Lecture 5	Povh	Krane	Enge	Williams
N-N scattering ℓ=0	**16.1**	**4.2/3**	**3.1-3**	**2.11, 9.9**
phase shift		**4.2**	**3.5**	
Properties of 2-nucleon potential				
Central	**16.1**	**4.4**		
spin dependence		**4.4**	**3.6**	
ℓ-s dependence		**4.4**		
exchange component		**4.4**	**2.6**	

Problems for Lecture 5

1 Calculate the classical impact parameter necessary to give an orbital AM of ℓ=1 for an *n-p* scattering event when E_{lab} = 10 MeV

2 Matching ℓ=0 wavefunctions at the boundary r = c + b of a square-well repulsive core potential leads to the transendental equation K cotKb = k cot [k(c+b) + δ]. Determine the radius of the repulsive core (c) from the information that the triplet S phase shift is zero for E_{lab} = 350 MeV. Use Vo = 73 MeV, and b = 1.337 Fm

3 Show for yourself that a proton with a KE of less than 10 MeV will have a small probability of having l = 1.

Review Lecture 4

The Deuteron

1 Nuclear potential short range attractive
Repusive core
Approximate with sqare will plut infinite repulsive core
assume ℓ=0

2 Solve Schroedinger Equation for ℓ=0 1D, knowing BE of deuteron as 2.23 MeV
Transcendental equation gives binding for range of pot. depths and widths
Calculate radius of deuteron from WF and equate with observation
Gives another transcendental equation
Combined, these give V_o, width of pot. well

3 *I* for deuteron is 1 (S=1), but p and n can couple to give S = 1 (triplet S state) or 0. No S=0 (singlet) GS state found . S=0 state is just unbound. Therefore Pot. well is shallower (60 MeV) for antiparallel coupling of s.

4 Mag. Dipole moment less than sum of MDM for p and n (since ℓ=0 there is no orbital contribution from motion of p)
Therefore postulate that the WF for deuteron is not entirely ℓ=0 (s-wave)

$\psi = a_s\psi_s + a_d\psi_d$ (*) where ℓ=2 (d wave) contribution is about 4%
(*) *Understand what this equation means*

5 ***I*** for deuteron is 1 and since ℓ=0, $S = s_n + s_p = ½ + ½ = 1$
For ℓ=2 component must recouple S=1 and ℓ=2 to give *I* = 1
This means twisting S, **so nuclear pot. must have a tensor part**. Depends on θ and S as well as r.

6 Measured Quadrupole moment is non zero, confirms deuteron must include ℓ=2 component in WF. (unlike ℓ=0 WF, ℓ=2 WF is not spherically symmetric)

7 Why there is no S=0 deuteron, and why there is not dineutron or diproton.
Paul exclusion principle does not permit 2 identical particles to have same set of Q numbers.

8 Introduction of Isospin as a quantum number. Nucleons have t = ½. T_z=+1/2 is neutron, t_z = -1/2 ia a proton.

Lecture 5

Scattering

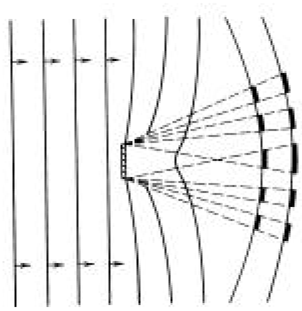
You might note that we can only find out about the (S=1,T=0) combination of the 2 nucleons from our study of the deuteron: the so called triplet S potential. To learn about the (S=0,T=1) (the singlet S) potential, and any dependence of the N-N potential on l, we need to study the effects of scattering between the nucleons. (the info about the unbound singlet (S=0) state of the deuteron that I gave you really comes from scattering data).

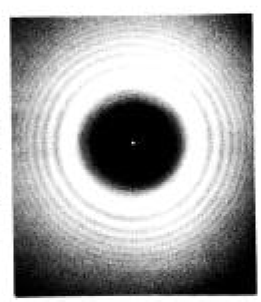
In studying the bound state of the 2-nucleon system (the deuteron) we have only found out the properties of the nuclear force for the case of p and n coupled to S=1, and for them in a state with $l = 0$. In scattering we can have nucleon pairs of all combinations (pp, pn, nn) with all possible spin alignments (S=0, S=1) , and passing with all values of ℓ (selected by the incident energy).

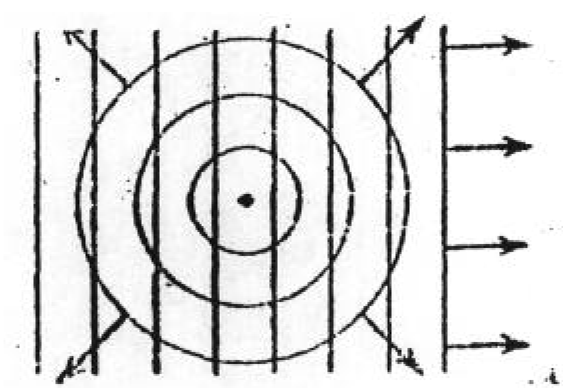
We already have indications that the nuclear force is **complicated**. We saw that there was a **tensor component** that was present for S=1 (not of course for S=0). We also concluded that for **S=0 the potential was less deep,** since the S=0 state of the deuteron is unbound.

