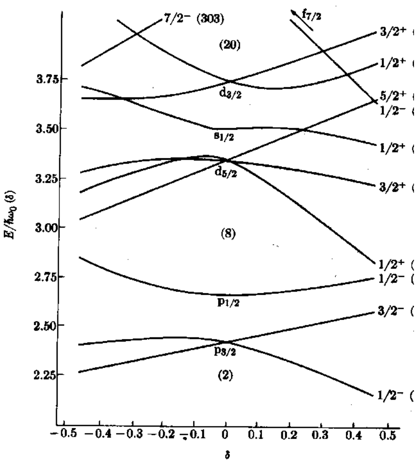
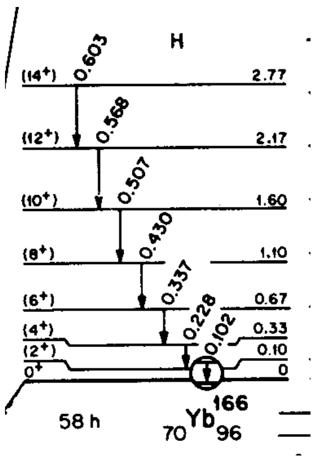
	Krane	Enge	Cohen	Williams
Lecture 9				
Shell Model	**Ch 5**	**6.2-**	**ch. 4**	**Ch 8**
quadrupole moments	**5.1**		**7.5**	**8.8**
deformed nuclei	**5.3**	**6.8**	**6.4**	
Collective models	**5.2**	**6.9-10**	**ch 6**	**8.10**

Questions Lecture 9

1 The GS configuratioPn of ^{18}F is $(1d5/2)_{\pi}(1d5/2)\nu$ (where π and ν indicate proton and neutron), outside a core of ^{16}O.
(a) What are the possible values of I^{π} for the states that could be formed?
(b) What would be the I^{π} for the states associated with the GS configuration of ^{18}O?

2 (a) On the basis of the simple shell model, what would be the GPS I^{π} for $_9F^{19}$, $_{10}Ne^{19}$, and $_{11}Na^{23}$?
(b) Explain why the measured values are ½ $^+$, ½$^+$, and 3/2$^+$ respectively.





3 Shown is the level diagram for ^{166}Yb.
(a) What model best explains this structure?
(b) Calculate the moment o inertia of ^{166}Yb on the basis of the spacing between the GS and 2$^+$ state, and between then 8$^+$ and 10$^+$ states.
(c) Explain the difference if any.

Review Lecture 8

Shell Model

1 Filling of the states. In doing this must not violate Pauli principle. *J* has 2j + 1 projections (mj) onto the z axis. So there are, for any state specified by QN j, 2j+1 sub states with different QN m_j.

2 Be able to define the configuration of the GS of even-even nuclei, and nuclei with one unpaired nucleon.

3 Understand the configuration of excited states with one unpaired nucleon.

4 Coupling of AM vector *j* of all unpaired nucleons to give spins of low-lying states.

5 The unavailabity of certain couplings that are forbidden by the Pauli principle..e.g. ^{18}O.

6 Magnetic dipole moments for single particle shell-model nuclei

$\mu_\ell = g_\ell\, \ell$ units of NM $g_\ell = 1$ for a proton $g_\ell = 0$ for a neutron

$\mu_s = g_s$ units of NM $g_s = 5.585$ for a proton $g_s = -3.826$ for a neutron

Note that it is ℓ that is important in producing the dipole moment. In the nucleus it is the total AM j that is a conserved quantity and ℓ and s are not good quantum. So that when the nucleus is put into a magnetic field it is vector j that starts to precess with fixed values of j_m along the z axis. Thus the observed mag moment is $\mu_{obs} = g\, m_{z\ max} = g\, j$. This will be the projections of μ_s and μ_l on to the z axis. The vectors ℓ and s have sharp projections on to the direction of vector j, so that the value of the intrinsic MM μ_{int} can be found.

7 Two couplings of ℓ and s stretched $\ell = j - ½$, and jackknife $\ell = j + ½$

The limits for μ are called the Schmitt limits.

