

1 Introduction

The study of Elementary Particle Physics continues the great tradition of Physics to explore the frontiers of human knowledge. In the case of Particle Physics we explore the realm of very small. Along the way we have discovered a basic understanding of the fundamental forces of nature. However MANY puzzles remain. We will look at these puzzles and limits to what we know as well as finding out what we we do know. I have very deliberately taken an experimental approach to this course. I really want to emphasize how much Particle Physics is an experimental science. Although this subject is an extremely active region of Theoretical Physics research, Particle Physics is very much founded on fact. Many beautiful and elegant theories have died when confronted with experimental fact.

Particle Physics is a dynamic field.

1.1 Basic Concepts

We start by reviewing or introducing some basic concepts. With a good understanding of just some of these ideas it is possible to understand many points of Experimental Particle Physics.

A very brief history of Particle Physics

- 500 BC Greeks postulate the existence of atoms
- 1600 - 1900 Chemical elements are discovered. They're identified as atoms.
- 1897 electron discovered
- 1900 Quantum Theory invented to explain Black Body Spectrum.
- 1905 Atomic Nucleus discovered by Rutherford.
- 1911 Planetary atom postulated with discrete atomic levels
- 1911-1928 Quantum Theory developed.
- 1928 Positron Postulated by Dirac.
- 1932 Neutron discovered.
- 1933 Neutrino Postulated.
- 1933 Positron discovered.

- 1935 Pion's existence postulated by Yukawa.
- 1937 Muon discovered.
- 1947 Pion discovered.
- 1947 Kaon discovered.
- 1953 Neutrino discovered.
- 1950 - 64 Strange Baryons discovered.
- 1957 Parity found to be violated.
- 1963 Eight-fold way predicts the existence of the Ω^- .
- 1964 Ω^- discovered.
- 1965 CP found to be violated in k^0 decays.
- 1968 Deep inelastic e scattering shows substructure in Protons. "Bound" quarks discovered.
- 1961-1970 Electroweak Unification postulated. The W and Z predicted and charm quark predicted.
- 1974 Charm Quark discovered.
- 1970-75 Quantum Chromodynamics postulated as the strong force.
- 1977 Bottom or Beauty Quark discovered.
- 1983 W and Z discovered. "Standard Model" firmly established.
- 1997 Top Quark discovered.
- 1999 Neutrino Oscillations observed.

The discovery of neutrino oscillations is the first indication of physics beyond the standard model. Physicists continue to look for more evidence of physics beyond the standard model since such discoveries provide hints of what theory the Standard Model is a subset of.

Sizes

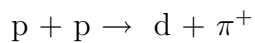
	size	Equivalent Photon Energy
ATOM	10^{-10} m = 10^5 fermi	0.00001 GeV = (10 KeV Xrays)
Nucleus	10^{-14} m 10 fermi	0.1 GeV
Proton	10^{-15} m 1 fermi	1 GeV
Quark	$\leq 10^{-18}$ m 0.1 fermi	100 GeV

Quarks and Leptons (e, μ, τ) are observed to be structure-less down to $10^{-18} = 100$ GeV.

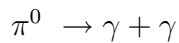
1.2 Reactions and Decays

We learn about particle properties by their interactions with other particles.

Example of a reaction



Example of a decay



The basic conserved quantities are Energy, Momentum, Charge, Angular momentum. These are always conserved and are the same on either side of the arrow. Initial state quantities = Final state quantities.

Conserved integral quantities imply the existence of a Quantum Number. We shall learn about other conserved quantities in this course.

1.3 Spherical Harmonics and Orbital Angular Momentum

I shall take some time to review the concept of orbital angular momentum.

The Classical definition of Angular momentum about a point is:

$$\vec{L} = \vec{r} \times \vec{p} \tag{1}$$

Where \vec{r} is the distance from the point to a particle of momentum \vec{p} . This quantity is non-zero if the distance of closest approach of the particle to the point is non-zero. If we look at an interaction between two particles it is clear we can define an angular momentum exactly as above. Now very interestingly the uncertainty principle can give us a non zero angular momentum even if the two particles originate from the same point. ie.

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad (2)$$

implies a potential non-zero angular momentum for any two interacting particles since the uncertainty principle allows a non-zero distance of closest approach. This type of angular momentum is called the “Orbital Angular Momentum” of interacting particles and is always present in a system of two or more particles.

In Quantum Mechanics the quantization of angular momentum in integral units of \hbar arises naturally from the mathematics of wave functions in spherical coordinates.

The Schrodinger Equation in a potential free region, a long way from the interaction point reduces to:

$$\nabla^2 \Psi = 0 \quad (3)$$

which can be solved by the method of separation of variables. In Spherical coords we look for solutions of the form:

$$\Psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi) \quad (4)$$

In Spherical coords $\nabla^2 \Psi = 0$ becomes:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Psi}{\partial r} \right) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left(\sin(\theta) \frac{\partial \Psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2 \Psi}{\partial \phi^2} = 0 \quad (5)$$

Since r, θ and ϕ are totally independent variables equation 5 can only be solved if:

$$\frac{1}{R(r)} \frac{d}{dr} \left[r^2 \frac{dR(r)}{dr} \right] = a \quad (6)$$

$$\frac{1}{\Theta(\theta) \sin(\theta)} \frac{d}{d\theta} \left[\sin(\theta) \frac{d\Theta(\theta)}{d\theta} \right] = b \quad (7)$$

and

$$\frac{1}{\Phi(\phi)} \frac{d^2 \Phi}{d\phi^2} = c \quad (8)$$

where a, b and c are each separation constants that satisfy.