In scattering, the effect of nucleons coming into close proximity with the nuclear force will affect the incident debroglie wave. It produces a phase change, so that interference effects will produce a probability distribution for the scattered particles that can be analysed in terms of the potential-well depth and shape.

The analogy with optical scattering around an object is very close. The wavelength (in this case the debroglie wavelength) and the object have to be of the same order.

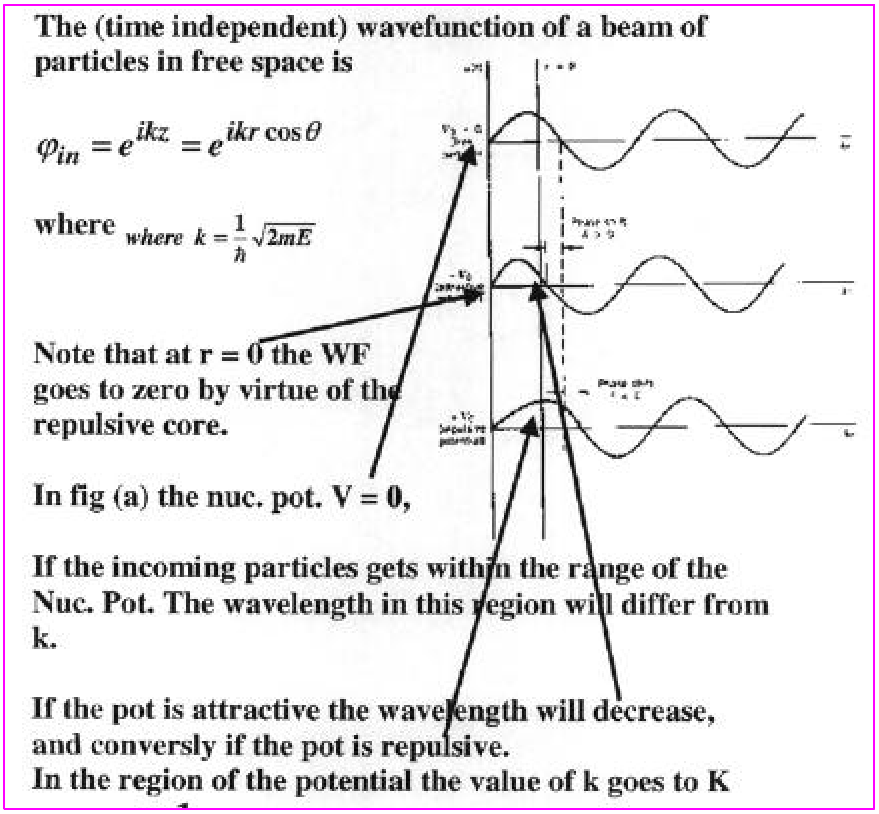






In N-N scattering, the incident particle is considered to be a plane wave with wavelength dependent on its momentum. After encountering the nuclear potential there is an outgoing spherical wave as shown. Well away from the potential we will see interference effects, which we will observe as an intensity distribution as a function of angle.

The interference is the result of a phase shift induced by the nuclear potential. The magnitude of this shift is directly relatable to the form of the potential.



How do we quantify the scattering probability? How can we categorise it as a function of say the energy of the incident particle energy? How do we categorise the scattering probability as a function of the angle of scattering? We need to have a means of comparing the data with theoretical predictions.

Nuclear Cross section

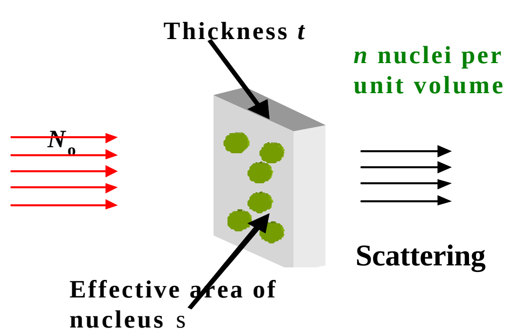
Hence I want now to define the term used to quantify this… **the scattering cross section**.
For scattering, as for all nuclear reactions, we use a characteristic called the Cross Section. Although for many situations the "cross section" is close to the physical cross section of the target nucleus, it should not be seen as that. It is like the "effective" cross sectional area of the interacting target/projectile cross section. If life was simple and we were throwing balls at coconuts the cross section is always the same. For nucleon scattering the cross section we will see that σ varies with several variables, particularly the energy of the incident particles

$$-\frac{dN}{N_o} = \frac{eff\ area\ of\ nuclei}{area\ of\ target}$$
$$= \frac{A\,dx\,n\sigma}{A}$$
$$= n\sigma\,dx$$