Examples ^{17}O $j = 5/2^+$ due to d5/2 neutron ^{13}C $j = ½^-$ p1/2 neutron

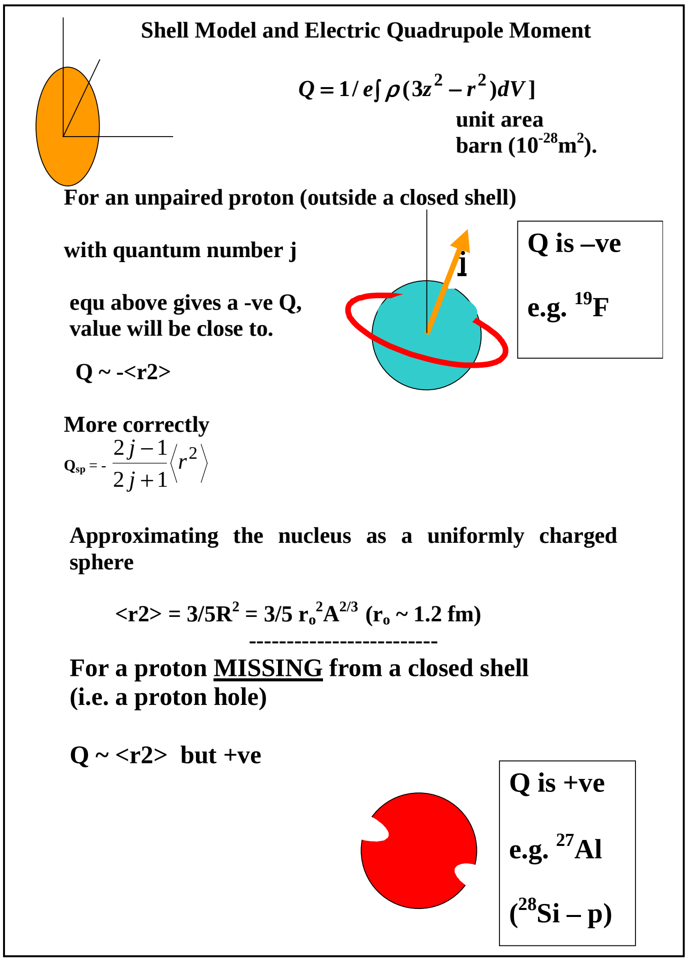
Quadrupole moments for single-particle nuclei

You may recall from lect 3 that the electric quadrupole moment was defined as:

$$Q = 1/e\int \rho(3z^2 - r^2)dV]$$ **units of area usually barn (10^{-28} m^2).**

Where ρ is the charge density distribution.

Can the shell model make any statement about the values of Q?



Well in the case of a nucleus with a **single unpaired proton** it should. Remember only protons can contribute to Q.
If the proton is an orbit of given j, its maximum projection m_j will put vector j close to alignment with the z axis. In which case the proton will be in orbit in the x-y plane. From the equ above this will give a -ve Q, and its value will be close to the expectation value of the radius of the nucleus.

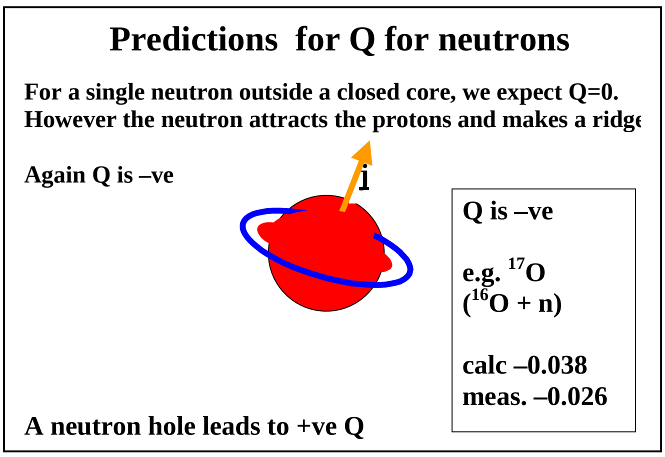
Ie Q = -<r^2>

In a more correct quantum mechanical treatment (see Meyer and Jenson "Elementary Theory of Nuclear Structure p 231)

$$Q_{sp} = -\frac{2j-1}{2j+1}\langle r^2\rangle$$

for a uniformly charged sphere <r^2> = 3/5R^2 = 3/5 $r_o^2 A^{2/3}$ (r_o ~ 1.2 fm)

If a single unpaired proton gives a -ve Q, a missing proton, (a proton hole should lead to a +ve Q.



