Lecture 2N (2003)	**Povh**	**Krane**	**Enge**	**Williams**
Nuclear Size				
Electron scattering	**5.4**	**3.1**	**1.3**	**3.2-4**
Mu-mesic x-rays		**3.1**		**3.6**
Nuclear Masses	**2.2**	**3.2**	**1.4**	**4.1**
Binding Energies	**2.3**	**3.3**	**1.4**	**4.2**
Separation energy	**3.4**	**3.3**	**1.4**	**4.3**
Nucleon pairing	[illegible]	[illegible]		[illegible]

Problems Lecture 2

1 300-MeV electrons are used to inelasticity scatter off a nucleus

(a) What is the wavelength of the incident electron?

(b) What is the wavenumber of the incident electron?

(c) How much momentum is transferred to the nucleus if the electron scatters through 30°?

(d) What is the momentum of the scattered electron?

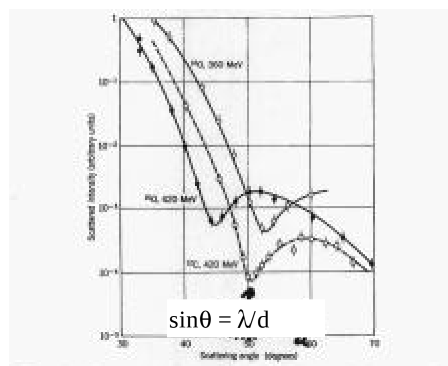
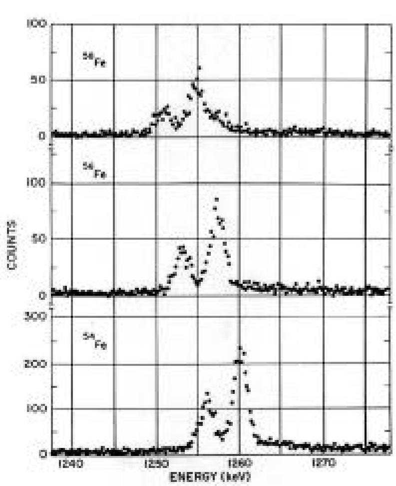


Figure 3.1 Electron scattering from ^{16}O and ^{12}C. The shape of the cross sectio[n] is somewhat similar to that of diffraction patterns obtained with light waves. Th[e] data come from early experiments at the Stanford Linear Accelerator Center (H. Ehrenberg et al., Phys. Rev. 113, 666 (1959)).

2 Estimate the radius of ^{12}C and ^{16}O from the 3 sets of data in fig 3.1 of Krane. How do your values compare with the values from $R = R_o A^{1/3}$. Where $R^o = 1.2$ fm?



3 Convince yourself that the energy of the K x-rays from the mu-mesic isotopes of Fe shown in the figure to the left, should increase as A decreases.

4 Compute the mass defects of ^{32}S, ^{20}F, ^{238}U

5 Evaluate the neutron separation energies of ^{7}LI, ^{91}Zr, and ^{236}U

6 Calculate the thresholds for the reactions: $^{20}Ne(\gamma,p)$, $^{16}O(\gamma,p)$, $^{208}Pb(\gamma,n)$

Review Lecture 1

1 Basic Nomenclature

Nucleons
Nucleus:
Nuclide.
Isotope
Isobar

$^{A}_{Z}X_{N}$

A is the nuclear mass number
Z is the charge or isotopic number
N is the neutron number

2. order of magnitude of quantum states from Heizenberg uncertainty principle

Molecules	
Atoms	**eV**
Nuclei	**MeV**
Nucleons	**100 MeV**

3. Dimensions

Atomic size ~10^{-10} m
Nuclear Size ~ 10^{-15} 10^{-14} m

4. Rutherford's proof of Nuclear model of atom by alpha scattering

$$\frac{dN}{d\Omega} \propto \frac{1}{\sin^4\frac{\theta}{2}} \, .$$

6. Problems 1-3

Lecture 2N

Now we might ask how to determine the size of the nucleus with some certainty. Naturally we are probing something we can't see, and we need to probe with an external probe and interpret the results. Scattering from the nucleus is a standard tool, and since the nucleus contains charges we can use coulomb scattering. The first response is to note that it has been already done nearly 100 years ago. However if you recall, Rutherford scattering assumed that there was a point charge. **If the probe is to get close enough to see the size of the nucleus we must incorporate this into the analysis**.

