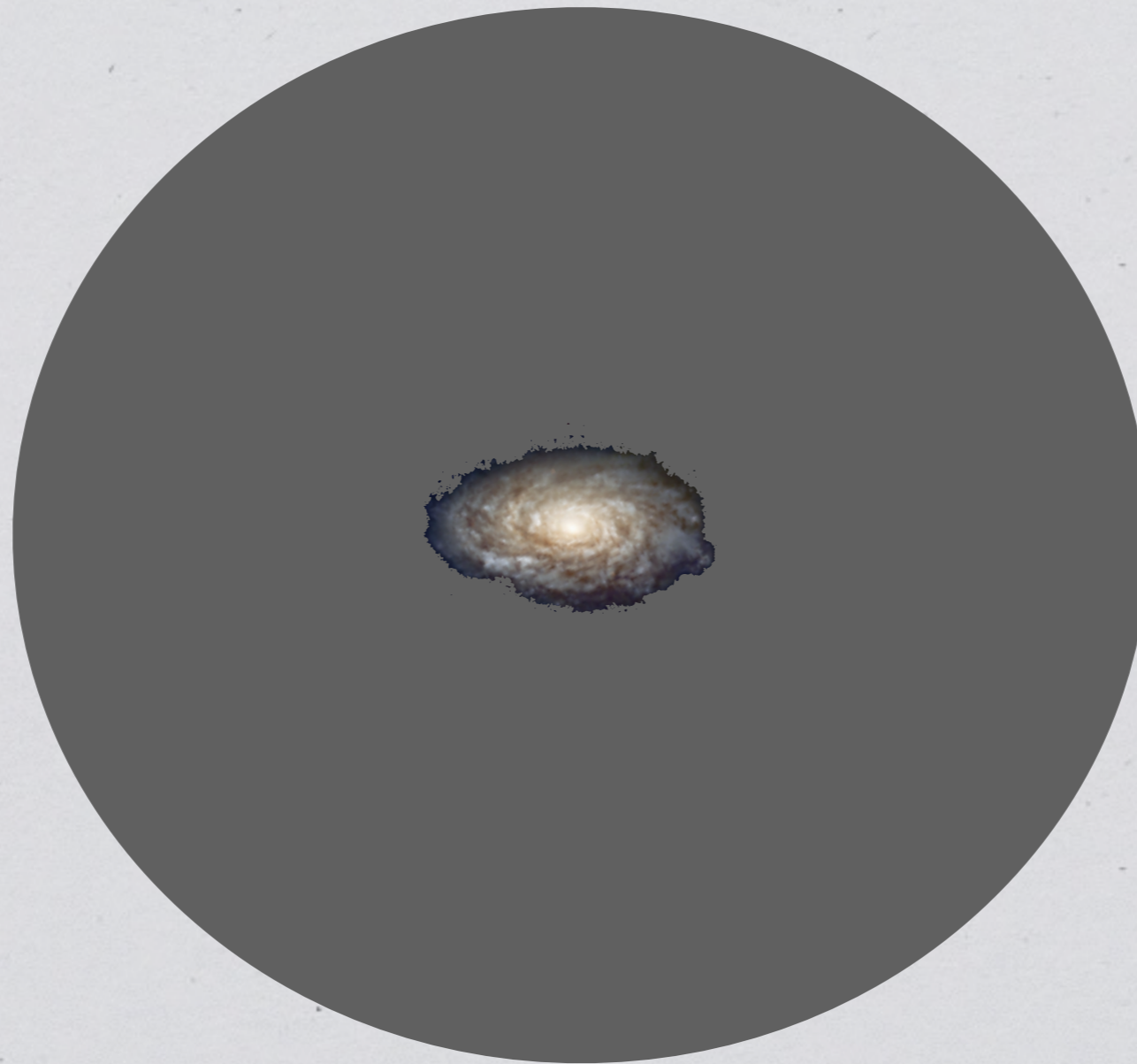
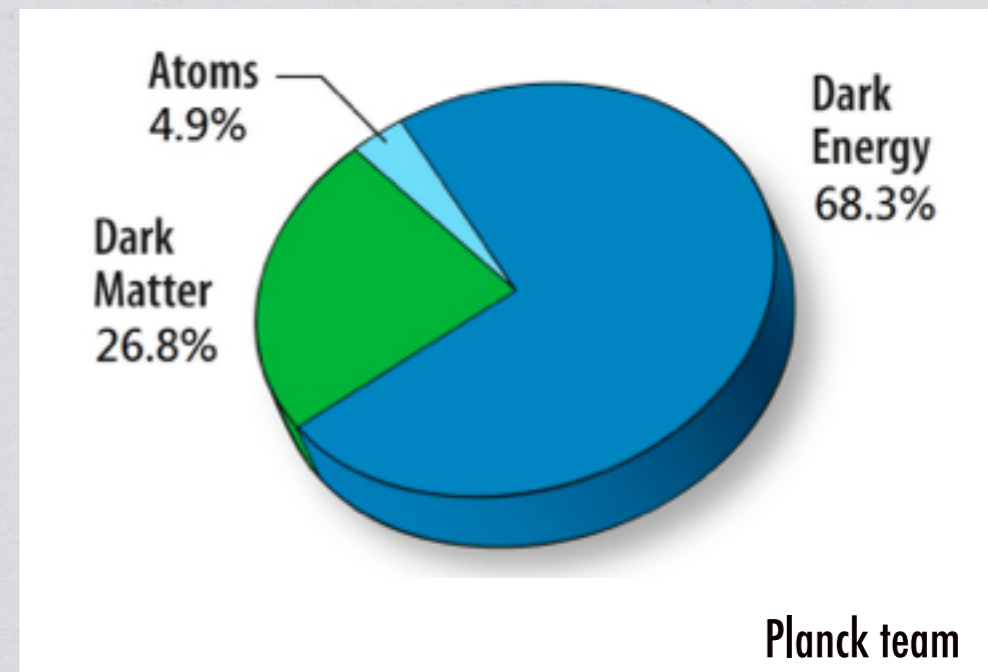
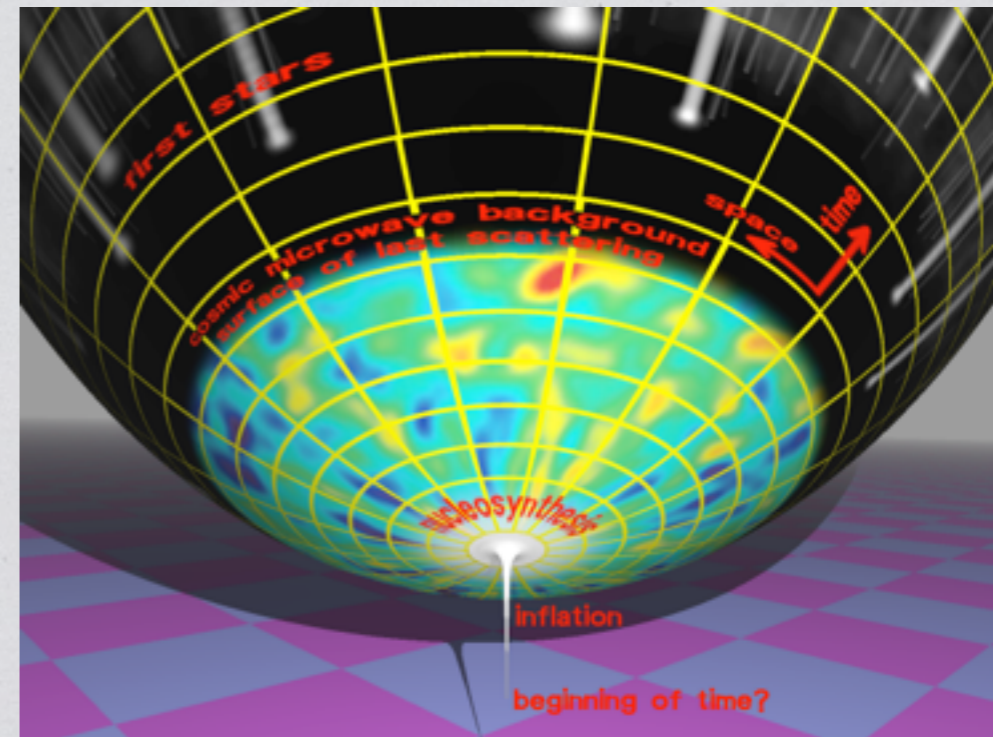