$$a + b + c = 0 \quad (9)$$

Now define $c = -m_l^2$ and equation 8 becomes:

$$\frac{d^2 \Phi}{d\phi^2} + m_l^2 \Phi = 0 \quad (10)$$

Which has the solution:

$$\Phi_{ml} = \frac{1}{\sqrt{2\pi}} e^{im_l\phi} \quad (11)$$

Where $m_l = 0, \pm 1, \pm 2, \dots$. The equation for $\Theta(\theta)$ becomes Legendre's equation:

$$\frac{1}{\sin(\theta)} \frac{d}{d\theta} \left[\sin(\theta) \frac{d\Theta(\theta)}{d\theta} \right] + \left[l(l+1) - \frac{m_l^2}{\sin^2(\theta)} \right] \Theta(\theta) = 0 \quad (12)$$

Where $l = 0, 1, 2, 3, \dots$ and $m_l = 0, \pm 1, \pm 2, \dots, \pm l$. The solution $\Phi_{lm_l}(\theta)$ can be expressed as a polynomial of degree l in $\sin(\theta)$ or $\cos(\theta)$. Together, and normalized, Φ_{m_l} and Θ_{lm_l} give the *spherical harmonics* $Y_{lm_l}(\theta, \phi)$.

Each distinct value of l corresponds to a unit of orbital angular momentum \hbar . Each value of m_l corresponds to the spin projection of the l th substate. In addition these functions have the symmetry property:

$$Y_{lm_l}(\theta, \phi) = (-1)^l Y_{lm_l}(-\theta, \phi + \pi) \quad (13)$$

This change in coordinates corresponds to interchanging two particles in a two body system.

1.4 Bosons and Fermions

Consider a wavefunction made of 2 or more identical particles $\Psi(n_1 n_2)$.

Then

$$|\Psi|^2 = |\Psi(n_1 n_2)|^2 = |\Psi(n_2 n_1)|^2 \quad (14)$$

ie we cannot distinguish the two situations. That implies

$$\Psi(n_1 n_2) = \pm \Psi(n_2 n_1) \quad (15)$$

ie. The wave functions can be either symmetric or anti-symmetric with the interchange of two particles.

The following rule holds always and divides all particles into two classes.

Bosons: Integral Spin $\{0\hbar, 1\hbar, 2\hbar, \dots\}$

Have $\Psi(n_1 n_2) = \Psi(n_2 n_1)$ ie symmetric upon interchange.

Fermions: $\frac{1}{2}$ integral spin $\{\frac{1}{2}\hbar, \frac{3}{2}\hbar, \frac{5}{2}\hbar, \dots\}$

Have $\Psi(n_1n_2) = -\Psi(n_2n_1)$ ie anti-symmetric upon interchange.

We split the wavefunction into at least two parts:

$$\Psi = \alpha(space)\beta(spin) \tag{16}$$

The

$$\alpha(space) \tag{17}$$

gives the orbital motion with respect to each other. Because of the features of spherical harmonics this gives a factor of $(-1)^l$ upon interchange of identical particles where “l” is the orbital angular momentum of the two particles.

The $\beta(spin)$ function requires either spins aligned (symmetric upon interchange) or anti-aligned (antisymmetric). These are the only two possibilities given

$$|\Psi(n_1n_2)|^2 = |\Psi(n_2n_1)|^2 \tag{18}$$

So for Bosons:

either $\Psi(n_1n_2) =$	$\alpha(l \text{ even})$ symmetric	$\beta(\uparrow\uparrow)$ symmetric
or $\Psi(n_1n_2) =$	$\alpha(l \text{ even})$ anti-symmetric	$\beta(\uparrow\downarrow)$ anti-symmetric

The result is always symmetric.

And for Fermions:

either $\Psi(n_1n_2) =$	$\alpha(l \text{ odd})$ anti-symmetric	$\beta(\uparrow\uparrow)$ symmetric
or $\Psi(n_1n_2) =$	$\alpha(l \text{ even})$ symmetric	$\beta(\uparrow\downarrow)$ anti-symmetric

The result is always anti-symmetric.

1.5 Side Note.

The requirement that a fermion wavefunction be antisymmetric automatically gives the Pauli exclusion principle. “No two fermions can occupy the same state”.

Occupying the same state implies $l = 0$ (no motion relative to each other) and spins aligned (the same state remember).

So

$$\begin{aligned}\Psi(n_1 n_2) &= \alpha(\textit{symmetric})\beta(\textit{symmetric}) & (19) \\ &= \Psi(n_2 n_1) & (20)\end{aligned}$$

Which is not allowed.

On the other hand Bosons can happily sit right on top of each other because $\Psi(n_1 n_2)$ is symmetric. So cool bosons enough and they all end up with the same wavefunction.

1.6 Example of an application to Particle Physics

The decay of the neutral ρ^0 meson which has spin 1. The π^0 has spin 0. Look at the decay:

$$\rho^0 \rightarrow \pi^0 \pi^0 \quad (21)$$

Since the π^0 has zero spin the decay $\rho^0 \rightarrow \pi^0 \pi^0$ needs the π^0 to have orbital angular momentum $l = 1$ to conserve angular momentum.

However this implies the final state wavefunction has the property $\Psi(\pi_1^0 \pi_2^0) = (-1)^1(\textit{symmetric spin}) = -\Psi(\pi_2^0 \pi_1^0)$. This is not allowed!

The decay $\rho^0 \rightarrow \pi^0 \pi^0$ has never been observed but $\rho^0 \rightarrow \pi^+ \pi^-$ is the main way the ρ^0 decays. This happens because the π^+ and π^- are not identical and so do not have this restriction.

This is typical of one sort of problem you'll be given in this course. Use selection rules to decide if a process is allowed.