What about the unpaired neutron case? Neutrons have no charge and one would expect no contribution to Q.
Well imagine a neutron whizzing around the nucleus. Because of the nuclear force it will pull out a tide of nuclear matter, again in the x-y plane. So again we might expect a -ve Q (actually a small effect)

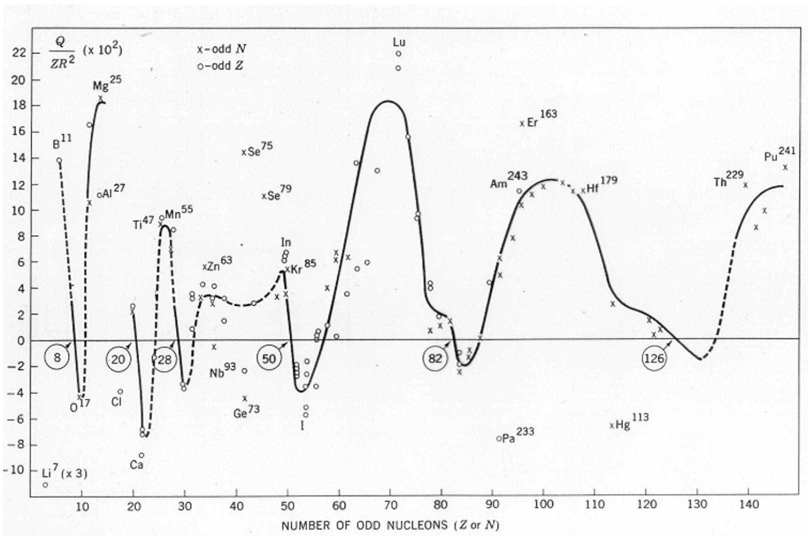
eg ^{17}O calc-0.038 measured -0.026

Table 5.1 Shell-Model Quadrupole Moments

Shell-Model State	Calculated Q (single proton)	Measured Q: Single Particle, p	Measured Q: Single Particle, n	Measured Q: Single Hole, p	Measured Q: Single Hole, n
$1p_{3/2}$	-0.013	$-0.0366(^{7}Li)$		$+0.0407(^{11}B)$	$+0.053(^{9}Be)$
$1d_{5/2}$	-0.036	$-0.12(^{19}F)$	$-0.026(^{17}O)$	$+0.140(^{27}Al)$	$+0.201(^{25}Mg)$
$1d_{3/2}$	-0.037	$-0.08249(^{35}Cl)$	$-0.064(^{33}S)$	$+0.056(^{39}K)$	$+0.45(^{35}S)$
$1f_{7/2}$	-0.071	$-0.26(^{43}Sc)$	$-0.080(^{41}Ca)$	$+0.40(^{59}Co)$	$+0.24(^{49}Ti)$
$2p_{3/2}$	-0.055	$-0.209(^{63}Cu)$	$-0.0285(^{53}Cr)$	$+0.195(^{67}Ga)$	$+0.20(^{57}Fe)$
$1f_{5/2}$	-0.086		$-0.20(^{61}Ni)$	$+0.274(^{85}Rb)$	$+0.15(^{67}Zn)$
$1g_{9/2}$	-0.13	$-0.32(^{93}Nb)$	$-0.17(^{73}Ge)$	$+0.86(^{115}In)$	$+0.45(^{85}Kr)$
$1g_{7/2}$	-0.14	$-0.49(^{123}Sb)$		$+0.20(^{139}La)$	
$2d_{5/2}$	-0.12	$-0.36(^{121}Sb)$	$-0.236(^{91}Zr)$		$+0.44(^{111}Cd)$

Data for this table are derived primarily from the compilation of V. S. Shirley in the *Table of Isotopes*, 7th ed. (New York: Wiley, 1978). The uncertainties in the values are typically a few parts in the last quoted significant digit.

Strongly deformed Nuclei



As you can see most nuclei are not spherical. Some have extreme deformations, particularly as A gets larger.

The Shell model can account very well for GS spins and parities, some excited states properties, selected magnetic moments and quadrupole moments, of nuclei that are near closed shells, however we shall see that it fails for nuclei with large deformations.

The shell Model so far

How good is it???