# Dark Matter



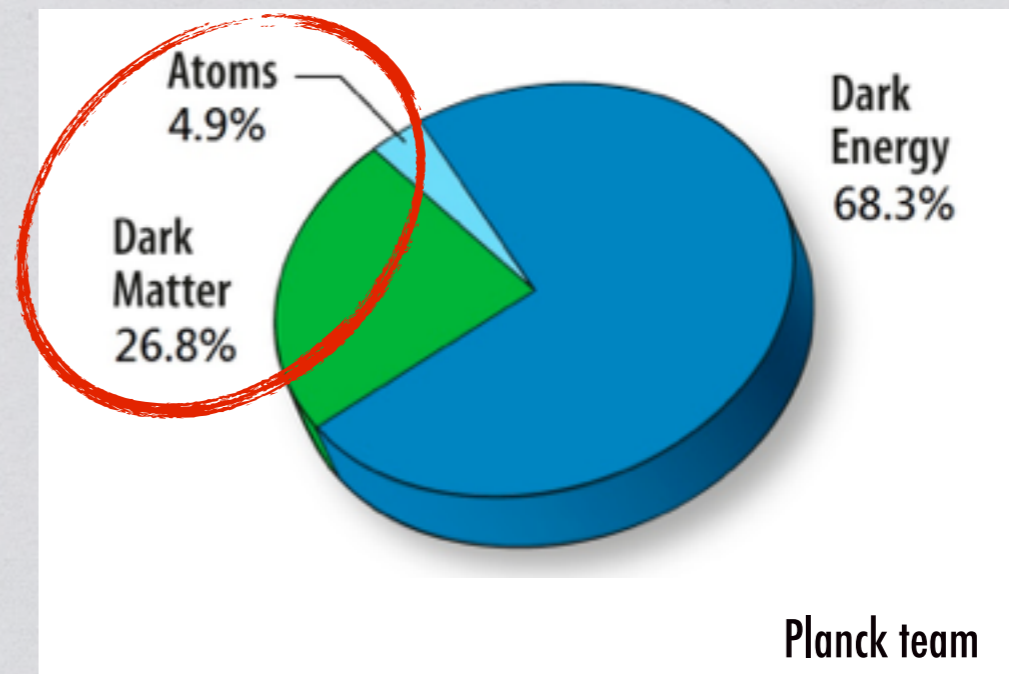
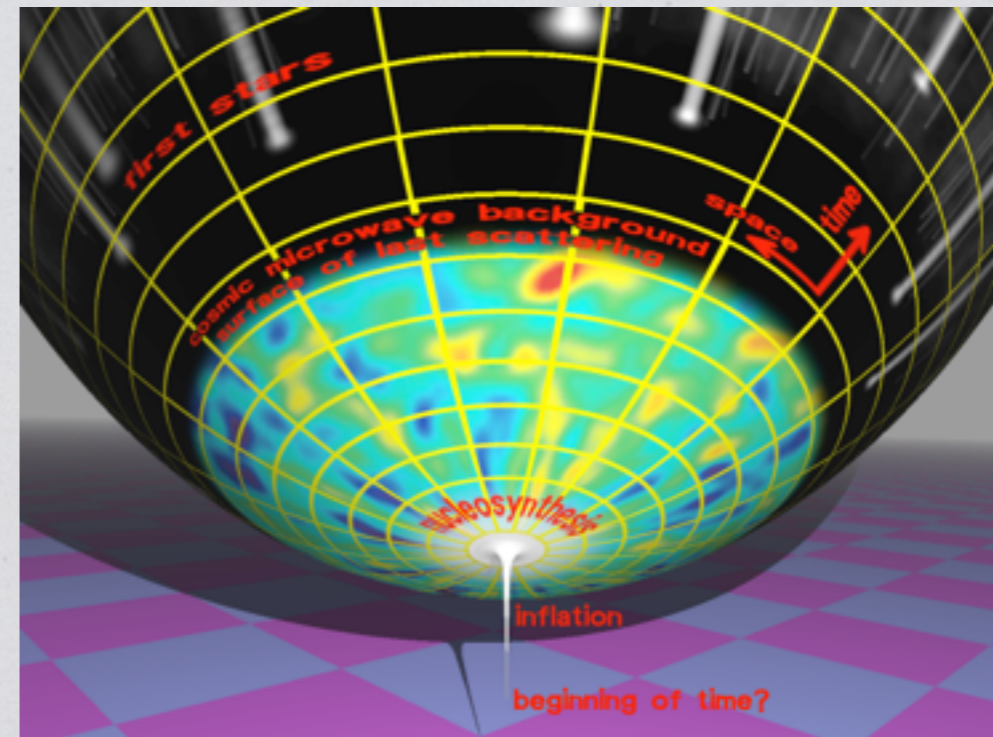
# A simple model for the Universe

- ★ Standard model:  
homogeneous,  
isotropic, expanding  
Universe
- ★ Astronomer's time unit:  
redshift  $z$  [ $z+1$ : inverse  
of expansion factor]
- ★ Simple composition:
  - ★ Dark Energy
  - ★ Dark Matter
  - ★ Baryons