So the number scattered in thickness *t* is

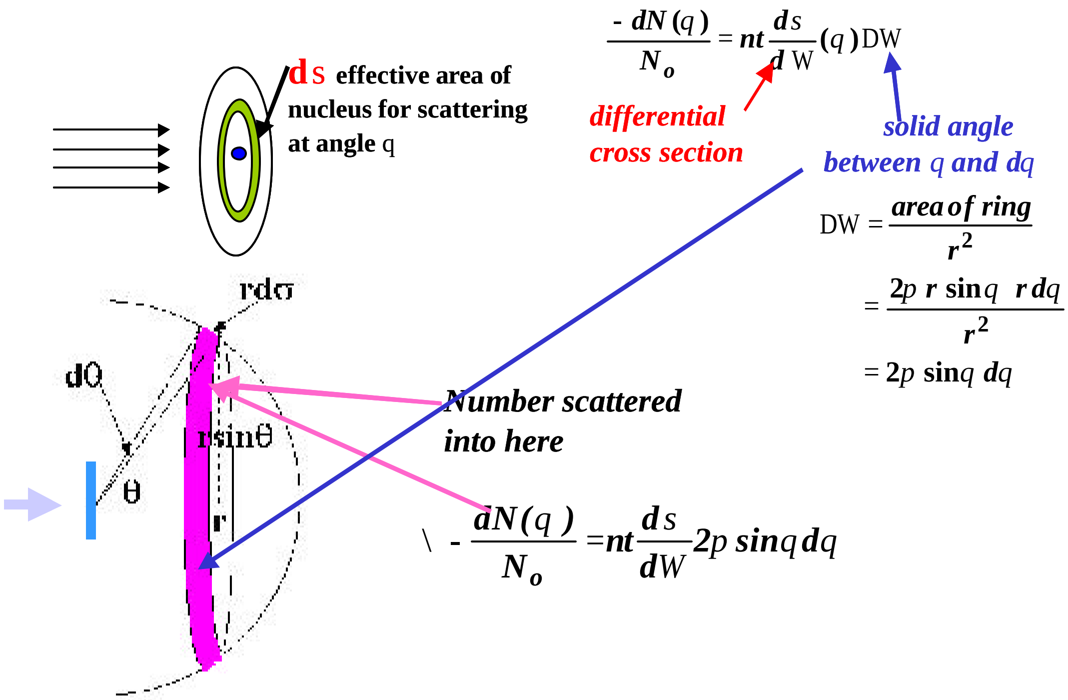
$$-\int_{N_o}^{N}\frac{dN}{N_0} = n\sigma\int_0^t dx$$

$$N = N_o(1 - e^{-n\sigma t})$$



Unit for cross section is barn. (b) 1 b = 10^{-28} m^2

Differential Cross section



Experiments

What we are going to do is direct a beam of neutrons onto a target of protons, or a beam of protons onto a target of protons, and see how many of the incident particles are scattered out of the beam. The experiment is in principle very simple.

The questions we need to ask are :

1 How do we do it?

- a where do we get the beam of protons of known energy from?
- b where do we get a beam of neutrons of known energy from?
- c where do we get a target of protons from?
- d how do we detect the particles that do not get scattered out, or conversely how do we measure the particles that are scattered?
- e how do we quantify the probability of scattering?

2 How does the scattering occur and what does it tell us about the nuclear potential? We will discuss this in some detail next lecture.

The first set of questions is the nuts and bolts of experimental nuclear physics. We need the toys to do the measurements to feed the data to the theorists.

a Protons are of course the nuclei of hydrogen atoms. So to get protons we simply strip off the electron (usually with a combination of heating and magnetic field). Having got a charged proton it is simply a matter of putting it between a high electrical potential and voila! energetic protons.

The part -3 proton accelerator is a simple example, which provides protons with energy of 200 keV

The Pelletron in the basement is an example of a more energetic accelerator, providing protons of 3 MeV. The positive terminal in this case is charged by charge carried up on a pelletised belt. The energy is determined by the potential at the source.

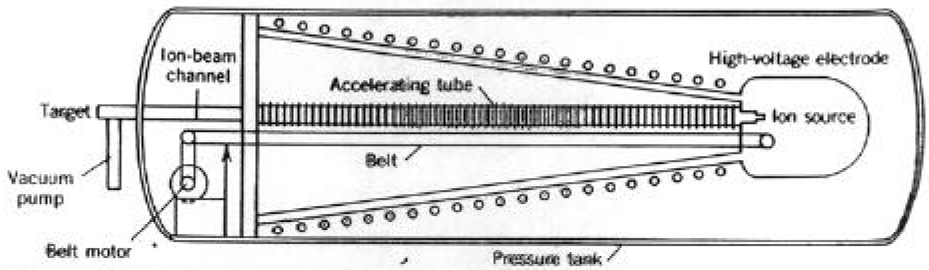


Figure 15.7 Diagram of Van de Graaff accelerator. The ion source is inside the high-voltage terminal, and both are contained in a pressure tank to inhibit sparking.

