Nuclear and Particle Physics
640-354
Lecture notes and figures

The subject provides an introduction to the modern picture of the atomic nucleus and elementary particle physics.

Students completing this subject will be able to:
- explain the modern picture of the atomic nucleus and the physics of elementary particles; and
- solve and analyse problems in these areas by applying simple quantum mechanical reasoning.

These lecture notes are copies of my originals. They are not meant to be definitive, nor are they guaranteed to be free of error.

They are made available on request, with the hope that they may assist you in this course.

I have also put in the reading room several copies of introductory Nuclear Physics texts. These are the property of either my graduate students, or myself.

We would appreciate it if they were not removed from the reading room.

These notes are on the web at
http://www.ph.unimelb.edu.au/~max/

Max Thompson
August 2003

Nuclear and Particle Physics 354

Lecture Times   **Mon., Wed., and Fri., @ 3.15 pm**

Location   **Hercus theatre**

Tutorial   **? ? ? ?**

Lecturers   **Prof. Bruce McKellar and Dr. Max Thompson**

There will be two assignments each worth 7.5% of the assessment

Text books

Prescribed text
W S C Williams,   **Nuclear and Particle Physics.**

Recommended text
Povh, Rith, Scholz and Zetsche   **Particles and Nuclei**
                    (a few copies on reserve in Physics library)

The following texts on Nuclear Physics will be in the 3rd-year Reading room

Introductory Nuclear Physics   *Kenneth Krane*

Concepts of Nuclear Physics   *Bernard Cohen*

Introduction to Nuclear Physics   *Harald Enge*

Lecture 1 (2003)				
	Povh		Krane	Enge
Williams				
Introduction				
Nomenclature	2.2	1.2	1.2	1.6
Constituents	2.1		1.10	1.6
Nuclear Size	5.3/4	3.1	1.2	3.1
Rutherford Scattering	5.2	11.6		1.2/3

Problems Lecture 1

1. Calculate the distance of closest approach of a α-particle of 6 MeV to a nucleus of gold.

2. If the mass of a nucleon is 1.7×10^{-27} kg, what is the mass density of the nucleus ^{16}O? What is the mass density of the nucleus ^{208}Pb? Compare both of these with the atomic mass density of the same material.

3. From the uncertainty principle $\Delta p . \Delta x \gg \hbar$, and the fact that a nucleon is confined within the nucleus, what can be concluded about the energies of nucleons within the nucleus?

Lecture 1

Why study Nuclear Physics?
Basically because understanding of the nature of the force between the most fundamental components of matter that are directly accessible is not understood. Unlike the case of atomic physics, where quantum electrodynamics provides an exact description of the EM force, the nuclear force is as yet not completely understood.

For that reason there are still very fundamental experiments being undertaken in an effort to clarify the nature of the force between the nucleons (protons and neutrons). In particular the effect of placing these nucleons in close proximity to many others, that is within the nucleus.

For those of you who are going to proceed to research in physics, this is a sufficient reason for studying nuclear physics, or indeed any branch of physics. For all of you, including those who will go into the real world and make money, the impact of nuclear physics in many areas of life is immense, and as physicists you should know more about it than the average Herald-Sun reader.

These areas include
- the environment: the continuing debate about the safety and environmental cleanliness of nuclear power, the feasibility of fusion power.
- medical applications: radiation therapy, diagnostic tools such as radioactive scans, CAT, PET. and total body composition studies.

However at this stage in the understanding of the nucleus, it is appropriate to question the more fundamental source of the nuclear force. In fact, it is now common knowledge that the nuclear components (protons and neutrons) are not the fundamental components of nuclear matter, but rather they themselves are compound particles consisting of quarks. These quarks not only combine to form the nucleons, but the whole range of mesons and other particles. So it is right to seek and explanation of the properties and interactions of nuclei and nucleons at a more fundamental level.

For that reason the present course is an amalgam (Nuclear and Particle). I will give the lectures on nuclear Physics and Prof. McKellar will present the study of subnucleon and particles physics. Hopefully the result will be a relatively coherent course.

The plan for the lectures is as shown.