# A simple model for the Universe

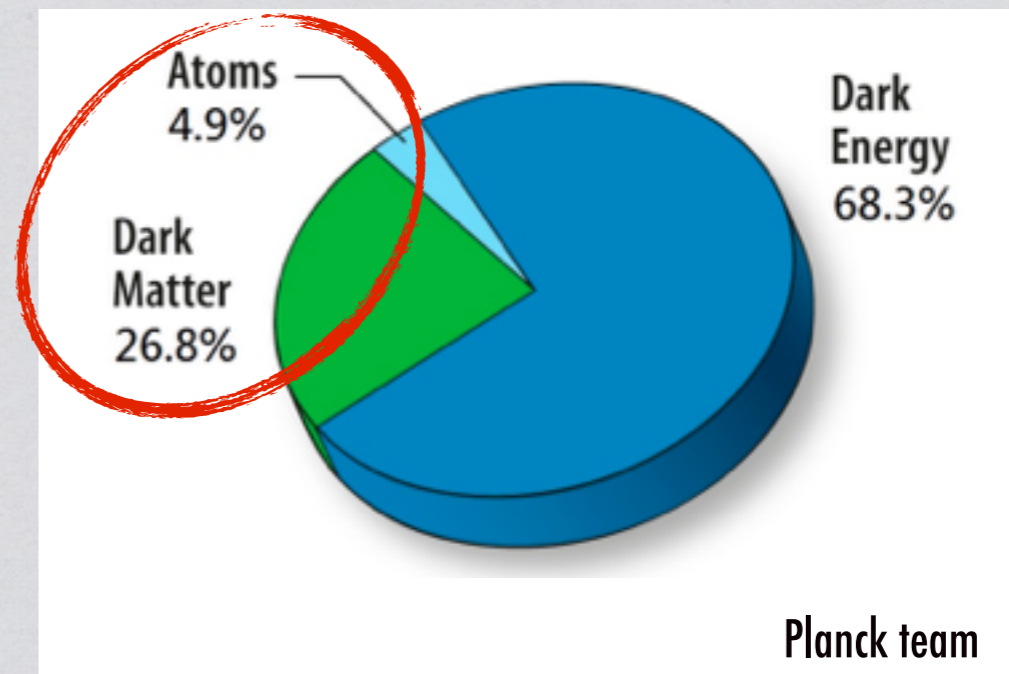
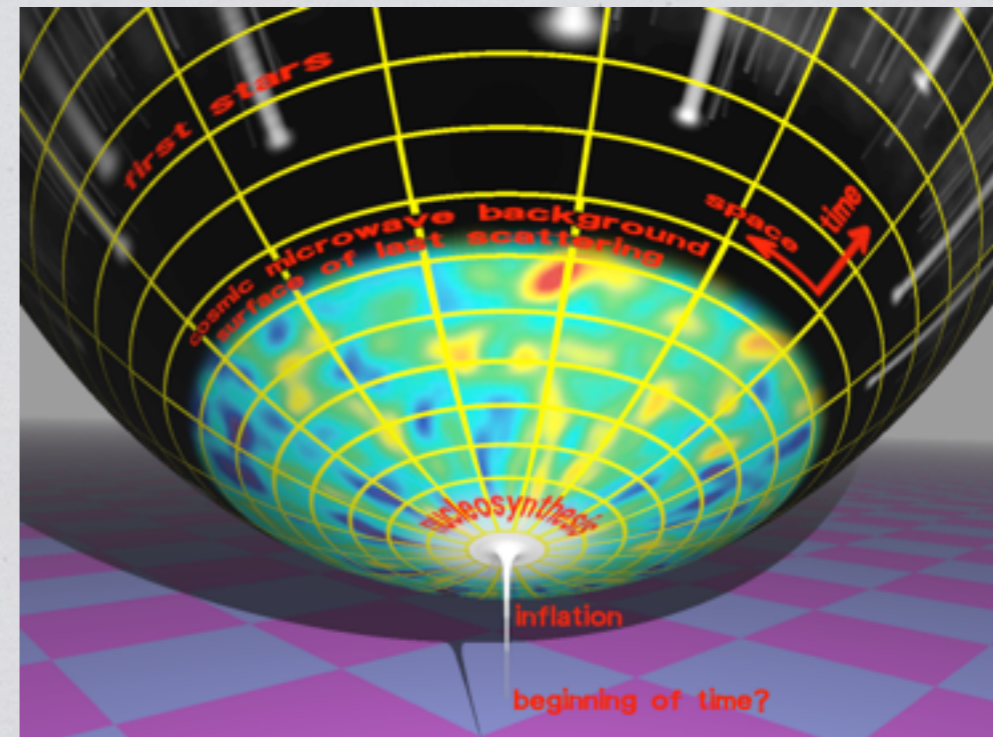
- ★ Standard model: homogeneous, isotropic, expanding Universe
- ★ Astronomer's time unit: redshift  $z$  [ $z+1$ : inverse of expansion factor]
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  - ★ Dark Matter
  - ★ Baryons



# A simple model for the Universe

- ★ Standard model: homogeneous, isotropic, expanding Universe
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- ★ Simple composition:
  - ★ ~~Dark Energy~~
  - ★ Dark Matter
  - ★ Baryons

Baryons are next class



# Outline

- ★ What dark matter *might* be
- ★ Dark matter halos
  - ★ Formation
  - ★ Mass function
  - ★ Growth and evolution
  - ★ Substructure
- ★ How warm is dark matter?

# What is dark matter?

- ★ We don't really know...
- ★ Must interact gravitationally only
- ★ We see small scale structure:
  - ★ DM non-relativistic at decoupling
- ★ Generic leading idea is that of a weakly interacting massive particle (~100GeV range)
- ★ Neutralino?



# Primordial fluctuations: The leading idea

- ★ Universe is initially very homogenous
- ★ But... quantum fluctuations at very early times amplified by inflation
- ★ *Scale invariant* and Gaussian density field  
[*simplest inflation model... active field of research*]

