → $\mu_s = -1.91$ NM

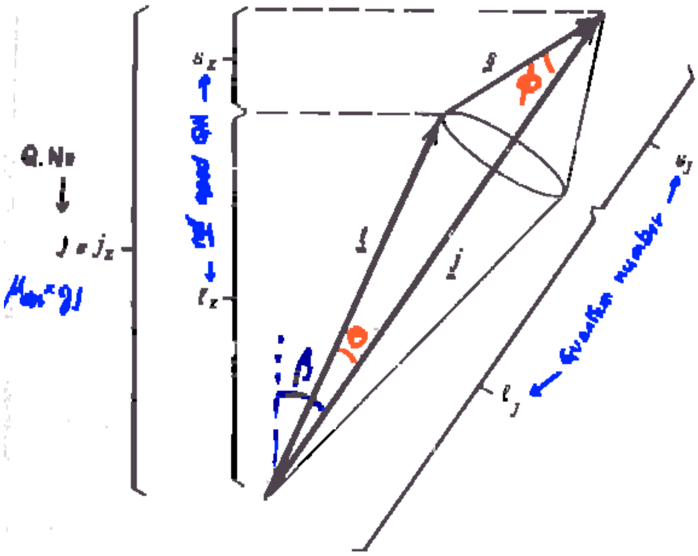
Note that it is ℓ that is important in producing the dipole moment. However in the nucleus it is the total AM j that is a conserved quantity (or a good quantum number, or is a constant of motion, or has fixed projections j_m) and ℓ and s are **not** good quantum numbers (i.e. do **not** have fixed projections on the z-axis; only on to *j*,). **So that when the nucleus is put into a magnetic field it is vector *j* that starts to precess with fixed values of j_m along the z axis.**

Thus the observed mag moment is

$\mu_{obs} = g \, m_{z\,max} = g \, j$

This will be due to the projections of μ_s and μ_l on to the z axis.

The vectors ℓ and s have sharp projections on to the direction of vector j,
so that the value of the intrinsic MM μ_{int} can be found.



$$\mu_{\text{int}} = \mu_l \cos\theta \; + \mu_s \cos\phi$$
$$= g_l |l| \cos\theta + g_s |s| \cos\phi \qquad \text{equ 1}$$

but because of precession they have varying projections onto the z axis. One can find the projection of μ_{int} on to the z axis to give μ_{obs}

$$\mu_{obs} = g\, m_{j_{\max}} = g\, j = \mu_{\text{int}} \cos\beta$$

$$\cos\beta = \frac{j}{|j|}$$

equ 2

$$so\ that\ \ gj = \mu_{\text{int}} \frac{j}{|j|} \ \ or\ \ g = \frac{\mu_{\text{int}}}{|j|}$$

μ_{int} can be found from equ 1

Using the cosine rule expressions for $\cos\theta$ and $\cos\phi$ can be found in terms of j, l and s. Leading to

$$\mathbf{g} = \mathbf{g}_\ell \frac{\mathbf{j(j+1)} + \ell(\ell+1) + \mathbf{s(s+1)}}{\mathbf{2j(j+1)}} + \mathbf{g_s} \frac{\mathbf{j(j+1)} + \mathbf{s(s+1)} - \ell(\ell+1)}{\mathbf{2j(j+1)}}$$

There are two possible ways s and ℓ can combine to give j stretched jackknife

here ℓ = j- ½ here ℓ = j + ½

from above

$\mu = gj = \frac{1}{2} g_s + (j - \frac{1}{2} g_l)$ $\mu = gj = 1/(j+1)[(-\frac{1}{2} g_S + (j + 3/2) g_l]$

These limits are called the Schmitt limits.
examples

^{17}O j = $5/2^+$ due to d5/2 neutron
g_s for neutron = -3.82 = 0

thus μ = - 1.91 NM (meas -1.89)

^{13}C j = $½^-$ p1/2 neutron
g_s for neutron = -3.82 = 0

thus μ = +0.67 NM (meas +0.7)

^{19}F j= $½^+$ proton in s1/2
calc 2.79 (meas 2.63)

^{39}K j = $3/2^+$ proton in d3/2
calc 0.09 (meas 0.39)

In general the agreement is really not so good. In the diagram are shown values for many single particle nuclei with a single proton or neutron. The limits are shown, and we see that few nuclei actually agree with the lines.

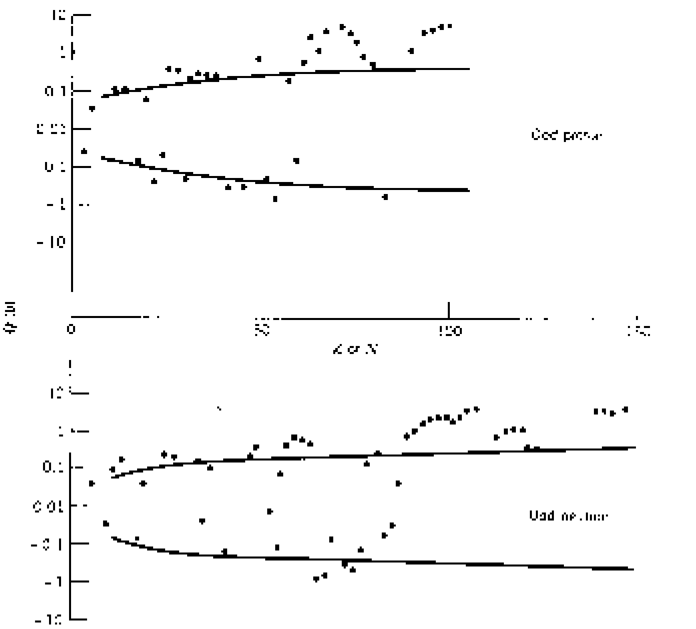


Figure 5.10 Experimental values of electric quadrupole moments of odd-neutron and odd-proton nuclei. The solid lines show the limits $Q \approx \langle r^2 \rangle$ expected for shell-model nuclei. The data are within the limits, except for the regions 60 < Z < 80, Z > 90, 90 < N < 120, and N > 140, where the experimental values are more than an order of magnitude larger than predicted by the shell model.

The reasons for this include (1) Shell model assumptions of spherical nuclei is not always true. (2) the value of g_s, (which is due to the meson cloud) for nucleons INSIDE a nucleus is not that of free nucleon. (3) The wavefunctions are never a simple as we assume.