1.7 Particles and Antiparticles

A consequence of relativistic quantum mechanics is the existence of antiparticles for every particle.

Antiparticles have the same mass but opposite charge and magnetic moment. All Bosons and fermions have anti-particles.

A consequence of the fact that fermions have half-integral spin is that they are only created or destroyed in pairs. Any process that created or destroyed a single Fermion would violate conservation of angular momentum since orbital angular momentum produces allows only integral values.

There is no such problem for Bosons which can be created with the conservation of angular momentum. So for example the reaction:



can proceed.

1.8 Quarks and Leptons

Quarks

Quarks are the constituents of Hadrons. A hadron is any particle that feels the strong force. Hadrons are either Baryons = (QQQ) (3 quarks) {*proton, neutron*} or a meson = $Q\bar{Q}$ (2 quarks) { π, k }.

Quarks are observed to be structureless and featureless on a scale of less than 10^{-18} m or above 200 GeV.

They carry fractional electric charge. Either $+\frac{2}{3} | e |$ or $-\frac{1}{3} | e |$. They occur in several varieties or flavours.

u(up), d(down), s(strange), c(charm), b(bottom), t(top). Each quark has an internal quantum number that is conserved in strong and electromagnetic interactions. They are grouped as doublets of generations.

u	$\frac{2}{3}$	7 MeV	c	$\frac{2}{3}$	1.5 GeV	t	$\frac{2}{3}$	174 GeV
d	$-\frac{1}{3}$	4 MeV	s	$-\frac{1}{3}$	150 MeV	b	$-\frac{1}{3}$	4.5 GeV

Since quarks have half integral spin, Baryons with 3 quarks have half integral spin and are Fermions. Mesons have 2 quarks and so have integral spin and are bosons.

Examples

Baryons	Mesons
uud = p (proton)	$u\bar{d} = \pi^+$ (pion)
udd = n (neutron)	$d\bar{s} = k^0$ (kaon)
uds = Λ (lambda hyperon)	$c\bar{c} = J/\Psi$ (meson)

All quark quantum numbers are conserved in strong and electromagnetic reactions.

Example

$$\begin{array}{rcccl}
 \pi^+ & p & \rightarrow & k^+ & \Sigma^+ \\
 u\bar{d} & uud & & u\bar{s} & uus \\
 S & 0 & 0 & +1 & -1
 \end{array}$$

Leptons

The leptons have also been observed to be structureless and point-like down to 10^{-18} or up to 100 GeV. They're

e	μ	τ
ν_e	ν_μ	ν_τ

They have integral charge 0 or ± 1 and do not feel the strong force. Indeed the ν only interact via the weak force. Each lepton has an internal quantum number that is strictly conserved in all interactions.

$$\begin{aligned}
 L_e, L_\mu, L_\tau &= +1 && \text{for } e^-, \mu^-, \tau^- \\
 &= -1 && \text{for } e^+, \mu^+, \tau^+ \\
 &= -1 && \text{for } \nu_e, \nu_\mu, \nu_\tau \\
 &= +1 && \text{for } \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau
 \end{aligned}$$

Examples: **Pair Production**

$$\begin{array}{rcccl}
 \gamma & \rightarrow & e^+ & e^- \\
 L_e & 0 & -1 & +1
 \end{array}$$

Pion Decay

$$\begin{array}{rcl} \pi^+ & \rightarrow & \mu^+ \quad \nu_\mu \\ L_e & 0 & -1 \quad +1 \end{array}$$

Muon Decay

$$\begin{array}{rcl} \mu^+ & \rightarrow & e^+ \quad \nu_e \quad \bar{\nu}_\mu \\ L_\mu & -1 & 0 \quad 0 \quad -1 \\ L_e & 0 & -1 \quad +1 \quad 0 \end{array}$$

While the decay:

$$\begin{array}{rcl} \mu^+ & \rightarrow & e^+ \quad \gamma \\ L_e & 0 & -1 \quad 0 \\ L_\mu & -1 & 0 \quad 0 \end{array}$$

has a branching ratio below 10^{-13} .

In 1999 SuperKamiokande observed that ν_μ neutrinos from cosmic ray interactions in the atmosphere *oscillate* into another (unknown) form.

$$\nu_{\mu} \rightarrow \nu_X$$

At this time (May 2000) we do not yet know what neutrino type ν_X is although the current prejudice is that they are ν_τ . Ray Volkas and Robert Foote from the theoretical particle physics group have postulated that ν_X are new massive neutrinos via a well formulated theory.

This process of neutrino oscillation of course violates lepton number conservation. In addition it requires that neutrinos have non-zero mass. Both these conclusions violate the Standard Model of particle physics and will be used to develop the superset of the standard model.

Finally, a consequence of fermion number conservation is that Baryon number is observed to be conserved.

Each Baryon is assigned +1 Baryon Number:
Each anti-Baryon is assigned -1 Baryon Number.

1.9 Cross Sections and Decay Rates

The strength of an interaction can be inferred by the decay rate of a particle decay or the size of the cross section for the particle reaction.

Decay Rates

$$P(t) = e^{-\frac{t}{\tau}} \quad (23)$$

where $P(t)$ = Probability of decay per unit time and τ = mean life or lifetime.
Faster decay rate \Rightarrow stronger interactions.

If a decay rate is very fast it is easier to determine the “width” of the particle. This is the uncertainty in the mass of the particle. These are related through the uncertainty principle.

$$\tau\Gamma = \hbar \quad (24)$$

(cf. $\Delta t \Delta E \geq \frac{\hbar}{2}$) Where Γ is the width of the particle.