You can see that the range of topics is quite extensive. The degree of theoretical rigor will vary from topic to topic. Overall I hope that you might at the end of the course be able to say that the course objective has been met.
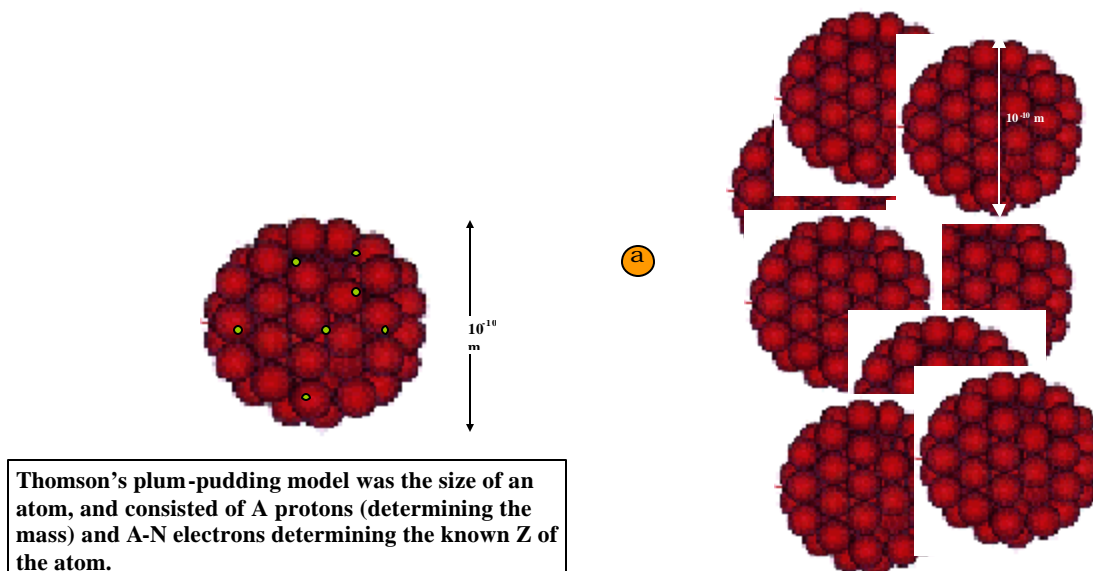
Lect.	date	Topic	Lecturer
1	28-Jul	Atoms to nuclei and nucleons	MNT
2	30-Jul	Structure of Nucleons	BMcK
3	1-Aug	Quarks and leptons	BMcK
4	4-Aug	Nuclear size	MNT
5	6-Aug	Static properties of nuclei	MNT
6	8-Aug	The deuteron	MNT
7	11-Aug	N-N Scattering	MNT
8	13-Aug	Feynman diagrams	BMcK
9	15-Aug	Reaction kinematics	BMcK
10	18-Aug	NN potential	MNT
11	20-Aug	Resonances	BMcK
12	22-Aug	Spin	BMcK
13	25-Aug	Isospin	BMcK
14	27-Aug	Nuclear Shell Model I	MNT
15	29-Aug	Nuclear shell model II	MNT
16	1-Sep	Nuclear collective model I	MNT
17	3-Sep	Nuclear Collective Model II	MNT
18	5-Sep	Flavour Symmetry I -- Mesons	BMcK
19	8-Sep	Flavour symmetry II -- Hadrons	BMcK
20	10-Sep	Colour and hadron wavefunctions	BMcK
21	12-Sep	beta decay	MNT
22	15-Sep	Weak interactions	BMcK
23	17-Sep	Neutrino oscillations	BMcK
24	19-Sep	alpha decay	MNT
25	6-Oct	Gamma decay	MNT
26	8-Oct	Accelerators	BMcK
27	10-Oct	Detectors	BMcK
28	13-Oct	Nuclear Reactions	MNT
29	15-Oct	Reaction Theories	MNT
30	17-Oct	electron-positron to hadrons	BMcK
31	20-Oct	deep inelastic scattering	BMcK
32	22-Oct	Configuration mixing	MNT
33	24-Oct	Photonuclear reactions	MNT
34	27-Oct	Elemental Analysis	MNT
35	29-Oct	B Physics -- Decays	BMcK
36	31-Oct	B Physics -- CP violation	BMcK

In these lectures we are discussing the fundamental building blocks of matter: a wonderful example of scientific enquiry.