Property	adequacy	limitations
Ground-state spin	✓ ✓ ✓	
Ground state parity	✓ ✓ ✓	
Excited state j^{π}	✓ ✓	Complicated for more than 1 unpaired nucleon
Magnetic dipole moment	✓	Only those near closed shells agree with Schmitt limit
Observed quadrupole moment	✓	Generally gives correct sign for closed-shell+1 nuclei

We understand how the Shell model can account very well for GS spins and parities of the GS of many nuclei.

- ✓ e.g. closed shells 0^+
- ✓ unpaired nucleon spin is spin of nucleus
  e.g. ^{5}He, ^{13}C, ^{17}O

Its ability to account for the measured magnetic dipole moments and electric quadrupole moments is limited, but acceptable for nuclei near closed shells. We will now look at variations of the model for deformed nuclei, away from closed shells.

Look at ^{19}F

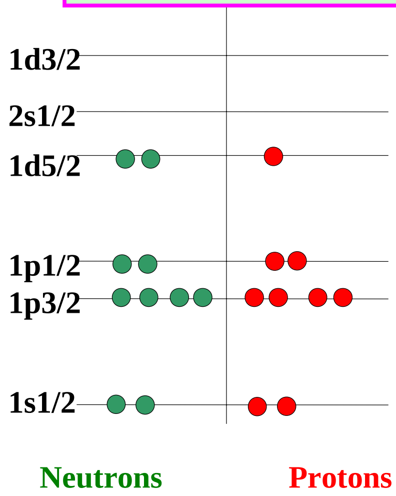
Ground-state j^{π}
Prediction….5/2$^+$
Observed…..1/2$^+$???????

Z= 9 N=10

What do we expect?===➔ 5/2!!

What do we see===➔ ½!!

Why?

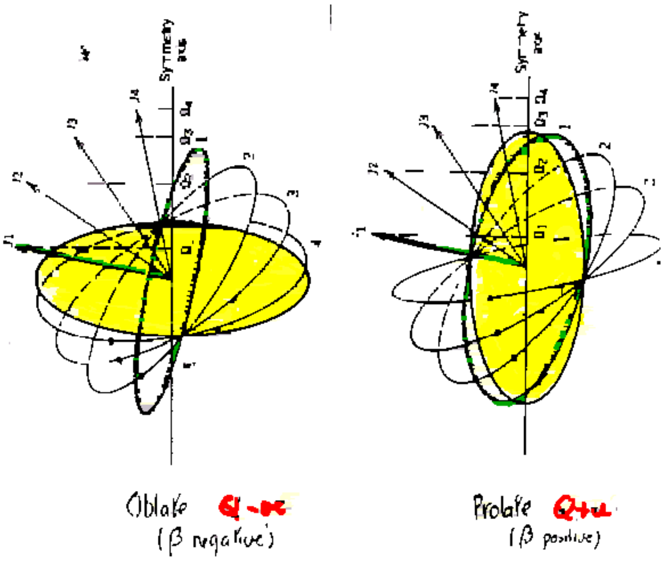


^{19}F is deformed, and if you recall our discussion of the shell model assumed that the nucleons move in a potential that is the average of all the potentials of the component nucleons. If the nucleus is spherical this potential is also spherically symmetric, and the solution we discussed was valid.

However as soon as we have a significant distortion of the mass (and charge) distribution, as indicated by the measured electric quadrupole moment, we have a potential that is no longer spherically symmetric, but axially symmetric.

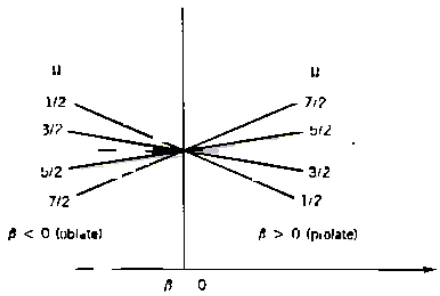
Essentially Nilsson who was a student of Neils Bohr of Copenhagen University solved this shell model for deformed nuclei.

For a deformed nucleus you can imagine, in a semiclassical picture, that as a nucleon moves around its orbit it will find the potential different due to the deformation of the nucleus. Q mechanically the probability distribution will lead to the same effect .



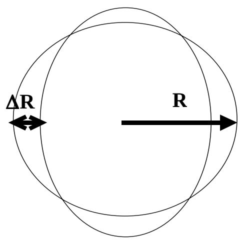
As a result, the energy of the state will change depending on the degree of deformation. Importantly the energy of the sub states of a given *j* will be different, since they correspond to different orientations of the orbit within the nucleus.