For example, the π which decays via $\pi \rightarrow \mu^+ \nu_\mu$ has a meanlife, τ of 2.6×10^{-8} secs. This corresponds to a width, Γ of:

$$\Gamma = \frac{\hbar}{\tau} = \frac{6.22 \times 10^{-22} \text{MeV} - \text{secs}}{2.6 \times 10^{-8} \text{secs}} = 2.5 \times 10^{-8} \text{eV} \quad (25)$$

Slow decay \Rightarrow weak interaction.

A different example is the decay of the π^0 . This decays as $\pi^0 \rightarrow \gamma\gamma$. For this reaction:

$$\tau = 0.87 \times 10^{-16} \text{secs} \Rightarrow \Gamma = \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22} \text{MeV} - \text{sec}}{0.87 \times 10^{-16} \text{secs}} = 7.6 \text{eV} \quad (26)$$

This is considerably faster than the decay of the charged pion as it occurs via the electromagnetic interaction.

Finally consider the decay: $\rho^0 \rightarrow \pi^+ \pi^-$. This decay has a width $\Gamma = 150 \text{MeV}$.

$$\tau = \frac{\hbar}{\Gamma} = \frac{6.582 \times 10^{-22} \text{MeV} - \text{secs}}{150 \text{MeV}} = 4.4 \times 10^{-24} \text{secs} \quad (27)$$

Very fast decay \Rightarrow strong interaction.

Cross Section

A cross section gives a measure of the probability of an interaction. For the reaction:

$$a + b \rightarrow c + d$$

where a is the projectile, b is the target, c and d are the final state reaction products, the cross section is defined as:

$$\sigma = \frac{\text{Yield}(c + d)}{N_a N_b} \quad (28)$$

where N_a = number of particles a fired at the target.

N_b = number of target particles b per unit area.

Yield(c+d) = number of times the reaction products (c+d) are produced.

The dimensions of σ are **area**. An interpretation of σ is the effective area blocked by each b for the process: $a + b \rightarrow c + d$.

Then the yield of the reaction is given by:

$$Yield = \sigma \times N_b \times N_a \quad (29)$$

The units used for σ are barns,

barn b	=	10^{-24}cm^2
millibarn mb = 10^{-3} b	=	10^{-27}cm^2
microbarn mb = 10^{-6} b	=	10^{-30}cm^2
nanobarn mb = 10^{-9} b	=	10^{-33}cm^2

Some typical cross sections:

Strong Interaction: $\pi^+ p \rightarrow \text{anything}$, $\sigma(1 \text{ GeV}) = 20 \text{ mb}$

Electromagnetic Interaction: $\gamma p \rightarrow \text{anything}$, $\sigma(1 \text{ GeV}) = 0.130 \text{ mb}$

Weak Interaction: $\nu_\mu p \rightarrow \text{anything}$, $\sigma(1 \text{ GeV}) = 10^{-11} \text{ mb}$

1.10 Special Relativity

In typical Particle Physics situations the kinetic energy of a particle is often at least as large as its mass. Consequently the low energy Newtonian Approximations to the equations of motion do not work.

In this course we will typically work with Relativistic formulae. The basis of these are the following. Let:

E = Total energy of the particle

m_0 = Rest mass of the particle
 p = momentum
 c = speed of light
 γ = Relativistic Gamma factor
 v = velocity of the particle
 β = $\frac{v}{c}$

Then the basic equations of Special Relativistic kinematics are:

$$E^2 = p^2c^2 + (m_0c^2)^2 \quad (30)$$

$$E = \gamma m_0c^2 \quad (31)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad (32)$$

Take (30), rewrite it as:

$$\begin{aligned}
 E^2 - (m_0c^2)^2 &= p^2c^2 \\
 \Rightarrow (E - m_0c^2)(E + m_0c^2) &= p^2c^2
 \end{aligned}$$

Let T= kinetic energy of the particle then

$$T = E - m_0c^2$$

ie. Kinetic Energy = Total Energy - Rest Energy. So:

$$\begin{aligned}
 p^2c^2 &= T(T + 2m_0c^2) \\
 \Rightarrow p &= \frac{\sqrt{T(T + 2m_0c^2)}}{c} \quad (33)
 \end{aligned}$$

**This is the relation between kinetic energy and momentum.

Turning it around gives:

$$T^2 + 2m_0c^2T - p^2c^2 = 0$$

Solving for T gives:

$$T = \frac{-2m_0c^2 \pm \sqrt{4(m_0c^2)^2 + 4p^2c^2}}{2}$$

Clearly only the +ve gives +ve T so

$$T = \sqrt{((m_0c^2)^2 + p^2c^2)} - m_0c^2 \quad (34)$$

Next $T = E - m_0c^2$ by definition so $T = \gamma m_0c^2 - m_0c^2 \Rightarrow$

$$T = m_0c^2(\gamma - 1) \quad (35)$$

$$\gamma = 1 + \frac{T}{m_0c^2} \quad (36)$$

$$\gamma = \left(\sqrt{\frac{p^2c^2}{(m_0c^2)^2} + 1} \right) - 1 \quad (37)$$

Then finally we can derive:

$$\beta = \frac{pc}{m_0c^2 + T} = \frac{pc}{E} \quad (38)$$

Work with $E, T, p, m_0, \gamma, \beta$ with these expressions.

1.11 Relativistic 4-vectors

Define a four vector $p = (\vec{p}c, E)$ where \vec{p} is usual momentum vector and E is the total energy of the particle. Then:

$$p^2 = |\vec{p}|^2 c^2 - E^2 = -(m_0c^2)^2 \quad (39)$$

is invariant under Lorentz transformations. This leads to some very useful relations.