The concept of the atom was first introduced by the Greek philosophers Democritus and Leucippus around 450-420 B.C. The word "atom" derives from the Greek $\alpha\tau o\mu o\sigma$ for "indivisible". The current chemistry picture of the atom was not fully realised until the 19th Century by Dalton. Chemists **were** able to determine the diameter of atoms to be of order 10^{-10} m (or 1 angstrom), a figure that is essentially the same from hydrogen to uranium. (Why is this?) But by the late 1800's the similarity of chemical properties of groups of elements raised doubts as to whether the atom was not *atomos*. In 1897 J. J. Thomson (unfortunately not a forebear) it soon became clear that the atom was not indivisible or (*atomos*). He reported the discovery of a radiation from a cathode ray tube that was composed by very tiny particles, smaller than an atom and negatively charged--electron. So the indivisible atom consisted of electrons and positive particles designated protons. These were mixed up like a plum pudding to make a neutral unit the size of an atom.

Now we come to Nuclear Physics. Early workers such Marie and Pierre Curie were studying the new radiations that came from heavy atoms like radium, and confirmed the electrons of Thomson, and a heavy charged particle "the alpha" particle. This was to prove useful to Rutherford, who was not happy with the plum-pudding model, and in 1911 set his graduate students Geiger and Marsden the project to investigate it. It was Rutherford's postulate that the atom was essentially empty space consisting of a massive minute nucleus with the appropriate numbers of electrons surrounding it.

You recall the classic experiment from 1st year. In essence, if the plum pudding model were correct, the 10^{-10} m atoms would be cheek to jowl and the α's would scatter successive scatterings as they penetrated the foil. Thus there would be a relatively small scattering angle[1].

Thomson's plum-pudding model was the size of an atom, and consisted of A protons (determining the mass) and A-N electrons determining the known Z of the atom.

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[1] See Fowlers (U of Virginia) lecture

http://www.phys.virginia.edu/classes/252/Rutherford_Scattering/Rutherford_Scattering.html

On the other hand if the nucleus was tiny (we are talking ~10^{-15} m), but had a large charge (Z), the scattering would be classical electrostatic scattering. In reality he found that essentially all the α's got through, with few deflected more than 1°, and essentially none were reflected.

Before discussing this in a little more detail, let's complete the history that leads up to the content of this course.

It was not until 1932 that the picture of the nucleus itself as being composed of protons with neutralising electrons was clarified. Chadwick showed that the nucleus contained a new heavy particle (essentially identical in mass to the proton), which was neutral, and hence was called the neutron.

Size cf atom. Marble at centre of oval

10^{-10} m
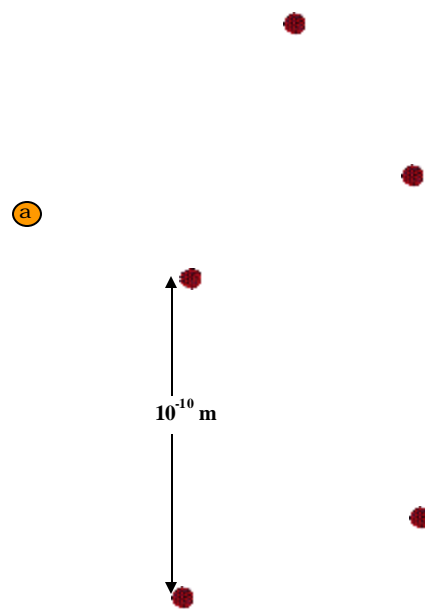
The Rutherford model of the foil

Over the next 30 or so years, nuclear physicists studied the properties of these nuclei made up of protons and neutrons (nucleons): their spin, their deformation, their magnetic moments, and importantly the spectroscopy of the quantum states. The fact that nuclei, like atoms, could be excited to higher quantum energy states confirmed that they like atoms had internal substructure (in this case the protons and neutrons), which could be modelled. We will study these models during the course.

By the 1960's the question was beginning to be asked as to whether the nucleons were fundamental particles, or whether they also had internal structure. The test for this was to look and see if scattering very high energy particles (e.g. electrons) off a nucleon was always elastic (energy in equals energy out), or whether one could observe evidence of energy being lost in the collision, and therefore transferred to internal components. With the advent of high-energy electron accelerators such evidence was found, and Murray Gell-Mann proposed the Quark model, which will be studied in the course. With the quark as the (current) fundamental particle, it was possible to account for not only the proton and neutron, but the plethora of other subatomic particles (such as mesons) that had been discovered and predicted. You will study these with Prof. McKellar.