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \langle \rho \rangle}{\langle \rho \rangle}$$



$$\delta(\vec{k}) = \frac{1}{V} \int \delta(\vec{x}) e^{i\vec{k}\cdot\vec{x}} d^3x$$

$$\langle |\delta(k)|^2 \rangle \equiv P(k),$$

$$P(k) \propto k.$$

# Primordial fluctuations: Growth

Evolved Power Spectrum =  
Primordial Power Spectrum

- \* Transfer function
- \* Growth Function

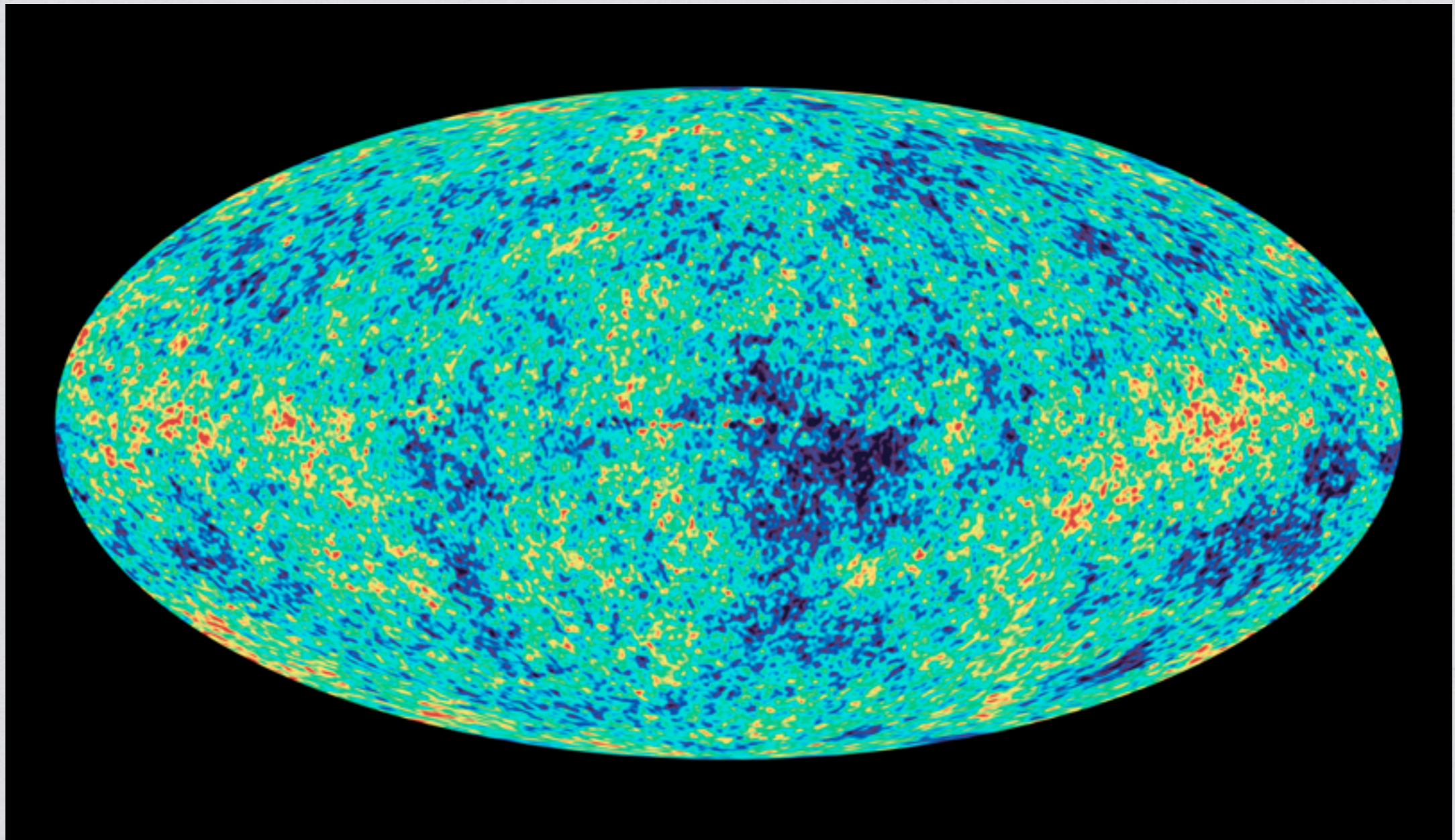
# Primordial fluctuations: Growth

$$\ddot{\delta} + [\text{Pressure} - \text{Gravity}] + \delta = 0$$

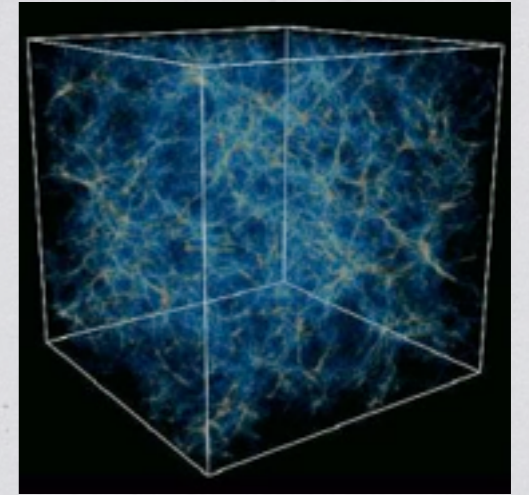
- ★ Transfer function  $T(k)$ : “early time” processing
- ★ Matter dominated Universe, all scales grow equally.  
Transfer function trivial
- ★ Radiation dominated Universe: Slow-growth only  
[expansion faster than collapse]
- ★ Small scales (smaller than horizon) erased by “free streaming” while DM relativistic
- ★ DM needs to be cold (or warm) otherwise  $T(k)$  is suppressed

# Primordial fluctuations: Growth

★ CMB allows us to quantify fluctuations at decoupling



# Growing the cosmic web

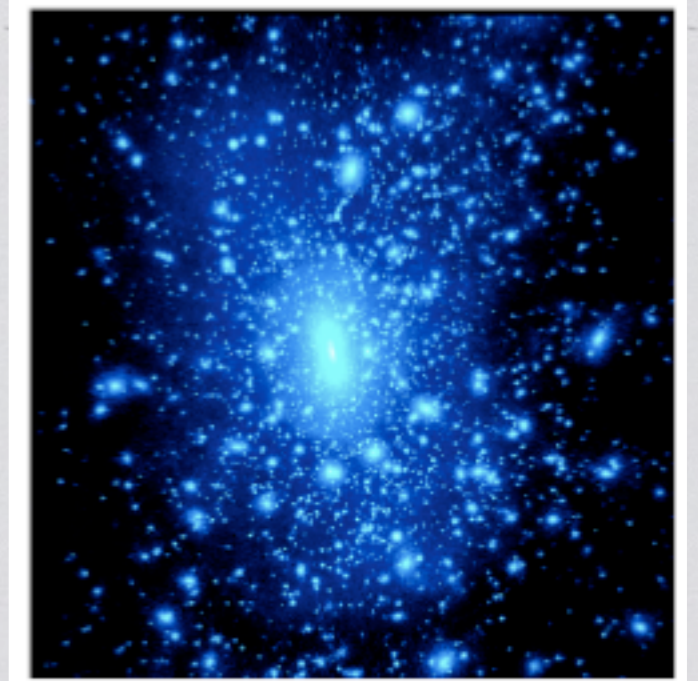


- ★ Choose a cosmological model [ $\Omega_m, \Omega_\Lambda, \Omega_b, h, P(k)$ ]
- ★ Choose computational setup [box size, resolution]
- ★ Create a discrete realization of the density field
- ★ Evolve it analytically to initial redshift
- ★ Run the N-body simulation
- ★ Identify halos, post-process/characterize them:
  - ★ Publish papers!