Scattering of high-energy (200-500-MeV) electrons was the preferred method, and we will shortly look at the experimental methods. However you might already have worked out how this will work, and what the experimental results will look like.

Harking back to Rutherford, I would expect that **if the electron doesn't make it close to the nucleus (that is for small scattering angles) the results should be the same**. It is only when the electron is close enough to discern that the nucleus is not a point that differences might occur.

A beam of 200-MeV electrons has a deBroglie wavelength of about 10 fm (< diam of a moderate nucleus). (you should check this). So we have here a typical problem of a wave scattering around a small object. (1997 VCE question). $\sin\theta = n\lambda/d$

7 PHYSCAT IA

Mary and John attend an orchestral concert. Unfortunately they have poor seats, and are sitting in the balcony behind a column, such that John cannot see the double bass section of the orchestra and Mary cannot see the flutes. The situation is illustrated below in Figure 6.

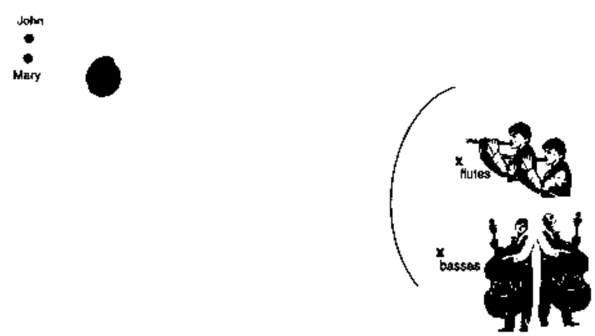


Figure 6

At the end of the concert Mary comments that she could not hear the flutes well. John, however, says that to him the orchestra sounded balanced, and he could hear every instrument including the double basses.

So I would expect a scattering cross section to look like:

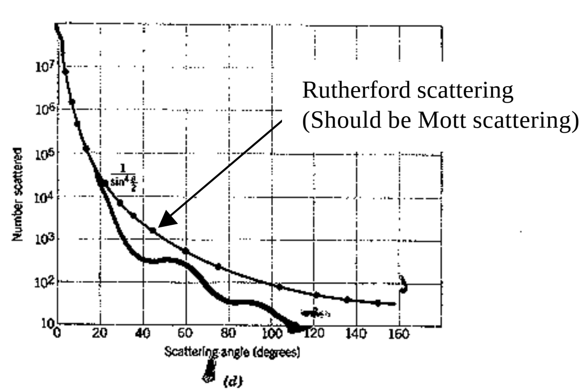


Figure 11.10 (*d*) The dependence of scattering rate on the scattering angle θ, using a gold foil. The $\sin^{-4}(\theta/2)$ dependence is exactly as predicted by the Rutherford formula.

Here are some real ones:

Note for ^{16}O the first minimum moves to a smaller angle as the wavelength decreases (energy increases).

However there is a lot more that can be got from these data, and it is necessary to look in a little more detail.

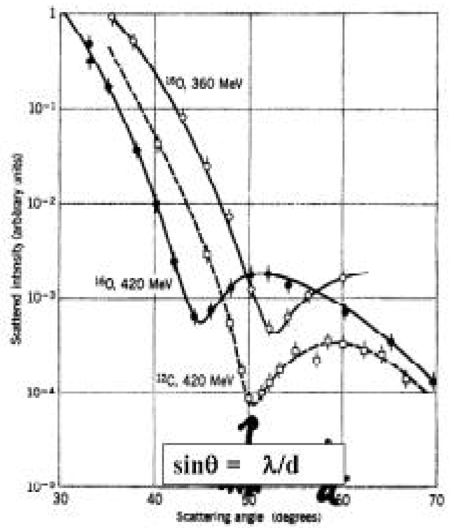


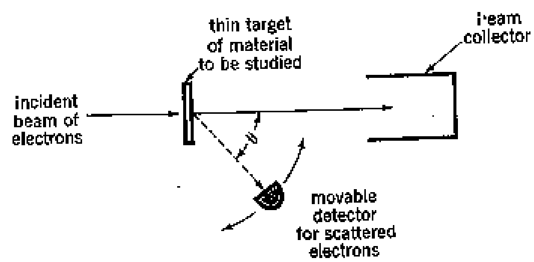
Figure 3.1 Electron scattering from ^{16}O and ^{12}C. The shape of the cross secti is somewhat similar to that of diffraction patterns obtained with light waves. T data come from early experiments at the Stanford Linear Accelerator Center (H. Ehrenberg et al., *Phys. Rev.* **113**, 666 (1959)).