Nuclear components:
        proton and neutron (**NUCLEONS**)

        $m_p = 938.280$ MeV/c^2 $= 1.67252 \times 10^{-27}$ kg
        $m_n = 939.573$ MeV/c^2 $= 1.67482 \times 10^{-27}$ kg

                compare electron 9.1091×10^{-31} kg.

The nucleus   a stable collection of nucleons.

Size: ~1 – 7 x 10^{-15} m = 1 – 7 fermi (fm)

$$(cf\ atom \sim 10^{-10}\ m)$$

A particular nuclear species is called a **NUCLIDE.**

A particular nuclide consists of **N** neutrons and **Z** protons

The charge on a proton is +e (1.60210 x 10^{-19} C) so the nuclide $_Z^A X$ has Z protons, N neutrons, and **A= N+Z** nucleons.

A is the nuclear mass number
Z is the charge or isotopic number (the element number in the periodic table)
N is the neutron number

Note that not all combinations of Z and N are stable. For light nuclei (A < ~20) to first order, those with N=Z are: usually N>Z. The reason for this is directly relatable to the nature of the nuclear force, and this lecture course.
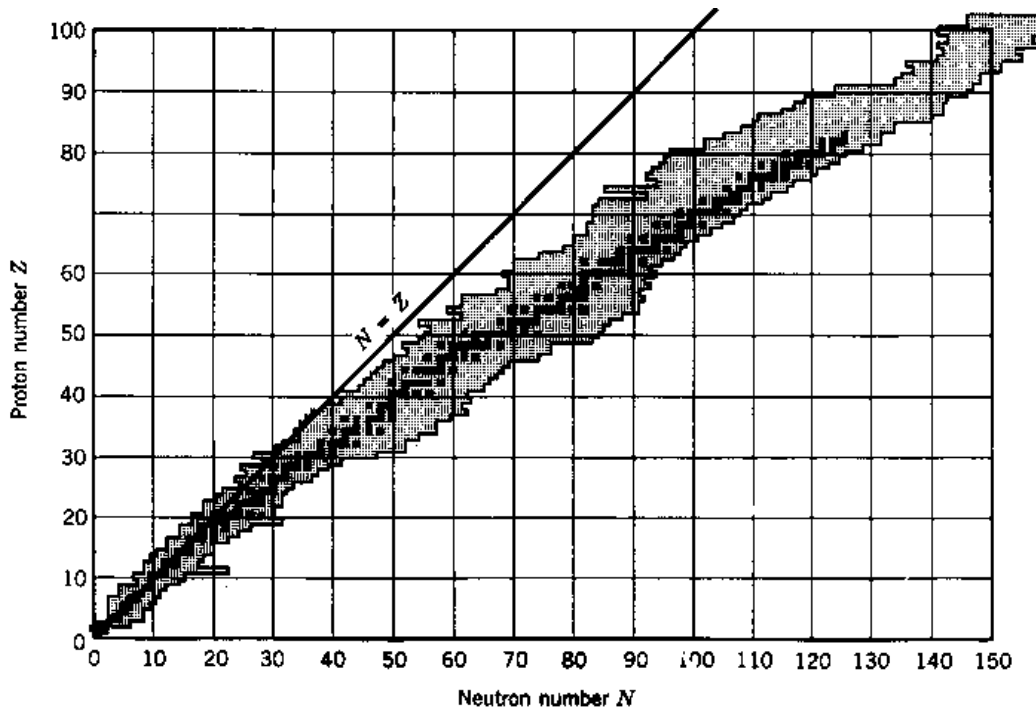

Figure 1.1 Stable nuclei are shown in dark shading and known radioactive nuclei are in light shading.

An **ISOTOPE** is one of a set of nuclides with the **same Z** and consequently different A. (*ie* isotopes are the same chemical element but different masses).

An **ISOTONE** is one of a set of nuclides with the **same N** and consequently different A
e.g. $_{18}A^{39}$, $_{19}K^{40}$, $_{20}Ca^{41}$ (all have N = 21)

An **ISOBAR** is one of a set of nuclides with the **same A but different N and Z.**

e.g $^{14}_{8}O$, $^{14}_{7}N$, $^{14}_{6}C$.