Example of 4-momentum use

We have two particles A and B that interact. How much energy is available for particle creation?

$$p^2 = (\vec{p}_A + \vec{p}_B)^2 - (E_A + E_B)^2 = -m_A^2c^4 - m_B^2c^4 + 2\vec{p}_A \cdot \vec{p}_B c^2 - 2E_A E_B \quad (40)$$

The center of momentum system (CMS) is defined as the reference frame where the total momentum is 0. ie $(\vec{p}_A + \vec{p}_B) = 0$. If the total energy in this system is E^* ,

$$\Rightarrow p^2 = (\vec{p}_A c + \vec{p}_B c)^2 - (E_A + E_B)^2 = -E^{*2} \quad (41)$$

Lets look at two special cases. (i) If the target particle m_B is at rest, $(\vec{p}_B = 0)$ so

$$E^{*2} = -p^2 = (m_A c^2)^2 + (m_B c^2)^2 + 2m_B c^2 E_A$$

the energy required to create a new particle is $m^*c^2 = E^*$

The initial energy E_A required to create it is

$$E_A = \frac{(m^*c^2)^2 - (m_Ac^2)^2 - (m_Bc^2)^2}{2m_Bc^2} \quad (42)$$

$$E_A \approx \frac{(m^*c^2)^2}{2m_Bc^2} \text{ for } E_A \geq m_Am_B \quad (43)$$

ie the energy available for particle creation grows $\propto \sqrt{E_A}$ in fixed target reactions. This is because a lot of the incident beam's momentum is used to give the created particles momentum too.

(ii) If the two particles have equal and opposite momentum ($\vec{p}_A = -\vec{p}_B$). This is the situation in many colliding beam experiments.

Then

$$\begin{aligned} E^{*2} &= -p^2 = -(\vec{p}_A + \vec{p}_B)^2 + (E_A + E_B)^2 \\ &\Rightarrow E^{*2} = (E_A + E_B)^2 \end{aligned}$$

So $E^* = 2E_A$

So all the incident Energy is available for particle creation.

1.12 Units in Particle Physics

The fundamental units in Physics are length, mass and time. In MKS these are expressed in terms of meters, kilograms and seconds.

In particle physics, the study of the small, these units are inconvenient. Here we work with particles with energies in the regions of MeV and GeV.

Here we work with particles with energies in the regions of MeV and GeV.

We express masses in terms of their Einstein equivalent rest energies m_0c^2 .

For example $m_0 = (\text{proton}) = 938 \text{ MeV}/c^2$.

$m_0 = (\text{electron}) = 0.511 \text{ MeV}/c^2$.

We express momentum in units of MeV/c. Typical lengths are $10^{-15} \text{ m} = 1 \text{ fermi}$, $1 \text{ barn} = 100 \text{ fermi}^2$.

$\hbar = 1.054 \times 10^{-34} \text{ Joule-secs}$

$$\begin{aligned}
&= 6.582 \times 10^{-22} \text{ MeV-sec} \\
c &= 2.99 \times 10^8 \text{ m sec}^{-1} \\
\hbar c &= 197.3 \text{ MeV-fermi}
\end{aligned}$$

Natural Units are defined to have $\hbar = c = 1$. All other quantities are then expressed in units of energy.

Conversion Factor	$\hbar=c=1$ Units	Actual Dimensions
1kg = 5.26×10^{26} GeV	GeV	GeV c ⁻²
1 m = 5.07×10^{15} GeV ⁻¹	GeV ⁻¹	$\frac{\hbar c}{\text{GeV}}$
1 fermi = 5.07 GeV ⁻¹	GeV ⁻¹	$\frac{\hbar c}{\text{GeV}}$
1 sec = 1.52×10^{24} GeV ⁻¹	GeV ⁻¹	$\frac{\text{GeV}}{\hbar}$

Other useful conversion factors $(1 \text{ GeV})^{-2} = 0.389 \text{ mb}$

1.13 Interactions and Fields in Particle Physics

This is a very brief, hand-waving introduction to this. See Girish Joshi's course for the *real* story.

Classically interaction between particles occur through the effects of a potential or field.

In Q.M. interactions occur through the exchange of quanta (bosons). Since the quantum carries energy and momentum, conservation laws require that it can only happen over a time limited by the Uncertainty Principle. That is

$$\Delta E \Delta t \leq \frac{\hbar}{2}$$

Such transitions are called VIRTUAL

Lets illustrate this concept through Electrostatics. Classically electrostatic force:

$$|F| = \frac{Q_1 Q_2}{r^2}$$

The force between charges comes from the exchange of virtual photons of momentum q, the change of momentum as it absorbs or emits a photon.

Then, the uncertainty principle links, the position (r), with the momentum of the photon (p), $p r \approx \hbar$.

Then for a photon the time of each transfer is $t = \frac{r}{c}$

$$\begin{aligned} \Rightarrow ptc &\approx \hbar \\ \text{so } p &= \frac{\hbar}{tc} \\ \left| \frac{dp}{dt} \right| &= \frac{\hbar}{t^2 c} \end{aligned}$$

This is the rate of momentum transfer for each transfer which is also the force, (F) on the particle. For a photon, $t = \frac{r}{c} \Rightarrow$

$$\left| \frac{dp}{dt} \right| = |F| = \frac{\hbar c}{r^2}$$

The number of photons emitted per unit time $\propto Q_1 Q_2$ and the uncertainty principle gives the r dependence.

Neither the field nor the virtual particles are directly observable.