# Building a (MilkyWay) halo

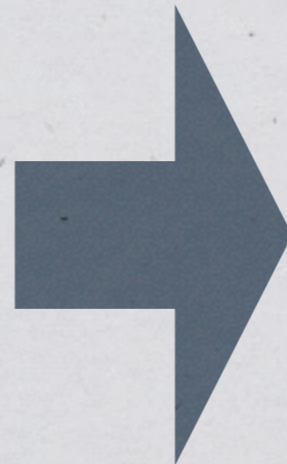
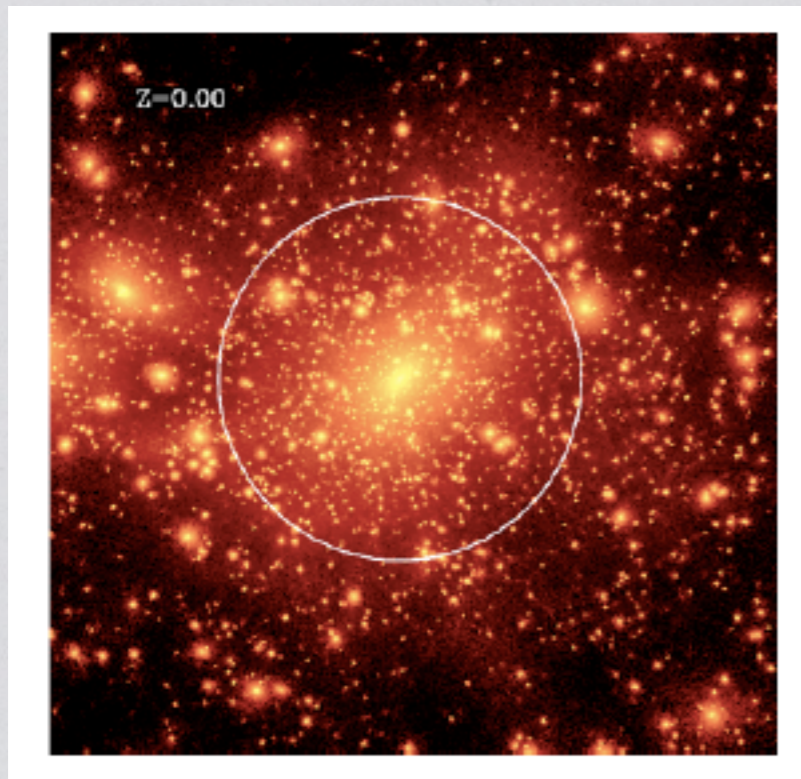


# What is a dark matter halo?

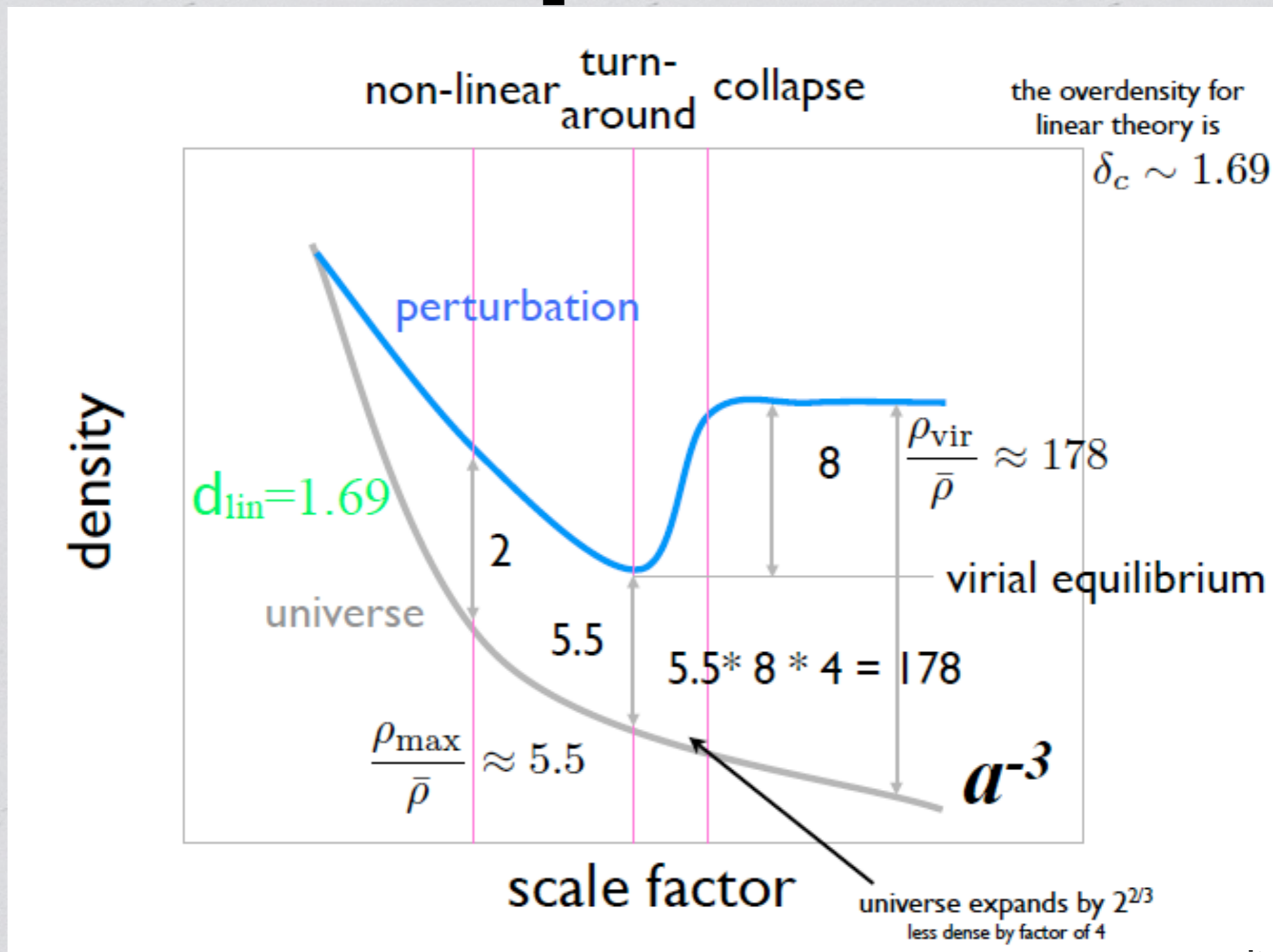


- ★ Halos are self-gravitating systems
- ★ Halos are non-linear peaks in the dark-matter density field whose self-gravity won over Hubble expansion
- ★ Operationally: A halo is a non-linear peak in the density field with boundaries defined by a given density contrast
- ★ [this can be used for defining halos numerically in simulations]