Nuclear Size – Rutherford Scattering

Lets see what we can learn about the size of the nucleus from Rutherford's experiment that we mentioned earlier, and then move on in lecture 4 to discuss modern determinations if the nuclear size. The references to the theory of Coulomb or Rutherford scattering are in the lecture summary.
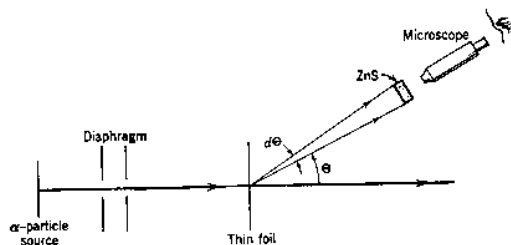
First the experimental equipment:

Figure 4-2 Arrangement of an α-particle scattering experiment. The region trav the α particles is evacuated.

- A source of high-energy charged particles…α-particles from Ra with a kinetic energy of about 7+ MeV (the same as used in the Part 3 nuclear prac.
- A very thin foil to produce the scattering. The classic foil is gold, since it can be made extremely thin (400 atoms was used), ensuring that multiple scatterings were unlikely. In fact Rutherford used materials with a range of different Z to check the theory.
- A detector of α-particles. Today we would use a SS detector such as in the Part 3 experiment. However in 1911 they were some 90 years too early, and had to rely on scintillations of the α's on a layer of ZnS. This detector system was incomplete, since the recording apparatus was missing. Hence
- A number of research students, whose job it was to observe the individual scintillations at the different angles, and record the results. This particular piece of apparatus has not changed in 90 years. Graduate students are still essential.

Now let's look at the microscopic level to understand the physics.

Rutherford assumed that the coulomb scattering was the result of an infinitely heavy, point charge. The scattering is a classical collision problem where, as shown in the figure some of the KE of the incident α is converted into PE at the point of minimum approach. Naturally for a conservative force, after the interaction the final KE is restored. The locus of the trajectory is a hyperbola.
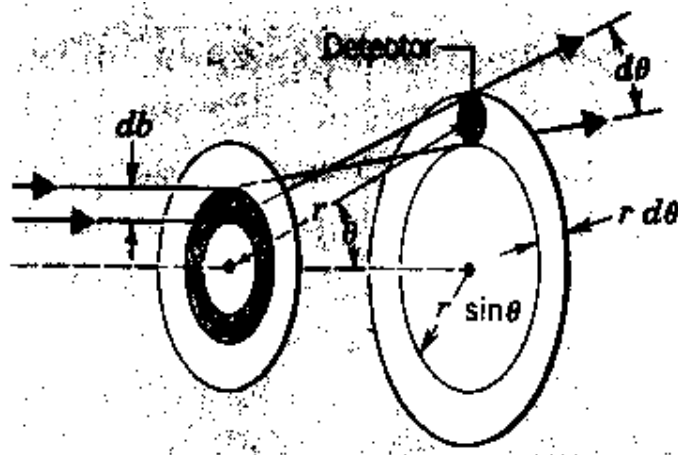
The angle of scattering θ, depends on how close the line of approach of the incident α is to the point scatterer. This is measured by the impact parameter b. The smaller b the larger will be the angle of scattering.

The relation between θ and b is $cot\dfrac{?}{2} = \dfrac{2b}{d}$, where **d is the distance of closest approach**, i.e. when we the collision was head-on, or when $b = 0$.

Exercise: calculate the distance of closest approach of an α of energy 6 MeV to a nucleus of Gold.

The probability of an α hitting a ring of width db distance b from the zero-impact line is
$P(b) = r\, t\, 2p\, b\, db$ where r is the number of atoms per unit volume and t is the thickness of the foil.

This is thus the probability of the α being scattered between angle q and dq, and since we know b as a function of q, this can be written as $P(b)db = \dfrac{p}{8} rt\, d^2 sinq\, \dfrac{dq}{sin^4\dfrac{q}{2}}$

I want you to derive an expression for d (the distance of closest approach) in the above equation, for an α of KE= $1/2mv^2$, and charge 2e, scattering off a charge Ze. I leave this as an exercise for you and to evaluate it. Note that the value of d is a first-order measure of the nuclear radius.