1.14 The Yukawa Theory

This theory was developed by Yukawa in 1935 to explain the short range nature of the strong force. Suppose that instead of a photon, the particle exchange involves a particle of mass m. The relativistic relation between the total energy E, the momentum p and mass m. The relativistic relation between the total energy E, the momentum p and the mass m is:

$$E^2 = p^2 c^2 + m^2 c^4$$

The differential equation describing the wave amplitude Ψ for such a free particle is obtained by putting in the Q.M. operators.

$$E_{op} = i\hbar \frac{\partial}{\partial t}; P_{op} = -i\hbar \nabla$$

which give the Klein-Gordon equation:

$$\nabla^2 \Psi - \frac{m^2 c^2}{\hbar^2} \Psi - \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} = 0 \quad (44)$$

We are interested in time independent, spherically symmetric solutions for a central force U(r). The θ and ϕ components are given by the spherical harmonics as given earlier.

$$\nabla^2 U(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial U}{\partial r} \right) = \frac{m^2 c^2}{\hbar^2} U(r) \quad (45)$$

The solution of this equation is:

$$U(r) = \frac{g}{4\pi r} e^{-\frac{r}{R}} \quad (46)$$

where

$$R = \frac{\hbar}{mc} \quad (47)$$

Here g is a constant of integration and is identified as the strength of the interaction. $R = \frac{\hbar}{mc}$ = range of the interaction. Historically the range of the strong interaction was measured to be ≈ 1 fermi which implies $m \approx 100$ MeV. In 1947 the pion with a mass of ≈ 140 MeV was discovered. It was identified, too simplistically, as the strong force quantum.

1.15 The Boson Propagator

Let's consider a particle scattered through an angle θ or equivalently the momentum transfer \vec{q} .

The potential $U(\vec{r})$ in coordinate space will have an associated amplitude $f(\vec{q})$ which is just the Fourier transform of $U(\vec{r})$.

$$f(q) = g_0 \int U r e^{iq \cdot r} dV \quad (48)$$

where g_0 is the intrinsic coupling strength of the particle to the potential. The volume integral is over all space.

$$q \cdot r = qr \cos \theta$$

$$dV = r^2 d\phi \sin \theta d\theta dr$$

θ and ϕ are the polar and azimuthal angles. We shall use units $\hbar = c = 1$.

$$f(q) = g_0 \int_0^\infty \int_0^\pi \int_0^{2\pi} U(r) (\cos(qr \cos \theta) + i \sin(qr \cos \theta)) r^2 d\phi \sin \theta d\theta dr \quad (49)$$

$$= \pi g_0 \int_0^\infty \int_0^\pi U(r) (\cos(qr \cos \theta) + i \sin(qr \cos \theta)) r^2 \sin \theta d\theta dr \quad (50)$$

Let:

$$y = \cos \theta \quad \Rightarrow \quad \frac{dy}{d\theta} = -\sin \theta \quad \Rightarrow \quad d\theta = -\frac{dy}{\sin \theta}$$

$$\Rightarrow f(q) = 2\pi g_0 \int_0^\infty \int_{-1}^1 -U(r) (\cos(qry) + i \sin(qry)) dy r^2 dr \quad (51)$$

$$\Rightarrow f(q) = 2\pi g_0 \int_0^\infty -U(r) r^2 \left[\frac{1}{qr} \cos(qry) \frac{i}{qr} \sin(qry) \right]_{-1}^{-1} dr \quad (52)$$

$$= 2\pi g_0 \int_0^\infty \frac{-U(r) r^2}{qr} [\sin(-qr) - i \cos(-qr) - \sin(qr) + i \cos(qr)] dr \quad (53)$$

$$= 2\pi g_0 \int_0^\infty -\frac{U(r) r^2}{qr} \times -2 \sin(qr) dr \quad (54)$$

$$= 4\pi g_0 \int_0^\infty \frac{U(r)r^2}{qr} \times \sin(qr) dr \quad (55)$$

Recall:

$$U(r) = \frac{g}{4\pi r} e^{-\frac{r}{R}}$$

and $R = m$ since $\hbar = c = 1$. So

$$f(q) = g_0 g \int_0^\infty \frac{e^{-mr}(e^{iqr} - e^{-iqr})}{2iq} dr \quad (56)$$

$$f(q) = \frac{g_0 g}{2iq} \int_0^\infty (e^{r(iq-m)} - e^{-r(iq+m)}) dr \quad (57)$$

$$= \frac{g_0 g}{2iq} \left[\frac{1}{iq-m} e^{r(iq-m)} + \frac{1}{iq+m} e^{-r(iq+m)} \right]_0^\infty \quad (58)$$

$$= \frac{g_0 g}{2iq} \left[0 - \frac{iq+m+iq-m}{(iq-m)(iq+m)} \right] \quad (59)$$

$$f(q) = \frac{g_0 g}{q^2 + m^2} \quad (60)$$

Where $g g_0$ = product of two vertex factors describing the coupling of the boson to scattered and scattering particles.