# A simple approximation for halos



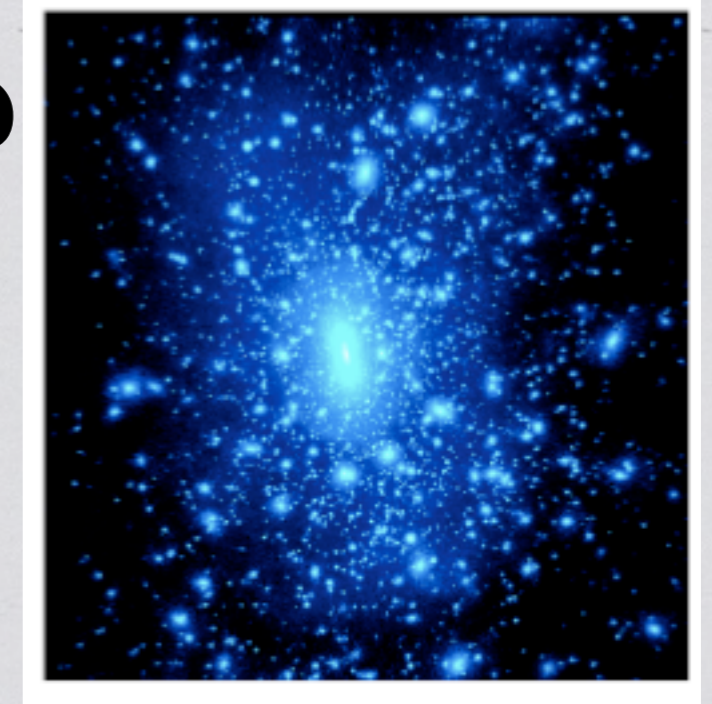
# Dark matter halo formation in spherical collapse



credit: R. Wechsler

# Where does a halo end?

★ The simple spherical collapse model can be used to define a “virial radius” [typical halo size]



★ In reality: lot of discussion!

[see Shull 2014, “Where do galaxies end?”]

★ Quotes from a meeting:

“The virial radius is actually at three virial radii”

“Let’s define the virial radius as the virial radius”

★ Bound, virialized material can exist outside the virial radius

★ There is continuous accretion

# The density profile of halos

- ★ Simple, quasi-universal profile for dark-matter halos clear from early simulations

$$\rho(r) \propto \frac{1}{r(1 + r/r_s)^2}.$$

What does it mean?

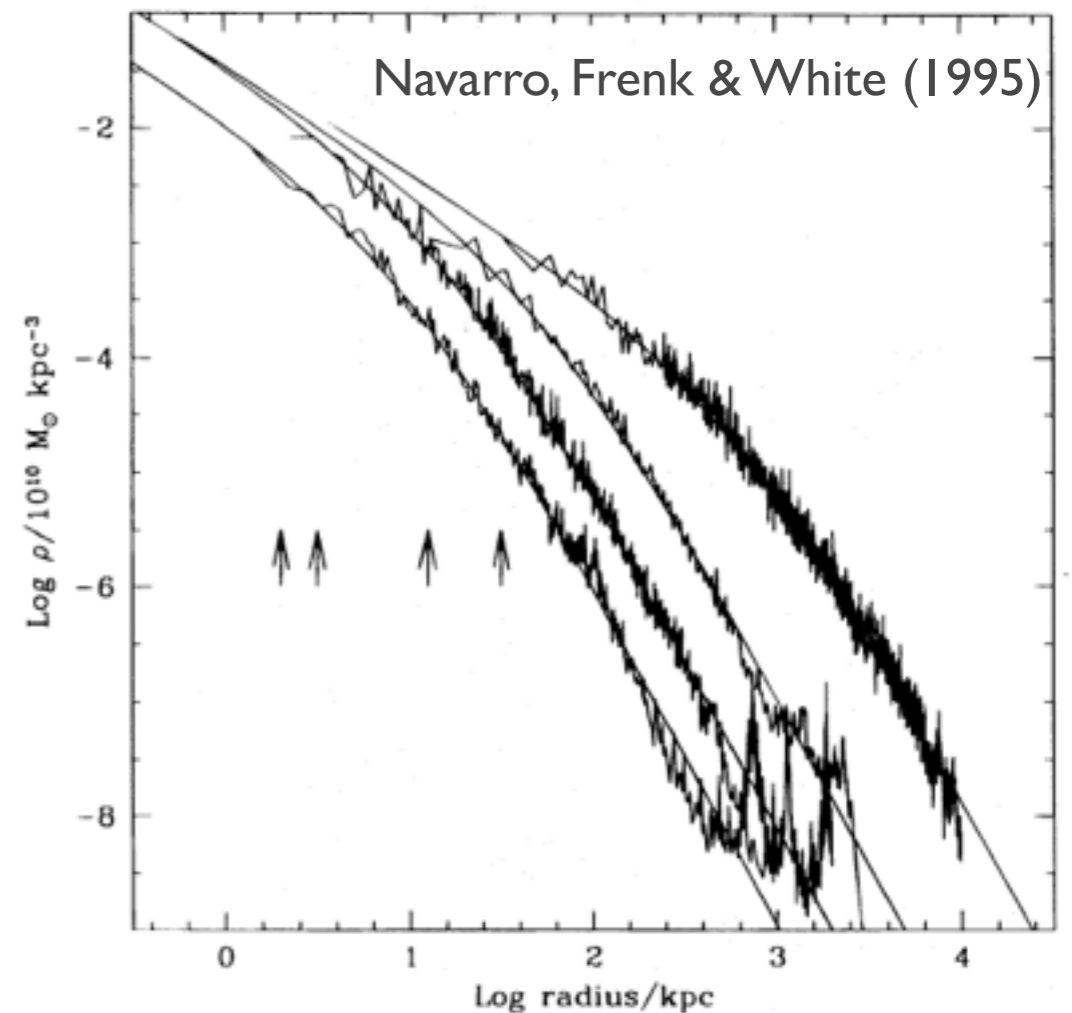


FIG. 3.—Density profiles of four halos spanning 4 orders of magnitude in mass. The arrows indicate the gravitational softening,  $h_g$ , of each simulation. Also shown are fits from eq. (3). The fits are good over two decades in radius, approximately from  $h_g$  out to the virial radius of each system.