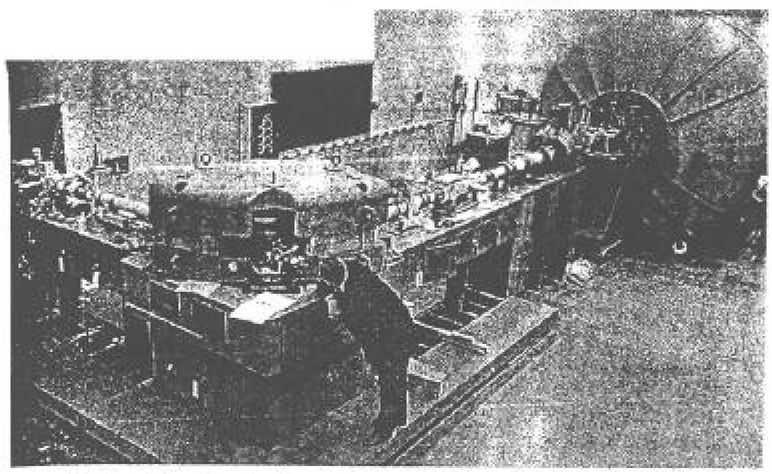


Figure 15.8 A Van de Graaff accelerator, showing the pressure tank at right, the emerging beam line, and a bending magnet to direct the beam to the experimental area. Courtesy Purdue University.

There are cyclotrons that accelerate protons by alternating HF electric fields, while confining the protons to move in a circular orbit.

572 NUCLEAR REACTIONS

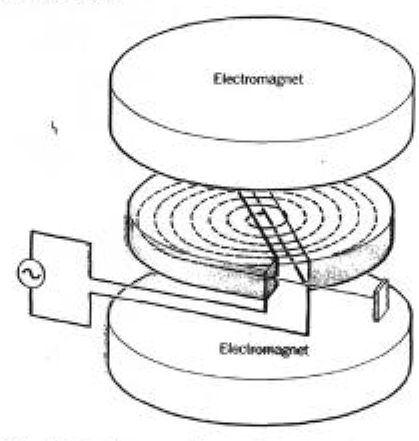


Figure 15.11 Simplified diagram of a cyclotron accelerator. The beam outward from the center, accelerated each time it crosses the gap betw dees, and is eventually extracted and directed against a target.

b You cannot accelerate neutrons, they have no charge, so if one wants a beam of neutrons they have to be produced by a nuclear reaction. The energy of the neutron is usually measured by measuring the time it takes (usually nano sec) to cover the distance from where it is produced to where it interacts.

c You can't collect protons as such, together to make a target from which to scatter. However a target of hydrogen is ideal (either gaseous or liquid). The electrons will be unseen by any neutrons incident on the sample (why?), and if protons are being scattered the effect of an electron on the proton with a mass 1/2000 times larger is like the effect on a truck of hitting a basketball.

d Detecting protons is not difficult. They ionise matter and this ionisation can be measured. In the part 3 lab, in the part-3 nuclear lab you use solid state detectors to detect α particles. They work equally well for protons. Scintillation detectors such as NaI also can be used.

Remember that the object of studying the scattering of p-n , p-p, is to find out the dependence of the nuclear potential on l, and S. You will recall that the only combination we got from the study of the deuteron was S=1 l=0 for p and n. S=0 with l=0 violated the Pauli principle. We can however get all the terms from scattering experiments.

What might we expect the scattering cross section to look like? First we should expect a classical diffraction pattern. However if λ is>> than the size of the scatterer, we should expect a very broad forward diffraction maximum, close to an isotropic scattering pattern. This is the case of a long wavelength sound wave being emitted from a small speaker.

Firstly lets look qualitatively at what we might expect.

The (time independent) wavefunction of a beam of particles in free space is

$$\varphi_{in} = e^{ikz} = e^{ikr\cos\theta}$$

where $K = \frac{1}{\hbar}\sqrt{2m(V_0 + E)}$

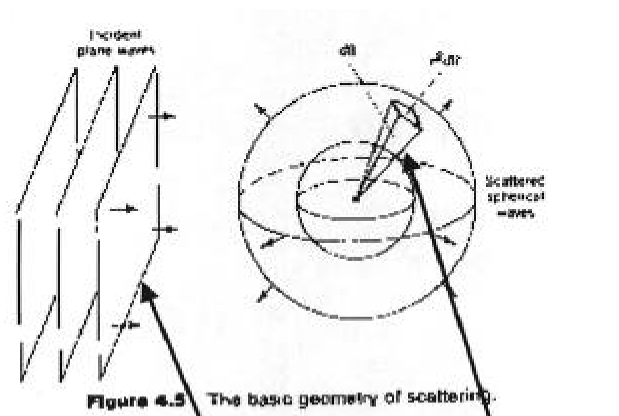


Figure 4.5 The basic geometry of scattering.

the number of particles at a given angle is the square of the amplitude of the WF, resulting from the interference of the incident beam (plane wave) and the scattered beam (spherical wave)