Before we do that let's see how these measurements are made. Our research group was involved in electron scattering experiments at Mainz University. We also work at Tohoku University where elastic electron scattering was a major research program.

What we need is:

- ✓ a source of high-energy (~100+ MeV) electrons
- ✓ a thin target containing the nuclei
- ✓ a means of measuring the angle and energy of the **elastically** scattered electrons.

FIGURE 1-1 Experimental arrangement for measuring the angular variation of electron scattering from nuclei. The angle θ is varied by moving the detector, and for each θ measurements are made of the ratio between the number of scattered electrons it detects and the number of electrons in the beam as determined by the collector. (Since very few electrons are deflected by large angles, practically all of the beam reaches the collector.) Typical results of these measurements are shown in Fig. 1-2. The detector is actually a very large and complex group of instruments capable of determining the energies of the electrons.



What is an electron linear accelerator?

A means of transferring EM power to electron KE.

Components: injector (100keV)

Klystron power for RF

Drift tube

Electron magnetic spectrometer

Need to be able to change angle

Need to measure energy of scattered electrons.

We want to consider only elastically scattered ones, not those that have given energy to the nucleus.

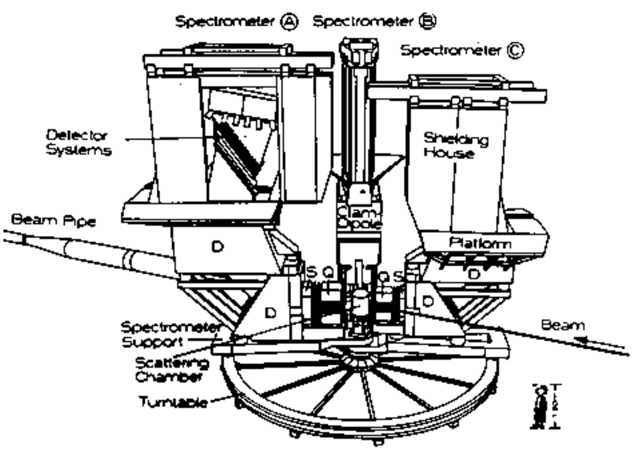


Figure 1: The three-spectrometer setup at MAMI

What can we actually learn that is quantitative from elastic electron scattering.?

I don't expect you to reproduce the detailed derivation on this, and indeed I do not intend to be too rigorous myself. However it introduces, or in some cases, revises stuff you did in Quantum Mech.

The simple diffraction picture I mentioned assumes a solid object**, here is the real situation.**

In simple terms we have a beam of electrons of defined momentum, scattering and exiting with the same energy, but of course a different vector momentum. The elastic scattering is the result of an electrostatic potential V(r).

Both the incident and scattered waves are plane waves with momentum $\boldsymbol{k_i}$ and $\boldsymbol{k_f}$. (Actually k is the wave number $2\pi/\lambda$) $\left(\frac{2\pi}{h}\boldsymbol{p} = \frac{\boldsymbol{p}}{\hbar}\right)$

The probability for scattering is given as the square of

$$F(k_i, k_f) = \int \psi_f^* V(r) \psi_i \, dv \qquad \text{equ 1}$$

Vectors $\boldsymbol{k_i}$ and $\boldsymbol{k_f}$ are defined in terms of the momentum transferred to the nucleus $\boldsymbol{q}$:
where $\boldsymbol{k_i} - \boldsymbol{k_f} = \boldsymbol{q}$

Since $\psi^*_i = e^{-ik_i . r}$ and $\psi_f = e^{ik_f . r}$
equu 1 can be written as