A qualitative explanation for this splitting of orbits of a given *j* is given below.



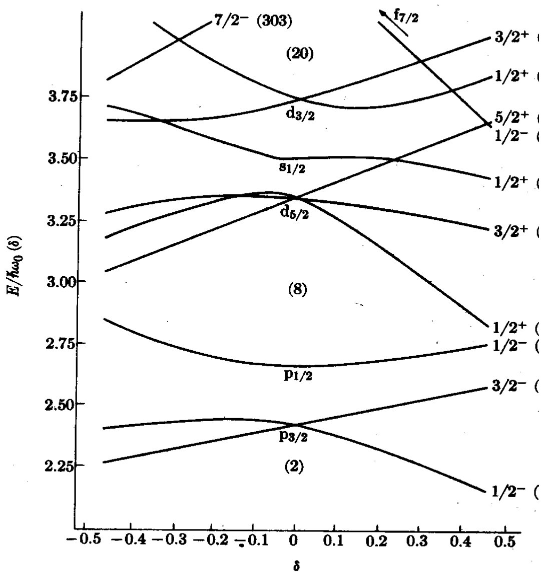
From the figure one can see that for a +ve Q nucleus (Prolate or football shaped) the orbit with the **largest projection of *j*** on to the symmetry axis, (here called Ω) **intercepts fewer nucleons** in its orbit. The nuclear potential, which is the average of all interaction, will be less than for a spherical nucleus. By contrast an orbital with the **lowest projection**, the nucleon will **interact with more other nucleons**: the potential will thus be deeper, and the energy of the orbital lower. So for +ve Q nuclei the substates with lower m_j (actually Ω in Nilsson's theory) will be lower in energy.

For –ve Q nuclei (oblate or cushion shaped), the opposite happens. This is shown in the lower part of the figure. For example, for + def. p ½ is bound more p3/2 less. For –ve def. the inverse. This is understandable in terms of the effective potential (force) when we consider the orientation of the orbit.



δ = ΔR/R

A measure of deformation



Now what does this suggest for GS spin of ^{19}F? $\mathbf{½^{+}}$**!!**

Extremely deformed nuclei

As we said no model covers every nucleus. When we look at the Q of nuclei we see many **heavy deformed** nuclei. The easiest model to describe them is a collective model.

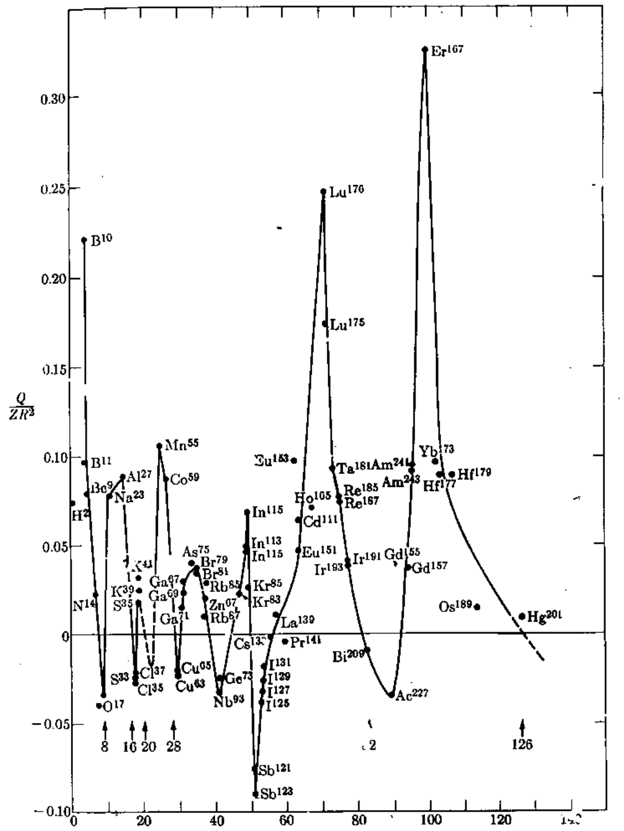
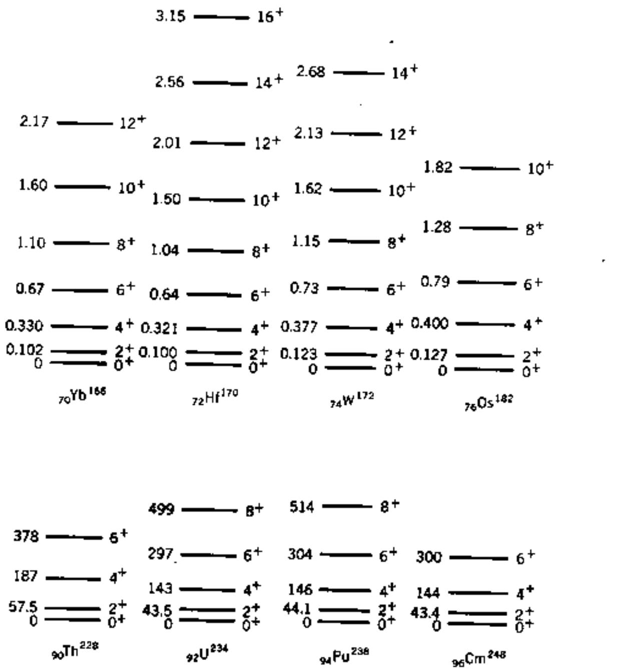