$$\frac{d\sigma}{dq^2} \propto |f(q^2)| \quad (61)$$

1.16 Electromagnetic Interactions

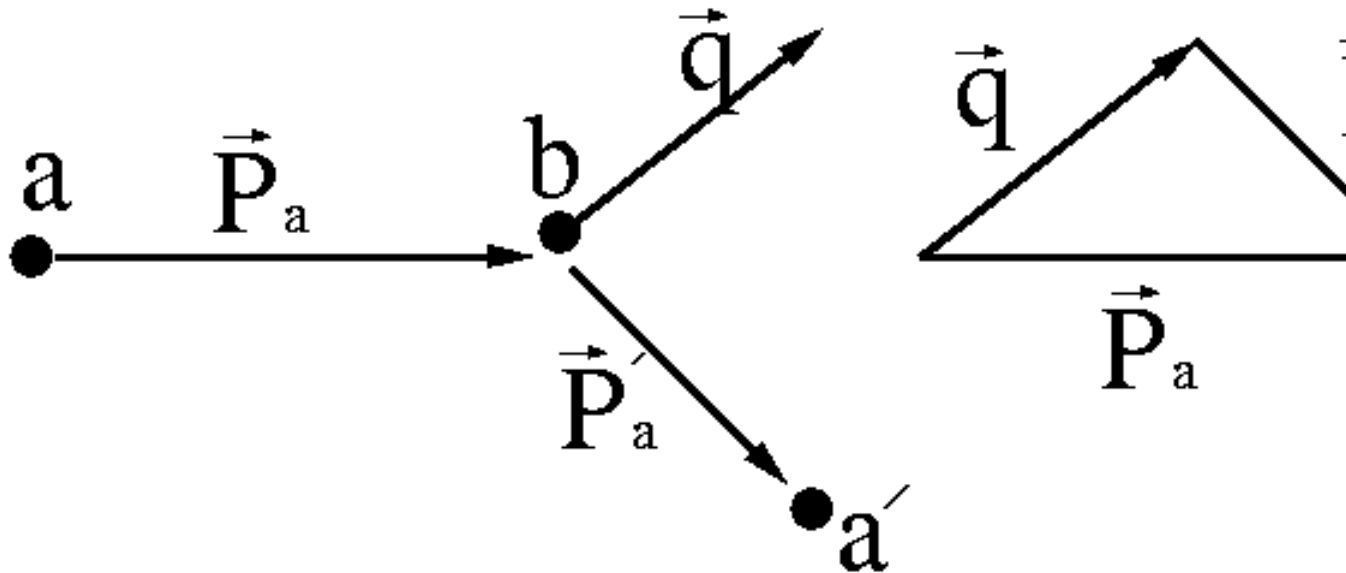
The coupling constant for E-M interactions is:

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137.036}$$

This is the so-called “fine structure constant”.

Some simple Processes

(a) **Photoelectric effect.**



The photon couples to the electron with amplitude $\sqrt{\alpha}$ $\Rightarrow \sigma(\text{photo-electron}) \propto |f|^2 \propto |\sqrt{\alpha}|^2 = \alpha$

(b) Rutherford Scattering.

q = momentum transfer (from the intermediate virtual photon)
 each vertex gives $\sqrt{\alpha}$ so

$$f(q) \propto \frac{\sqrt{\alpha}\sqrt{\alpha}}{q^2 + m_\gamma^2} = \frac{\alpha}{q^2} \quad \text{as } m_\gamma = 0 \quad (62)$$

$$\text{so } \frac{d\sigma}{dq^2} \propto |f(q)|^2 = \frac{\alpha^2}{q^4} \quad (63)$$

This is a second order process

(c) Pair Production near a Nucleus

The nuclear charge is Ze :

$$f \propto \sqrt{\alpha}^3 Z \Rightarrow \sigma \propto \alpha^3 Z^2$$

The field theory used to make exact calculations is called Quantum Electrodynamics or QED.

(d) Self Energy Process

The sum of all these terms is divergent. However QED is renormalizable which means that all these divergent integrals are dumped into m_e and the charge e , and replaced by their experimentally determined values.

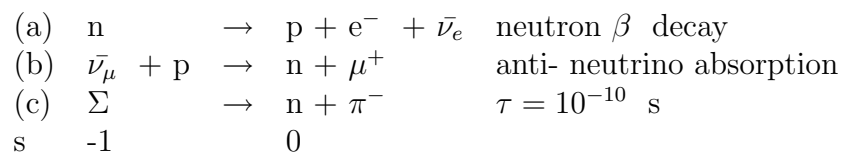
The result is that QED is incredibly precise and has been verified to the limit of experimental precision. (Currently 13 significant figures!)

1.17 Weak Interactions

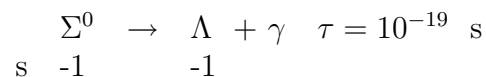
Weak interactions take place between all quark and leptons. However it is normally swamped by E/M and strong interactions unless these are forbidden by conservation laws.

Therefore weak interactions are only observable in neutrino interactions (these have no electric or strong charges).

Or quarks with a flavour change, ($\Delta s = 1$, $\Delta c = 1$, forbidden in strong and electromagnetic decays). eg.