$$F(q) = \int e^{iq.r} V(r) dv \qquad \text{. equ 2}$$

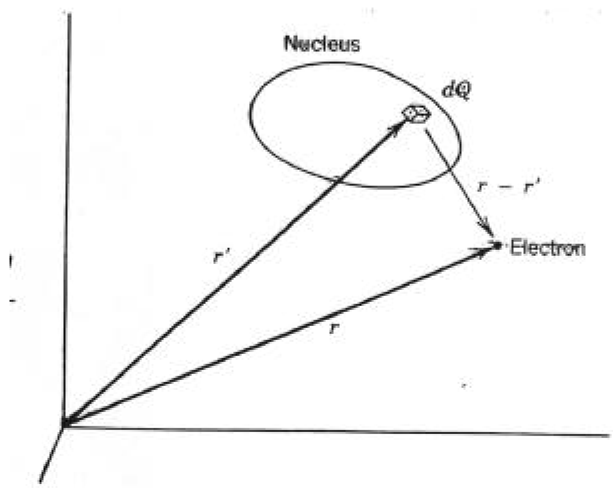
The form of the electrostatic potential V(r) is **not** in this case simply $\frac{Ze^2}{4\pi\varepsilon_0 r}$ as it would be between a point nuclear charge +Ze and an electron charge -e. . In this case the effect of each tiny nuclear charge ΔQ spread through the nucleus with a charge density ρ(r').

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_o}\int \frac{\rho_e(r')dv'}{|r - r'|}$$

Putting this specific form for V(r) into equ 2 one gets

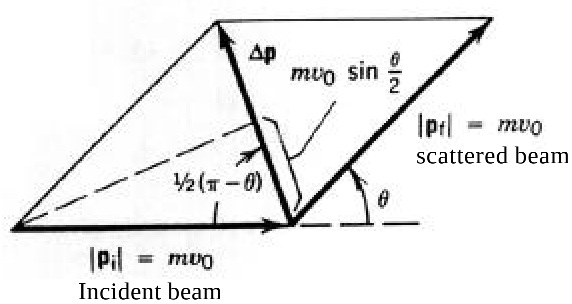
$$F(q) = \int e^{iq.r'} \rho(r') dv' \qquad .$$

Which is **the overlap of a wave of wavelength dependent on the momentum transfer with the total nuclear charge**.



If you look back to Rutherford scattering, you will notice that here as then, the incident and scattered momentum magnitude is unchanged and Δp is 2psinθ/2. So q is the wave number corresponding to this, so is a function of θ the scattering angle.

$$q = \frac{2p}{\hbar}\sin\theta/2$$



So F(q) can be measured as a function of θ (or q), so that the only unknown in the relation is ρ, the charge distribution of the nucleus. **F(q) is known as the form factor,** and is the **Fourier transform of** ρ**(r') .**

Here are some typical electron scattering cross sections

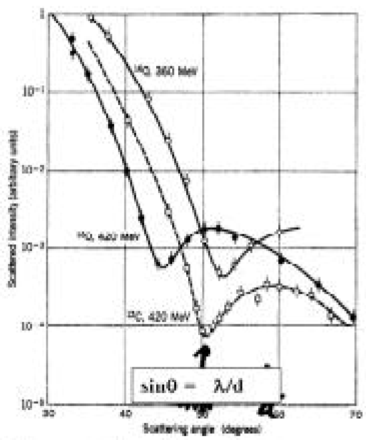
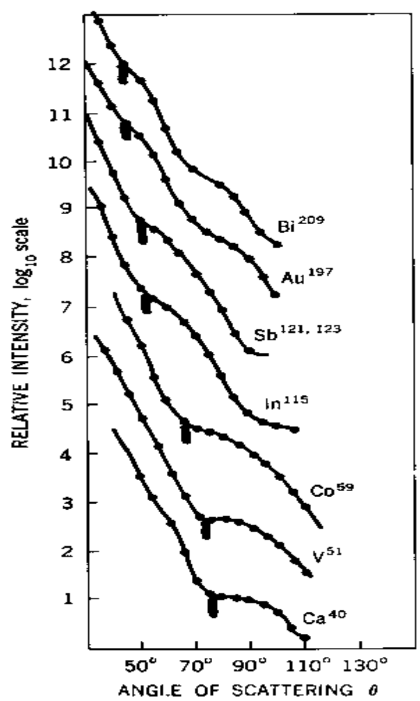
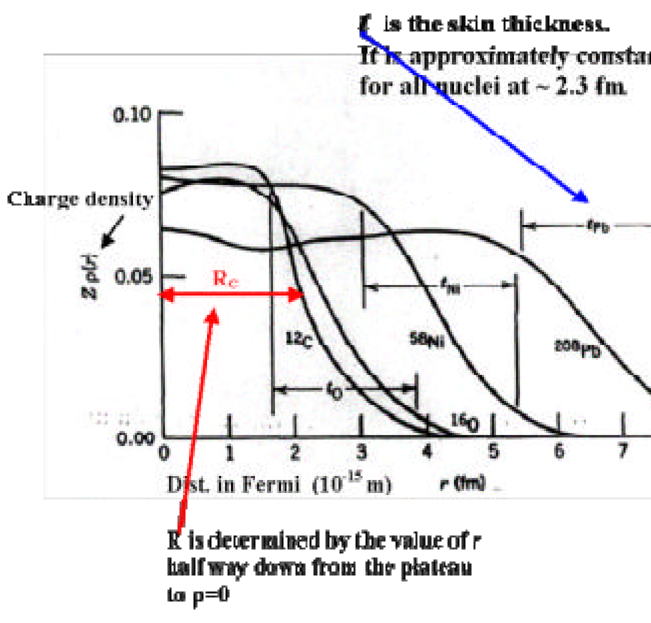