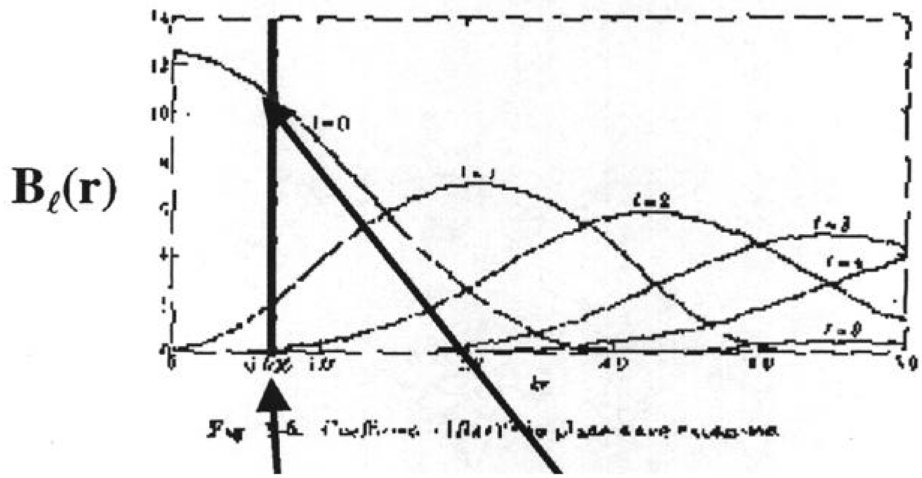
This change is manifested as a phase change vetween the unperturbed and perturbed wavefunction , δ. So a measurement of δ will tell us the nature of the scattering potential. How can this be achieved?

The very process of scattering says that some of the particles in the incident beam will, as a result of encountering the nuc. Pot., be removed from the beam, and appear at a different angle. The first suggestions should be to measure the number of scattered particles (the scattering cross section), and relate this to the theoretical calculation.

The interference resulting from a plane wave and a spherical wave is most easily calculated if we express the plane wave as a sum of spherical harmonic functions

$$\psi_{in} = e^{ikz} = e^{ikr\cos\theta} = \sum_{\ell=0}^{\ell=\infty} B_\ell(r)Y_{\ell,0}(\theta)$$

The radial function B is a sum of spherical bessel fns and looks like Fig 3.5 (Enge).



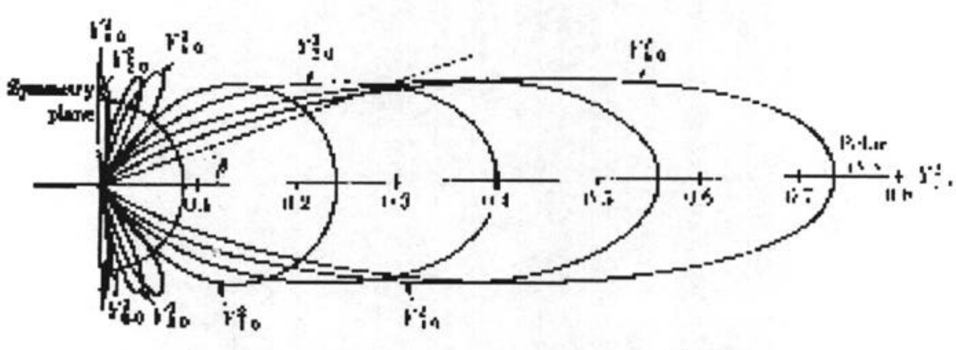


Fig. 3-6. Polar diagram of the functions $Y_{\ell,0}^2(\theta)$

Now this looks very complicated, and I do not intend to pursue it further. In reality it is conceptually quite simple.

If we consider scattering of nucleons with energies less than a few MeV (as we have calculated), we confine the problem to l = 0, and the scattering is isotropic. You are quite familiar with the situation from normal optical interference.
The reason scattering of nucleons with energies <a few MeV is that their deBroglie wavelength is much bigger than the nucleon size: the Angular distribution is isotropic. (In other words the angle of the first diffraction minimum given by $\sin\theta = \lambda/d$ is at an angle greater than π (in the CM system)). As we use higher energy probes, we would expect that the scattering cross section would begin to show forward peaking.