# The density profile of halos

- ★ Simple, quasi-universal profile for dark-matter halos clear from early simulations

$$\rho(r) \propto \frac{1}{r(1 + r/r_s)^2}.$$

- Cusp at the center (with finite mass)
- Divergence at large radii (but there is tidal truncation)

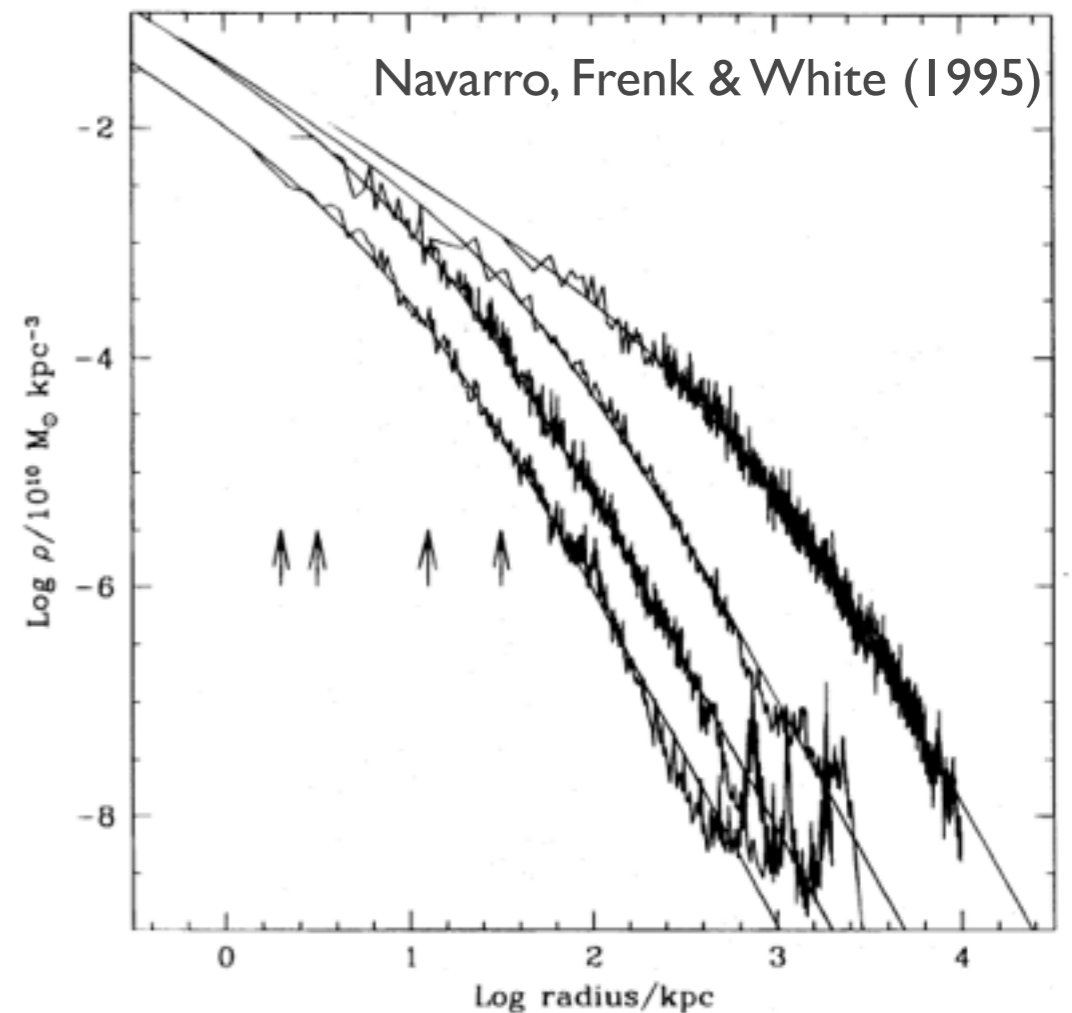
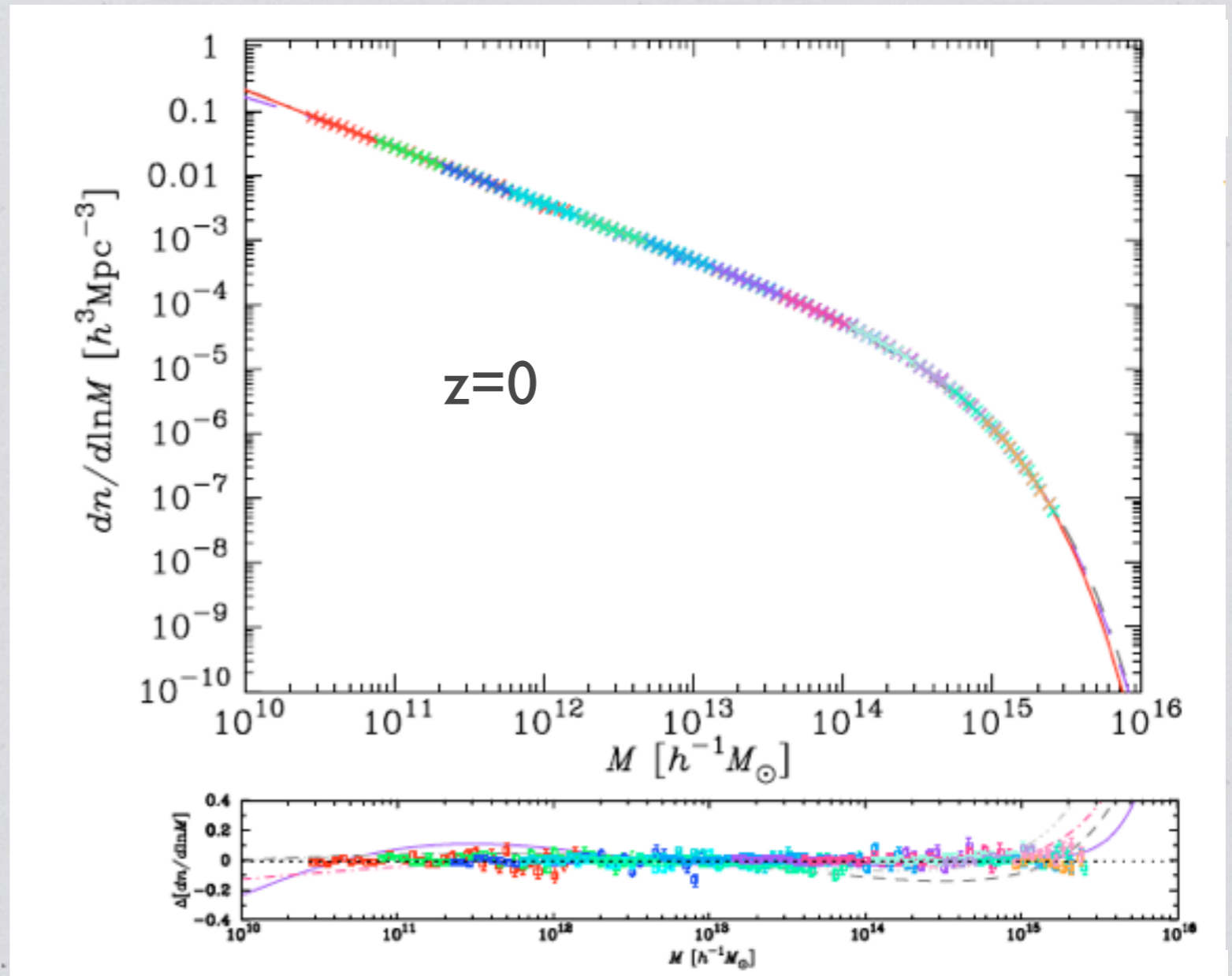


FIG. 3.—Density profiles of four halos spanning 4 orders of magnitude in mass. The arrows indicate the gravitational softening,  $h_g$ , of each simulation. Also shown are fits from eq. (3). The fits are good over two decades in radius, approximately from  $h_g$  out to the virial radius of each system.