Figure 3.1 Electron scattering from ^{16}O and ^{12}C. The shape of the cross secti is somewhat similar to that of diffraction patterns obtained with light waves. T data come from early experiments at the Stanford Linear Accelerator Center (H Ehrenberg et al., *Phys. Rev.* 113, 666 (1959)).

FIGURE 1-2 Angular distributions of 185-MeV electrons scattered from various nuclei. The curves through the data are theoretical fits. [*From B. Hahn, D. G. Ravenhall, and R. Hofstadter, Phys. Rev.,* **101:** 1131 (1956).]



Thus from F(q)'s experimentally measured dependence on θ, it is possible to determine ρ(r').



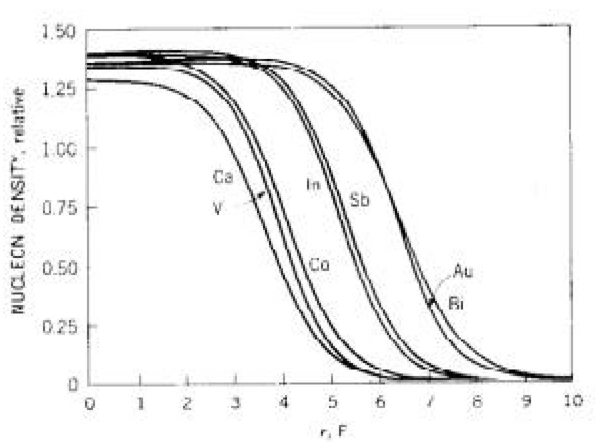


FIGURE 1-4 Nucleon density in various nuclei as obtained from the fits to the data shown in Fig. 1-2. [*From B. Hahn, D. G. Ravenhall, and R. Hofstadter, Phys. Rev.,* **101:** 1131 (1956).]

Now these plots are quite revealing. The Left-hand one is the **charge distribution**, as a function of radius, and the RH one is the **nucleon distribution**, allowing for the relative neutron/proton numbers on the assumption that their distribution is the same.

Note that the central region of the **nucleon density** is essentially the same for all nuclei, light or heavy. Nucleons do not congregate in the centre of the nucleus, but have a fairly constant distribution out to the surface. Thus the number of nucleons per unit volume is essentially constant. $\frac{A}{4\pi R^3} \sim const$. So that $R = R_0 A^{1/3}$

R_o = 1.2 fm from electron scattering studies.

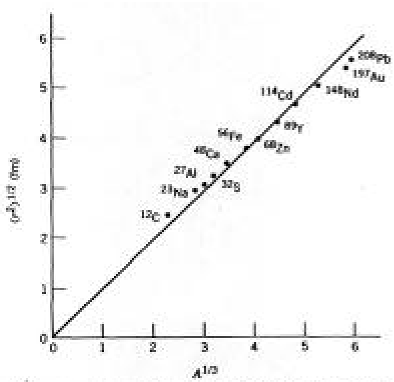


Figure 3.5 The rms nuclear radius determined from electron scattering experiments. The slope of the straight line gives $R_0 = 1.23$ fm. (The line is not a true fit to the data points, but is forced to go through the origin to satisfy the equation $R = R_0 A^{1/3}$.) The error bars are typically smaller than the size of the points ($\pm$ 0.01 fm). More complete listings of data and references can be found in the review of C. W. de Jager et al., *Atomic Data and Nuclear Data Tables* **14**, 479 (1974).