$d\Omega = 2\pi sin\theta\, d\theta.$
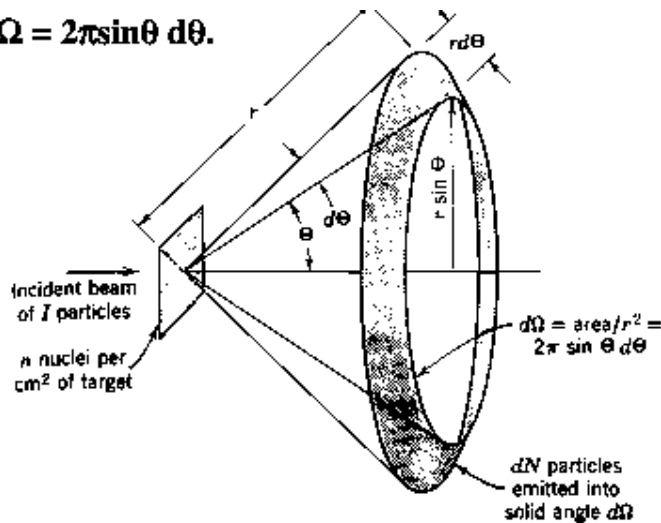
Note that this is the probability of an α incident with impact parameter b being scattered into an angle between θ and $\theta + \Delta\theta$. For a beam of α's of intensity I, incident uniformly on the foil, the solid angle dW subtended at angle q is:
$dW = 2psinq\, dq$. So in the above equation we substitute for dq, and find that the probability of an α ending up at angle q per unit solid angle (dW) is:

$$\dfrac{dN}{d\,O} = (\dfrac{1}{4p\,e_o})^2 (\dfrac{Ze^2}{Mv^2})^2\,\overset{}{\underset{s}{}}$$

This says in essence that for a given foil (Z) and given α energy, the probability of detecting an α at a given θ is given by $\dfrac{dN}{dW} \propto \dfrac{1}{sin^4\dfrac{q}{2}}$.

It was this extreme dependence on scattering angle that told Rutherford that the nucleus was essentially of point size.
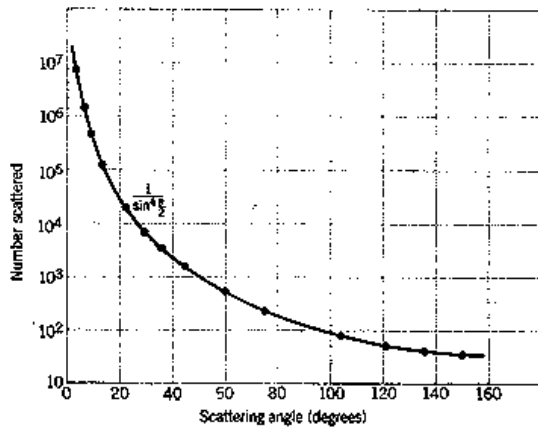
When you evaluate the distance of closest approach, you will find that the nucleus of Au is~10^{-14} m in radius.

Figure 11.10 (*d*) The dependence of scattering rate on the scattering angle θ, using a gold foil. The $\sin^{-4}(\theta/2)$ dependence is exactly as predicted by the Rutherford formula.

Evidence of Substructure in Atomic and sub-atomic states

Before going on to discuss how to measure the size of nuclei using electron scattering, I want to consider the nature of the physics of the systems we are considering. Firstly confirm that it is a quantum system we are studying, and what order of magnitude quantum states we might expect. But also to indicate how evidence of substructure can be interpreted.

From your early studies of statistical mechanics you are aware that the kinetic theory of gases relied on treating atoms classically, as if they were solid little balls. It worked. The size of the system was macroscopic, and classical physics sufficed.

You proved the universal gas laws, and may even remember that the model predicts the molar specific heat for a diatomic gas as $c_v = 7/2RT$. This expression comes from an average energy per molecule of ½ kt per degree of freedom. As an example for H_2 there are 7 degrees of freedom (3 translational, 2 vib. and 2 rot).

The experimental value of c_v for H_2 at room temperature was 5/2RT. It only reached a value of 7/2RT at relatively high temperatures. In fact at quite low temperatures it becomes 3/2RT. I earlier textbooks the explanation of this phenomenon was that the rot. and vibrational energies were "frozen" (we now would say quantised). We now know that at low
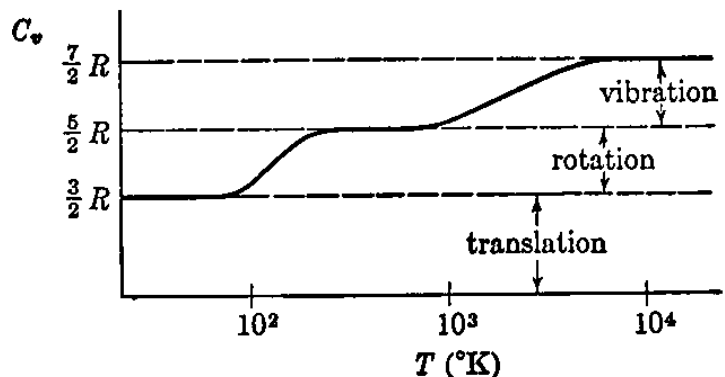
2-18. Molar specific heat of molecular hydrogen (I

11

temperatures, ½ kT is less than one quantum. So even for this fairly large scale system quantisation was evident.