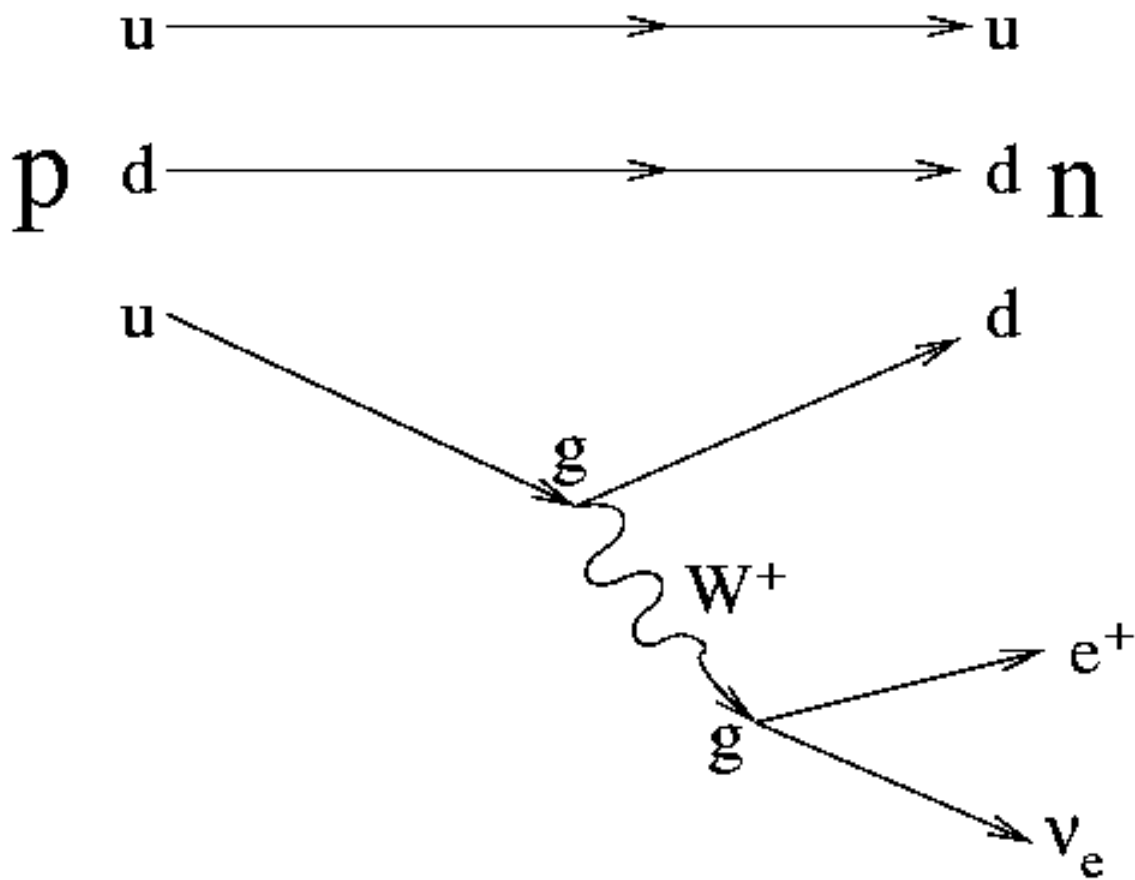


Compare this last decay with the E/M decay of Σ^0



This decay conserves strangeness but violates isospin \Rightarrow E/M decay. Relative strength of weak / strong action:

$$\sqrt{\frac{10^{-10}}{10^{-19}}} \approx 10^{-5}$$



The weak interactions are mediated by massive bosons, the W^\pm and Z^0 . For example, neutron β decay.

For example, neutron β -decay

Another example, neutrino absorption

Then early in 1973 neutral currents showing the existence of the Z^0 were observed in

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

Since then many W^\pm 's and Z^0 have been observed in $p\bar{p}$ colliders at CERN and FERMILAB and at electron-positron colliders at LEP at CERN and SLAC (Stanford USA).

If we assign a single number, g , to the couplings of the W^\pm and Z to quarks and leptons,

the amplitude is:

$$f(q^2) = \frac{g^2}{(q^2 + M_{WZ}^2)} \quad (64)$$

For $q^2 \ll M_{WZ}^2$ the amplitude is independent of q and so looks pointlike (the amplitude has no dependence on q^2). In 1935 Fermi proposed just such a theory to explain β -decay. The strength of the Fermi theory is given by the quantity, G .

$$G = \frac{g^2}{M_W^2} \approx 10^{-5} GeV^{-2} \quad (65)$$

G is determined from the rates of beta-decays of atomic nuclei and from the decay rate of the muon

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

The electroweak theory of Glashow, Salam and Weinberg proposed that

$$g = \frac{\sqrt{\alpha}}{2 \sin \theta_w} \quad (66)$$

for the charge changing weak interaction and

$$g = \frac{\sqrt{\alpha}}{2 \sin \theta_w \cos \theta_w} \quad (67)$$

for the neutral current weak interaction. Where θ_w is the Weinberg angle.

1.18 The Strong Force

First we find the magnitude of the strong interaction. Look at the reaction:

$$k^- + p \rightarrow \Sigma^{0*}(1385) \rightarrow \Lambda + \pi^0 \quad \Gamma = 36 MeV \Rightarrow \tau = 10^{-23} s$$

Compare with E/M decay

$$\begin{aligned} \Sigma^0(1192) &\rightarrow \Lambda + \gamma \quad \tau = 10^{-19} s \\ \Rightarrow \frac{\alpha_s}{\alpha} &\approx \sqrt{\frac{10^{-19}}{10^{-23}}} \approx 100 \\ \alpha_s &= \frac{g_s^2}{4\pi} \approx 1 \end{aligned} \quad (68)$$

This gives the strength of the coupling of the strong charges, g_s of the quarks. The mediating boson is called a gluon.

A gluon is massless and electrically neutral. The strong charges on the quark are called quark colors.

This is just the name for an extra degree of freedom of quarks. Whereas in QED there are just two kinds of electric charge +1 and -1, in Quantum Chromodynamics, (QCD) there are six.

$$\begin{array}{llll} \text{red} & \text{blue} & \text{green} & \text{on quarks} \\ \overline{\text{red}} & \overline{\text{blue}} & \overline{\text{green}} & \text{on anti-quarks} \end{array}$$

These labels are in analogy with the 3 primary colours of light.

Also unlike the QED photon, gluons also carry color.

Because $\alpha_s \approx 1$ multiple gluon exchange is highly probable and we can't use perturbation theory ie.

$$1 \approx \alpha_s \approx \alpha_s^2 \approx \alpha_s^3 = \dots etc$$

At very energies α_s is reduced enough to apply perturbation theory. \Rightarrow the existence of jets of quarks at high energy.

The other different property of QCD, colored mediating bosons, has interesting properties. Each gluon interacts with every other gluon so:

This leads to color flux tubes and the color potential has the form:

$$V_s = -\frac{4}{3} \frac{\alpha_s}{r} + kr \tag{69}$$