You see what we have in nuclei is close packing of nucleons, and R_o might be thought of as the radius of a nucleon. You might note the difference between nuclear sizes and atomic sizes. For nuclei $R = R_0 A^{1/3}$, so heavier nuclei are bigger.

For atoms we find that the size of atoms is essentially constant, mainly because for an atom there is a central force (the charged nucleus) that provides a larger central force as Z increases. So the atomic radii are pulled in. In a nucleus we do not have a central force; the A nucleons are all bumbling around in close proximity to only a few neighbours.

You also notice that the surface of the nucleus is not well defined. The density falls from its constant value to zero over a distance of about 2.3 fm (1 fermi = 10^{-15} m). This so-called "**skin thickness**" is essentially the same for all nuclei.

The distribution of nuclear density is given by $\rho(r) = \frac{\rho_o}{1 + exp(r - R)/\tau}$ where $R = R_0 A^{1/3}$ and $\boldsymbol{\tau}$ is the skin thickness.

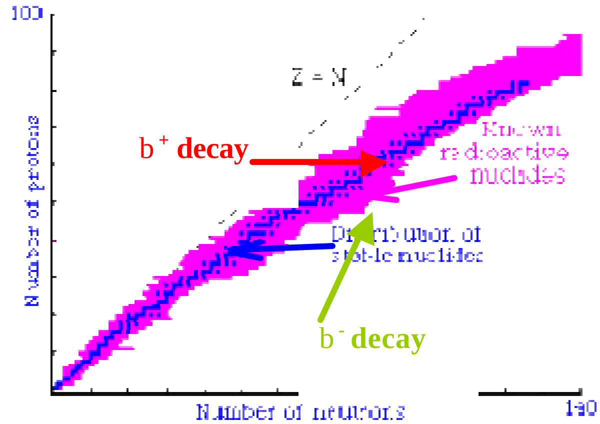
There are other, less accurate ways to determine the nuclear size. I want to mentio n them only because they are clever, and because they give you a chance to see how minute effects can lead to significant results.

So it looks as if the nucleus is made by just packing in protons and neutrons, so that they form a sphere of volume proportional to A. That is, the nuclear force has a very short range, and each nucleon attracts as many near neighbours as possible . If life were this simple we would not be studying nuclear physics. Let's look at what leads to the stability of just a few of the possible combinations a little more closely.

Nuclear Stability

Pairing

Firstly I have shown you the chart of nuclides. Not any combination of N and Z is stable. **For light nuclei N=Z** is most likely, though N±2 and Z±2 are OK. For **heavier nuclei we find that N>Z** is more likely for stability. **The reason for this should be evident** to some of you, and we will come back to this when we look at our first model of nuclei.



Note that nuclei below the stability line would have too many neutrons for stablity. If such a nucleus were made it would in essence change a neutron to a proton by beta minus decay. Conversely a nucleus above the stability line would have an excess of protons and would decrease the proton number by positron decay. Thus such unstable nuclei would move towards the stability line along an isobaric line (keeping N+Z constant).

Not only do we find that nuclei with N>Z are dominant (you correctly deduced this as being due to the repulsive effect of the coulomb repulsion, so that more N are required to separate the protons more). But there is a preference for stable nuclei to have even numbers of P or N or both

A	N	Z	Stable
even	even	even	155
odd	odd	even	53
odd	even	odd	50
even	odd	odd	4*

* $^{2}_{1}H$ $^{6}_{3}Li$ $^{10}_{5}B$ $^{14}_{7}N$

Nucleon Binding energies

However there are some other systematic effects that show up in nuclear properties that will help us get some handles on the nature of the nuclear force. **In order to see these we need to consider nuclear masses and the binding energies of the nucleons within the nucleus.**

We have said that the nucleus contains essentially all the mass of an atom, yet so far I haven't given any masses. To first order the mass of the electron in the hydrogen atom has 1/2000th of its mass. Here is a table of masses of the nucleons and the electron

	Kg	MeV/c^2	amu
proton	1.67252×10^{-27}	938.256	1.0072766
neutron	1.67482×10^{-27}	939.559	1.0086654
electron	9.10908×10^{-31}	0.511006	0.0005486
amu	1.66043×10^{27}	931.480	1

1 atomic mass unit (amu)
= 1/12th of the mass of an ATOM of ^{12}C

6 p @ 1.0072766 = 6.0436596

6 n @ 1.0086654 = 6.0519924

6 e @ 0.0005486 = 0.0032916

12.0989436!!! compare 12.00000

Note the different ways we can measure the masses. Because $E=mc^2$ we should feel quite happy to express the mass in terms of its energy equivalent. Hence column 2.