Fig. 5–18. Quadrupole distortion Q/ZR^2 ($\approx \Delta R/R$) for odd-A nuclei plotted vs. the odd nucleon number Z or N. (From F. Segrè, *Nuclei and Particles*, New York: W. A. Benjamin, Inc., 1964. Reproduced by permission.)

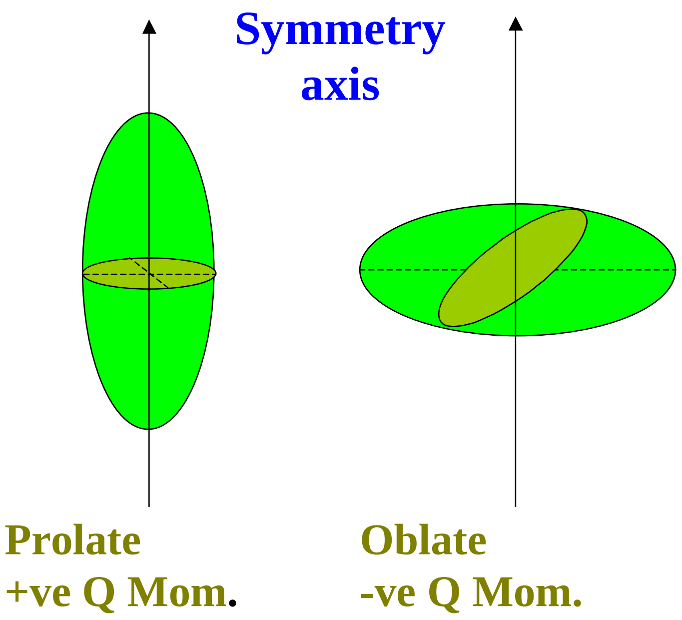
Lets look at the level structure of some even-even nuclei (fig 6.17 cohen) . A=140-200 or so. Stripping away some of the low-lying states we see.

How does this lovely sequence come about?



States of Even-Even Spheroidal Nuclei—The Ground-state Rotational Ban

These nuclei have significant deformation,, so they look like …



The GS since even– even is 0^+. What about the 2^+ 4^+ etc.

It turns out that the excited states correspond to rotation of the nucleus as a whole about one of the axes other **than the symmetry axis**. Rotation about the symmetry axis cannot be seen, but rotation about the other axes can. (We can see if a football is rotating end over end.)

Classically AM **L= Iω**

And $E = \frac{1}{2} I \omega^2 = \dfrac{L^2}{2I}$

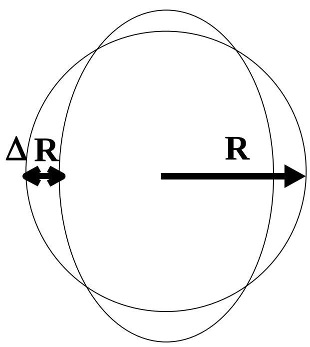
In a Q mech system $AM = \sqrt{I(I+1)}\hbar$

So energy of states is

$$E = \frac{L^2}{2I} \rightarrow \frac{\hbar^2}{2I} I(I+1)$$

Where I = 0, 2, 4, 6, etc
If GS is even parity and its config does not change, then all the rot. states must have even parity and since $\pi = (-1)^I$ odd values of I are forbidden.