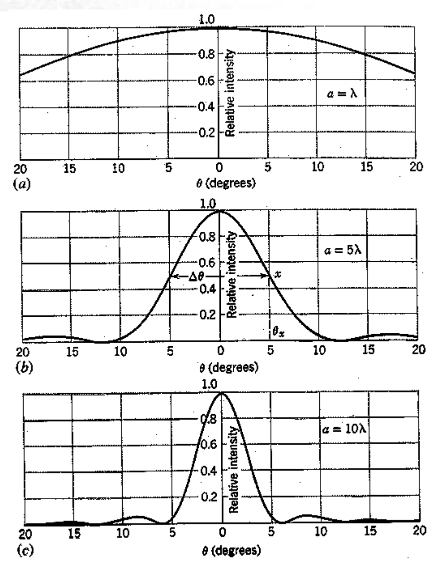


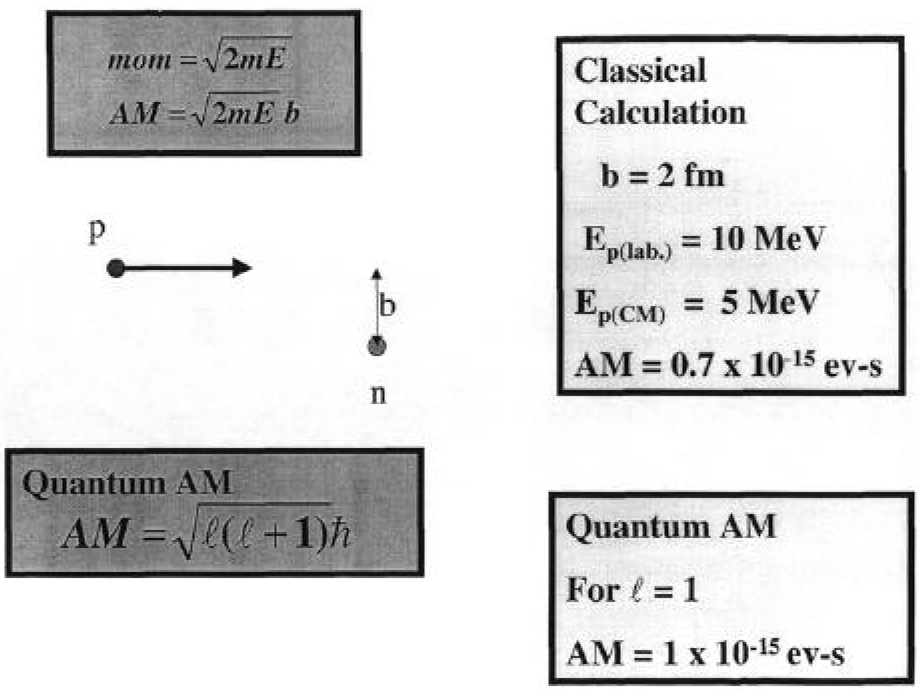
Figure 11 The relative intensity in single-slit diffraction for three different values of the ratio a/λ. The wider the slit, the narrower is the central diffraction peak.

ℓ=0 Scattering

We are now in a position to see what quantum scattering theory predicts for scattering of say protons from neutrons. We could do this for other than l=0. In fact if we want to find out the full nature **of all the terms in the nuclear potential** this would have to be done. I refer you to Chap 3 of Enge or Chap 4 of Krane, if you wish to follow this further, and the notes from Eisberg and Resnik that I have given you..

As long as kr < 0.7 the ℓ=0 contribution is by far the most important. We have seen that 2 nucleons with energy ~ 1 MeV passing within the range of the nuc. Pot. will have ℓ=0. So let's limit our discussion to low-energy scattering. You will see that the angular distn. is determined by $Y_{\ell 0}$, (fig 3.6), ℓ=0 we assume no angular dependence $Y_{\ell m} \rightarrow (1/4\pi)^{1/2}$.
You should note that the assumption of $l = 0$ is quite valid up to nucleon energies of a few MeV.

To see this imagine a neutron of momentum p approaching a proton at an impact parameter of b (about 1 fm. The range of the nuclear potent ial).



The Quantum AM is $\hbar\sqrt{\ell(\ell+1)}$. Which if we put ℓ=1 gives about 10^{-15} ev-sec. The classical AM = pb. At 10 MeV (lab) = 5 MeV (CM) the classical AM is b.$(2mE)^{1/2}$ which gives ~ 0.6 x 10^{-15} ev-s. Much less than the case for ℓ=1. So at 10 MeV we can assume, on this semi-classical argument that the relative AM is ℓ=0.

I want you to check this
Let us now see what the predicted cross section is for low-energy n-p scattering.

For ℓ=0 The incoming **plane** wave e^{ikz} can be expressed in terms of a **spherical wave** :

$$j_{in} = \frac{\sin kr}{kr} = \frac{e^{ikr} - e^{-ikr}}{2ikr} \qquad \text{equ 1}$$

The first term (with the time dependence added) represents a spherical wave emerging from the scattering centre. The other term is a wave converging on the centre.

The diverging wave has experienced the effect of the nuc. Pot. And has suffered a phase change which is for mathematical reasons written as 2δ

So

$$j = \frac{e^{i(kr+2d)} - e^{-ikr}}{2ikr}$$

$$= e^{id}\,\frac{\sin(kr+d)}{kr}$$

equ 2

So for l = 0 $\quad j_o = B_0(r)Y_{0,0}(q) = \frac{\sin kr}{kr} = \frac{e^{ikr} - e^{-ikr}}{2ikr}$

The outgoing part may be modified by the N-N potential $\quad j_o = \frac{e^{i(kr+2d_0)} - e^{-ikr}}{2ikr} = e^{id_0}\,\frac{\sin(kr+d_0)}{kr}$

To find the effect of this phase change well away from the interaction we add the incoming spherical wave and the outgoing one

$$j = e^{-ikz} + \frac{e^{i(kr+2d_0)} - e^{-ikr}}{2ikr} = e^{-ikz} + \frac{e^{i(kr+d_0)}}{kr}\sin d_0$$

This is a wave of amplitude $\frac{\sin d_0}{kr}$ moving away from the scattering centre.

The number of particles, with vel v by this wave per sec. carried in all directions is $\quad N_{sc} = \frac{4p\sin^2 d}{k^2}v$

thus $\quad s_0 = \frac{N_{sc}}{flux} = \frac{4p\sin^2 d_0}{k^2}$

-

Note again that for ℓ =0 there is no angular dependence.

The t heoretical cross section is obtained by finding δ. This value is obtained by joining the wave function of the scattered wave back at the potential.