Column 3 is probably the most convenient unit for expressing masses of nucleons and nuclei: the Atomic Mass Unit. Because nuclei come in modules of A nucleons it is convenient to have a unit for a nucleon that is about "1". It is **1/12 of the mass of a neutral atom of carbon 12**. Note this includes the mass of the 6 electrons.

Hopefully some of you have already seen that from the overhead that the sum of the parts is not equal to the whole….it is greater than it by about 100 MeV/c^2 or 1%. How come?

About 1% of the mass of the individual components has disappeared when combined into the nucleus of carbon; **that is, the 6 protons and 6 neutrons have a combined binding energy of about 93 MeV**. It would take 93 MeV of work to pull the carbon apart into its constituent components.

This is a significant mass loss. For an atomic system, when chemical reactions are involved the binding energies involved are of order eV. That is the energies of outermost bound electrons in the atom. For hydrogen you recall the BE of the electron to the proton is 13.6 eV…about 1 part in 10^8 of the mass of the atom. Now in the smaller nuclear system the BE is of order 1 part in 100 (or 1%) of the masses involved. It is of interest to go down one more step in the substructure scale, and postulate that if the nucleon is the binding together of 3 quarks, what fraction of their energy is lost in binding?

Entity	components	Masses	B. E.	F raction of mass
H atom	P e	M_p= 1 GeV M_e = 0.5 MeV	13.6 ev	10^{-6} %
12C	6 P + 6 n	M_p=M_n = 1 GeV	100 MeV	1%
neutron	d u u	M_Q = 100GeV? M_p = 1 GeV	~299 GeV	99%

Free quarks have not been found (except by McCusker from Sydney University in the 1970's), but it is likely that their mass is of order 100 GeV/c^2. If so, then three of them combined into a nucleon of mass about 1 GeV/c^2, we can see that 99+% of the mass has been released as BE.

Tabulation of Nuclear masses

The masses of all stable and most unstable nuclei are now tabulated. App. C of your text lists them in units of amu. Because the masses are very close to the value of A for the nucleus, many times the tabulation lists only the difference from the value of A. This is the **mass defect or mass excess**.

Measurement of Nuclear masses

Before discussing some of the characteristics of the nuclear force that can be derived from a study of nuclear masses, I feel it important that you have some idea how nuclear masses are measured. Enge spends a deal of time on this in section 3.2, and although of interest, it is a bit too specialised for an introductory course.

In essence the mass is measured by measuring the momentum of a charged ion whose velocity is known. Fig 3.13 shows the schematic of a mass spectrometer.

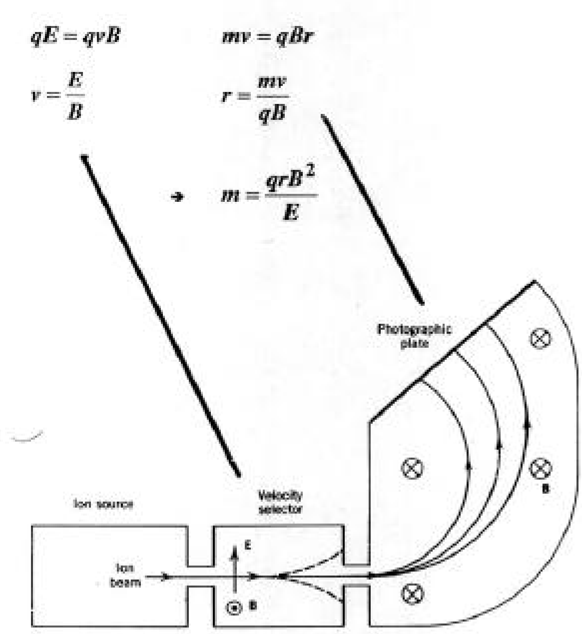


Figure 3.13 Schematic diagram of mass spectrograph. An ion source produces a beam with a thermal distribution of velocities. A velocity selector passes only those ions with a particular velocity (others being deflected as shown), and momentum selection by a uniform magnetic field permits identification of individual masses.

The accuracy of the measurements needs to be better than 10^{-6}, since this is the order of the BE involved in nuclei.

The spectrometer can also be used to determine the relative abundance of the isotopes of an element.