So in units of $\dfrac{\hbar^2}{2I}$, the energies if the states are separated by 6, 20, 42, etc almost exactly what is seen.



We can actually quantify this

If we assume it is a rigid body, so

$$I_{rigid} = 2/5\ MR^2(1 + 0.31\beta)$$

$$\beta = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{R_{av}}$$

Now, for your typical mass = 170 nucleus, this gives
$h^2/2I$ = 6 keV ,right order of magnitude, but too small (15 keV).

In fact the nucleus is not solid, but behaves like an elastic fluid, and the radius stretches at higher energy (faster rotations). Thus the Moment of Inertia increases, and the E of the rotational state decreases below the value predicted by the simple formula.

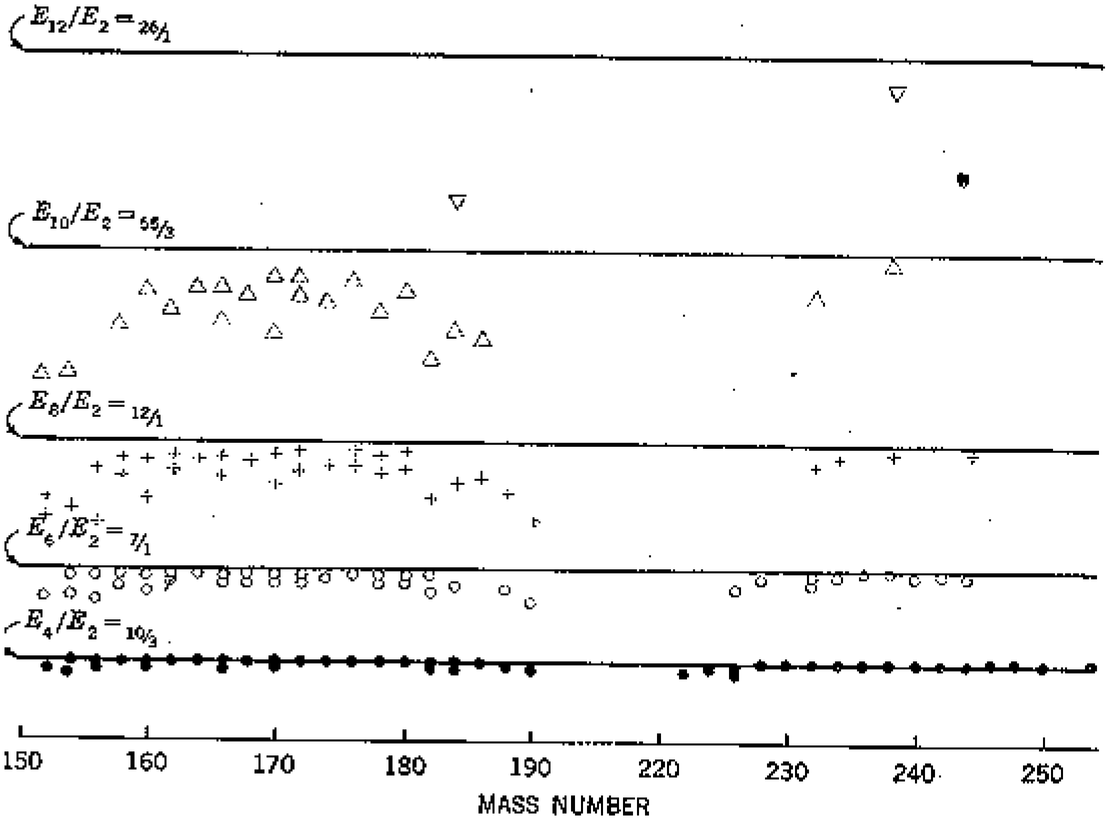


FIGURE 6-18 Ratio of energies of various members of ground-state rotational bands in spheroidal even-even nuclei to the energy of the lowest (2^+) member. Subscripts are I of the states, and the horizontal lines are the predictions of (6-7). [*From O. Nathan and S. G. Nilsson in K. Siegbahn (ed.), "Alpha, Beta, Gamma Ray Spectroscopy," North-Holland Publishing Company, Amsterdam,* 1965; *by permission.*]