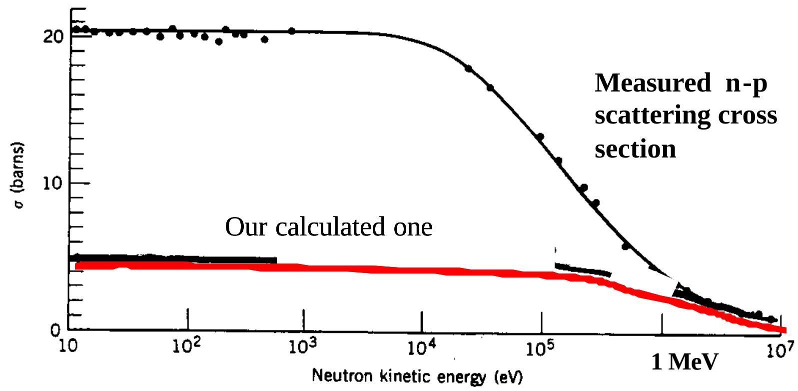
For r>c we have ψ = Bsin(kr + δ), and for b<r<c we have ψ = AsinKr
Matching the magnitude and the deriviatives gives the transendental equation

K cotKb = k cot [k(c+b) + δ]

Using the values that we used in solving the deuteron

V_0= 73 MeV (determines K for a given E)
b= 1.34 fm
c= 0.4 fm
gives values for δ as a function of E.

The theoretical value is about 5 mb. The result for the measured cross section is shown



What went wrong?

We used N-N parameters for S=1, from our Deuteron discussion

Thus only calculated the situation ÝÝ (S=1). In scattering we can also have Ýß (S=0).

Coupling to ÝÝ (S=1) is 3 times as likely as to Ýß (S=0).

Thus the measured cross section is s = ¾ s_3 + ¼ s_1.
We calculated s_3 ~ 5 b.

So 20 b = ¾ (5 b) + ¼ s_1 ➔ ¼ s_1 = ~16 b ➔ s_1 ~ 64 b.

The S^3 and S^1 parts of the N-N potential are very different.

We see that the theoretical value is significantly below the measured value at low energies. Why is this? And what does it tell us?

We did the calculation using the parameters that gave the bound deuteron. This arrangement of n-p was parallel spins for n and p: S=1. However in scattering we are not limited to S=1, we also have S=0. As we noted before, the nuclear potential for S=0 is smaller than for S=1.Recall the S^1 state of the deuteron was unbound. If we assume that the large difference is due to singlet scattering we can determine the relative strengths of σ_3 and σ_1.

As the neutron, with spin ½ approaches the proton in the target (with spin s = ½ , the probability of coupling to 1 is 3 times that of coupling to 0.

Thus the measured cross section is
s = ¾ s_3 + ¼ s_1.
We have calculated σ_3 to be about 5 b, since we used S=1 parameters.
So 20 b = ¾ (5 b) + ¼ s_1
¼ s_1 = 20 - ~4 = 16 b
So that σ_1 ~ 64 b.

This is a huge difference, and an important indication of the difference in the singlet and triplet components of the nuclear potentials. This confirms our observation from the fact that the S^1 excited state was less bound than the S^3, that the nuclear force must be spin dependent.

In fact it is not possible to quantify this difference from these scattering experiments. It was resolved by scattering VERY low-energy neutrons (0.01 eV) from hydrogen MOLECULES, which come in 2 forms Para- (proton spins S=0) and Ortho- (Proton spins S=1). These neutrons have deBroglie wavelengths of 0.05 nm (much larger than the separation of the nuclei in the H mol. So that the neutron interacts coherently with BOTH nuclei, and scatters coherently. The resulting interference pattern (scattering cross-section) allows the S=1 and S=0 potentials to be quantified, and the S=0 one is 40% weaker.

The reason scattering of nucleons with energies <a few MeV is not much help in determining the complete nature of the N-N potential is that they do not get close to the nucleon. In wave-mechanical terms their deBroglie wavelength is much bigger than the potential size: the Angular distribution is isotropic. In terms of the optical analogue, the angle of the first diffraction minimum given by $\sin\theta = \lambda/d$ is at an angle greater than π (in the CM system. As we use higher energy probes, we would expect that the scattering cross section would begin to show forward peaking since now the wavelength is smaller that the potential width..

So let us look at some results for n-p scattering when E_p is much higher. From our optical analogue we would expect the cross section to peak at smaller angles as λ/d decreases.

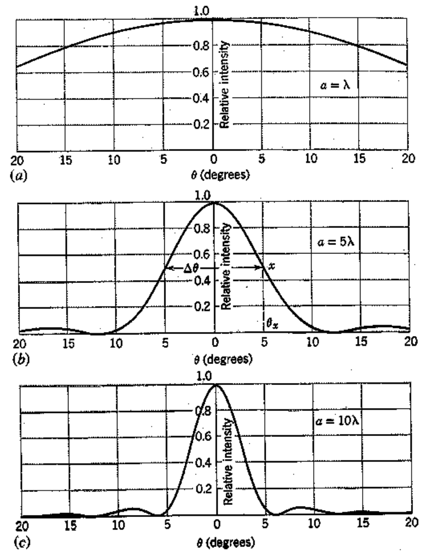
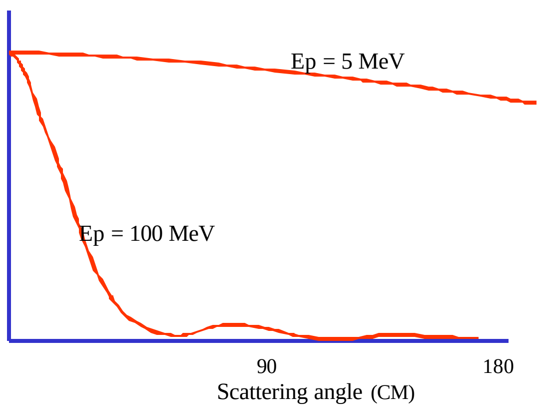
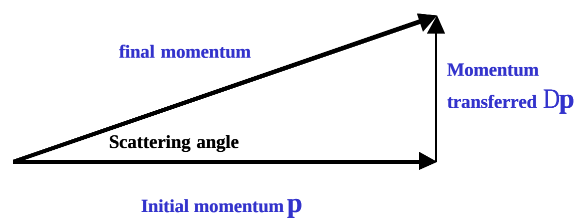