NUCLEAR PROPERTIES 63

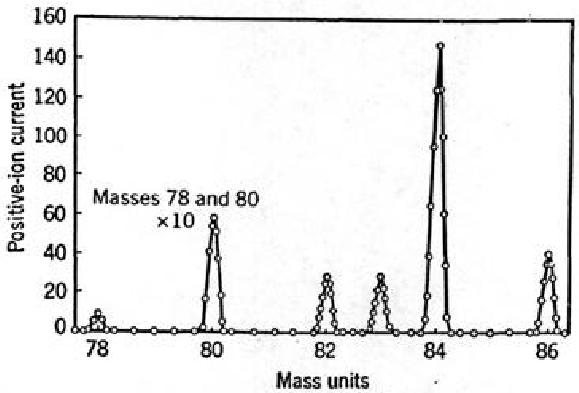


Figure 3.14 A mass-spectrum analysis of krypton. The ordinates for the peaks at mass positions 78 and 80 should be divided by 10 to show these peaks in their true relation to the others.

And if designed on a large scale, to provide samples on elements consisting of single isotopes. This was the method used in the Manhattan project to provide the enriched ^{235}U for nuclear weapons. They had acres of huge mass spectrometers that dumped ^{235}U onto little piles, which were collected and accumulated. Not however the healthiest of jobs, since it was a pretty radioactive nucleus.

More about binding energies

Separation Energy of a nucleon (S_n, S_p)

We indicated above that the mass of the components of ^{12}C exceeded the mass of the nucleus by some 100 MeV. That is the total binding energy of the nucleus was 100 MeV.

Perhaps a more interesting and revealing figure is the BE for the least bound nucleon (p or n). This is usually termed the **Separation energy S_n.**

For example the separation energy of the least bound neutron in nucleus $^{A}_{Z}X$.

$$^{A}_{Z}X \rightarrow {}^{A-1}_{Z}X + n$$

$S_n = m(A-1, Z) + m_n - m(A, Z)$
consider ^{40}Ca
$m(^{40}Ca) = 39.962591$ amu
$m(^{39}Ca) = 38.970718$ amu
$m(n) = 1.008665$ amu
so $S_n = 0.016792$ amu (x 931.480 MeV/c^2) = 15.64 MeV

Note that this is also the threshold energy for the photoneutron reaction $^{40}Ca(\gamma,n)$.

There is something that we can learn from an **examination of the separation energies of adjacent nuclei.** Let's look at the case for the isotopes of Ca.

Neutron Separation Energies

	isotope	Neutron No.	S_n(MeV)
(97%)	^{40}Ca	20	**15.7**
(7x10^4 yr)	^{41}Ca	21	8.4*
(0.64%)	^{42}Ca	22	11.5#
(0.15%)	^{43}Ca	23	7.0*
(2.1%)	^{44}Ca	24	11.1#
(~3 yr)	^{45}Ca	25	7.4*
(3.3%)	^{46}Ca	26	10.4#
(5 day)	^{47}Ca	27	7.3*
(0.19%)	^{48}Ca	28	9.9#
(10 min)	^{49}Ca	29	**5.1**

*Ave (odd N) 7.5 MeV
#Ave (even N) 10.7 MeV

pairing energy ~ 3.2 MeV

Calcium is a good set of data since there are many isotopes. Most of these are stable, and indeed at some stage we had a large fraction of the world's supply of the separated isotopes of mass 48 here in Melbourne. You will notice a systematic trend in these data. I choose to leave out mass 40 and mass 49 data. Both rely on very unstable nuclei in order to derive Sn.

Note that in general **the binding of the neutron in a nucleus with even number of neutrons is 10.7 MeV,** while the binding of one with an **odd number of neutrons is only 7.5. MeV.** This confirms our observation that **even numbers of nucleons gives stability**, but it further says that **the strength of that extra stability is about 3 MeV**. In reality we know that nucleons obey fermi statistics just as do electrons. That is, nucleons have an intrinsic spin like electrons. And just like electrons in atoms, **nucleons in nuclei like to pair off with their spins opposing**.

So even without any specific discussion of the nuclear force we are beginning to understand some of its properties.

Some properties of the Nuclear Force

observation	implication
Constant nucleon density	Short-range force with a repulsive core.
Few odd-odd nuclei	pairing → stability
S_n for Ca isotopes	pairing energy ~3 MeV