More importantly, if we were to take these solid little balls and examine them by say scattering electrons off them, as long as the electron energies were low, the scattering would be elastic, and we could consider them to be just solid balls. However you know that as soon as Frank and Hertz fired electrons of a few 10s of ev at them, the electrons lost energy: the atoms had absorbed energy from the incident particle. Not only that but the energy was in discrete values, and was emitted subsequently as photons. **These little balls had internal structure that was quantised.** The spacing of the quantum states is as you recall, in the region of the energy of a visible photon (~ev). So when probed with electrons with energies of this order, the internal structure can be seen (Frank and Hertz electron scattering off Hg 1913).

Quantisation and size

Molecules
  Quantisation in the Kinetic theory of gases

Atoms
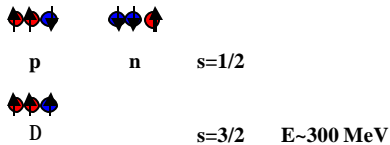 **Quantisation as revealed by Frank and Hertz**
  Substructure due to electrons

Nuclei
  Rutherford showed elastic scattering
  Is the nucleus quantised?
  Heizengerg uncertainty principle $\Delta x . \Delta p \approx \hbar$

  For atom $\Delta x \sim 10^{-10}$ m ➔ $\Delta E \sim$ eV
  For nucleus $\Delta x \sim 10^{-14}$ m ➔ $\Delta E \sim$ several MeV
  Quantum states of order MeV
  Substructure due to nucleons

Nucleon
  Assume $\Delta x \sim 10^{-15}$ m ➔ $\Delta E \sim$ 100 MeV
  Quantum states of order 100 MeV
  (Δ-resonance at about 300 MeV
  substructure due to quarks

  p   n   s=1/2

  Δ s=3/2   E~300 MeV

We are now at the next stage down: we have considered the nucleus as a classical ball, and in Rutherford scattering observed elastic scattering of MeV α particles. If this nucleus has substructure, what order of magnitude is the spacing of the quantum states? We may then be able to probe any quantum substructure by using suitably energetic probes such as electrons, and again look for evidence of inelastic scattering..

We can get some idea using Heizenberg's uncertainty relation. $\Delta x . \Delta p \sim \hbar$.
For an atom $\Delta x \sim 10^{-10}$ m ➔ Δp ➔ $\Delta E \sim$ ev
For a nucleus $\Delta x \sim 10^{-14}$ m ➔ Δp ➔ $\Delta E \sim$ MeV

The quantum states of a typical nucleus are of order MeV. So if we want to study the structure (that is the protons and neutrons that constitute it) by scattering we need a probe of energy >>MeV.

Just out of interest, we might assume that the nucleons are structureless. But again if we consider their size as 10^{-15} m, we expect quantum states at ~ 100 MeV. Indeed the first excited state of the nucleon is at about 300 MeV and is called the Δ resonance. It is, in terms of the substructure of the nucleon, a rearrangement of the quarks that form the substructure of nucleons (the GS of the nucleon is p = uud, n = ddu, u has q = +2/3e, d has q = -1/3e. The Nucleon GS has the spin of the quarks coupled to s= ½, and the Δ has one d or one u flipped to give s = 3/2)

Now we might ask how to determine the size of the nucleus with some certainty. Naturally we are probing something we can't see, and we need to probe with an external probe and interpret the results. Scattering from the nucleus is a standard tool. Since the nucleus contains charges we can use coulomb scattering. The first response is to note that this was done this nearly 100 years ago, by Rutherford. However if you recall he assumed that there was a point charge. If the probe is to get close enough to see the size of the nucleus we must incorporate this into the analysis. Scattering

of high-energy (200-500-MeV) electrons was the preferred method, and next lecture we will look at the experimental methods.