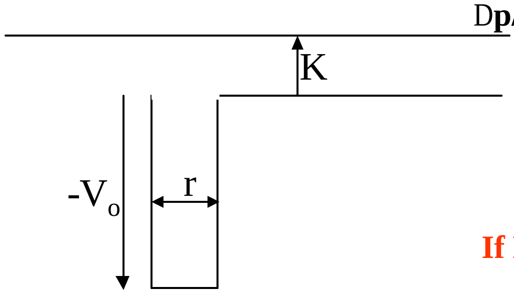
Figure 11 The relative intensity in single-slit diffraction for three different values of the ratio a/λ. The wider the slit, the narrower is the central diffraction peak.

Expected n-p scattering cross section



The classical reason for this expectation is that the impulse given to the nucleon as it gets into the range of the nuclear potential is small if it is moving quickly (the SPEED bus-effect).





$\Delta p/p \sim F\Delta t/p$

$\sim F(r/v)/mv$ **r = width of potential**

$\sim V_0/mv^2$ **(F = -dV/dr)**

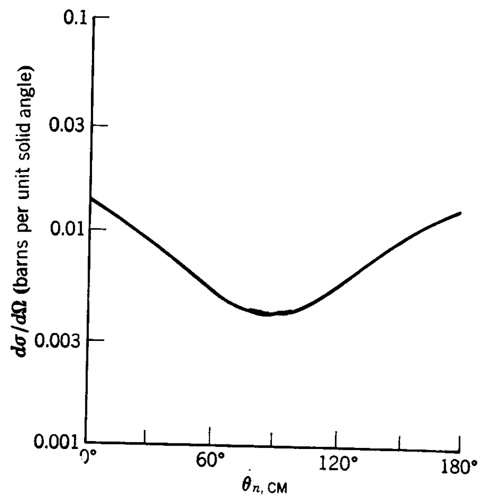
$\sim V_0/K$

If $K > V_0$ angle of scattering is small

When they did n-p scattering at 90 MeV they got the following:

(Note we are in the n-p CM ref. Frame.)

Not what was expected! There is no peaking at forward angles. There is an equal number of neutrons (or protons) scattered through large angles as there are through small.



Scattering of neutrons by protons at 90 MeV. Results plotted in the CM reference frame.

The expected forward-peaked cross section would have involved an interaction as shown in the top diagram of the figure below. The explanation of what was observed is shown in the lower diagram.

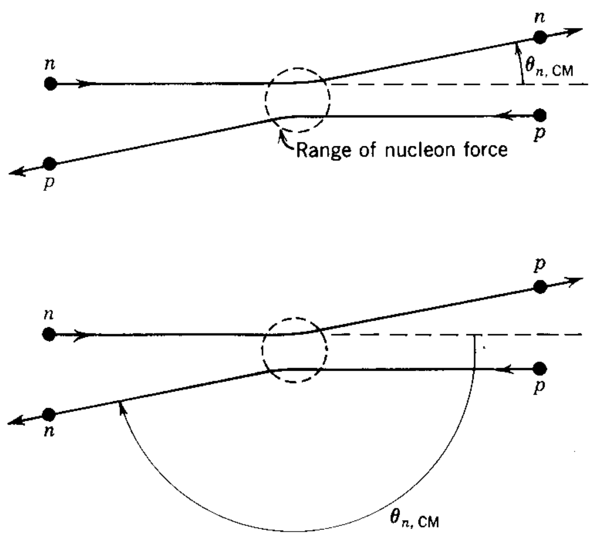


Figure 17-6 *Top:* Neutron-proton scattering as seen in a frame of reference in which the center of mass of the system is stationary. If the kinetic energies of the nucleons are large compared to the depth of the nucleon potential, the momentum transfers are small and the neutron and proton scattering angles are small as well. *Bottom:* The same, for a scattering in which the neutron changes into a proton and vice versa when they interact. Although the momentum transfers are still small, because of the exchange the scattering angles are large.

The interpretation of the data was that in about half of the scatterings, whilst in the region of the N-N potential, a p changed to a n, and vice verca.

At this stage we can begin to see that the nuclear potential is quite complicated. It depends not only on r, but on S, and now we see it has the possibility of exchanging p and n.. The interpretation of this, and the discussion of all the components of the N-N potential will be left to the next lecture.