# The halo mass function from simulations

- ★ Consensus to ~5% level on DM halo mass function at  $z=0$  [at fixed cosmology]

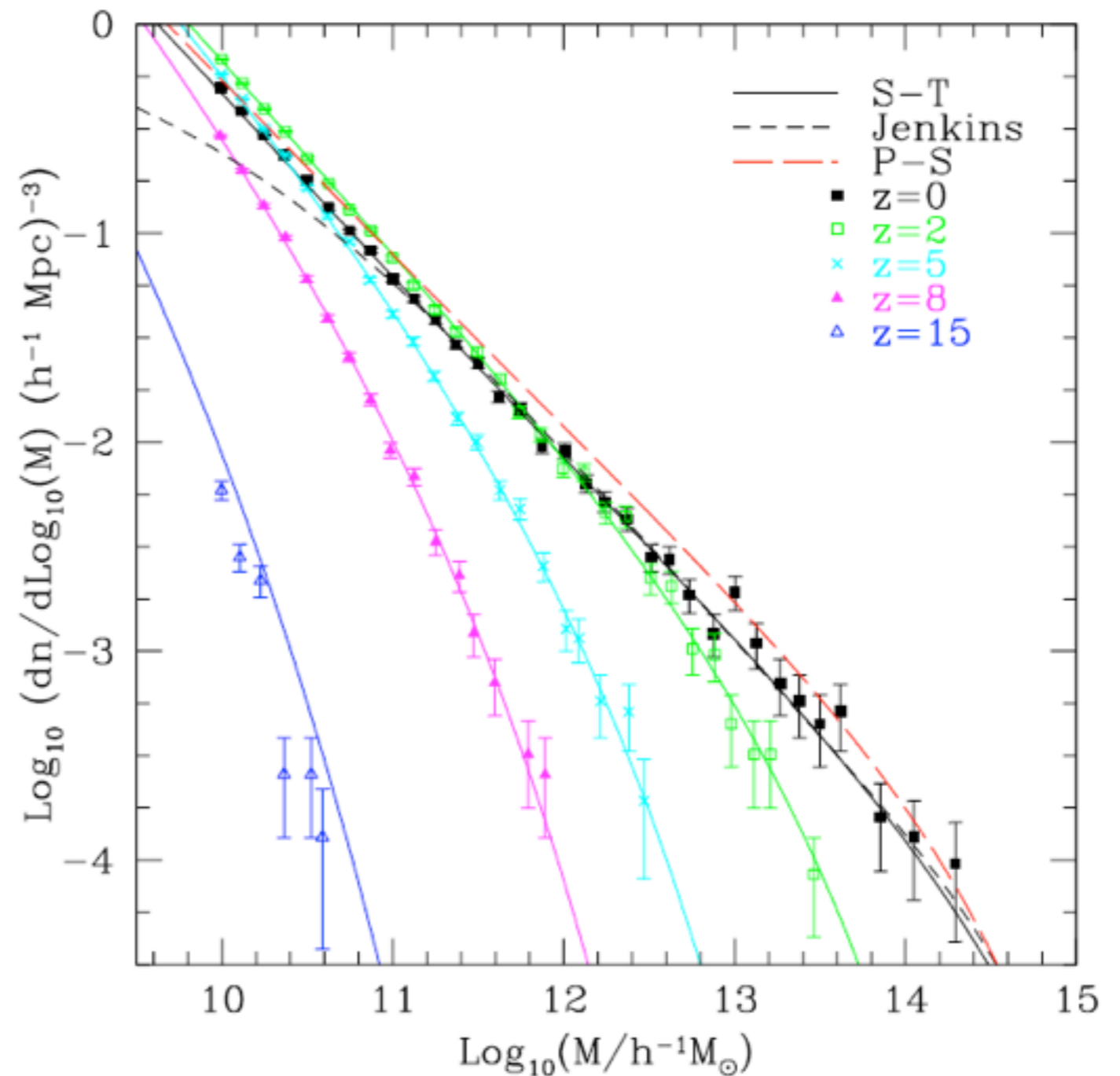


Warren et al. 2004

# The halo mass function from simulations

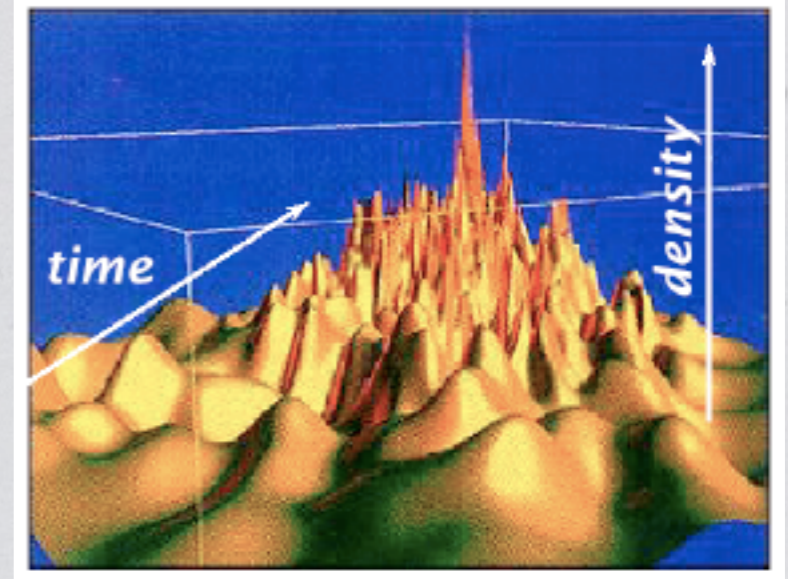
★ Strong evolution with redshift

★ Note flattening at  $z=0$  for low masses



Reed et al. 2003

# The halo mass function from Press-Schechter



- ★ Halos arise from density fluctuations (Gaussian Random field) which grow with redshift
- ★ Smooth density field on scale such as to enclose mass  $M_h$
- ★ Count peaks with density above a critical threshold (typically 1.69 in linear growth)
- ★ This is the basic idea for Press Schechter mass function

# The halo assembly time

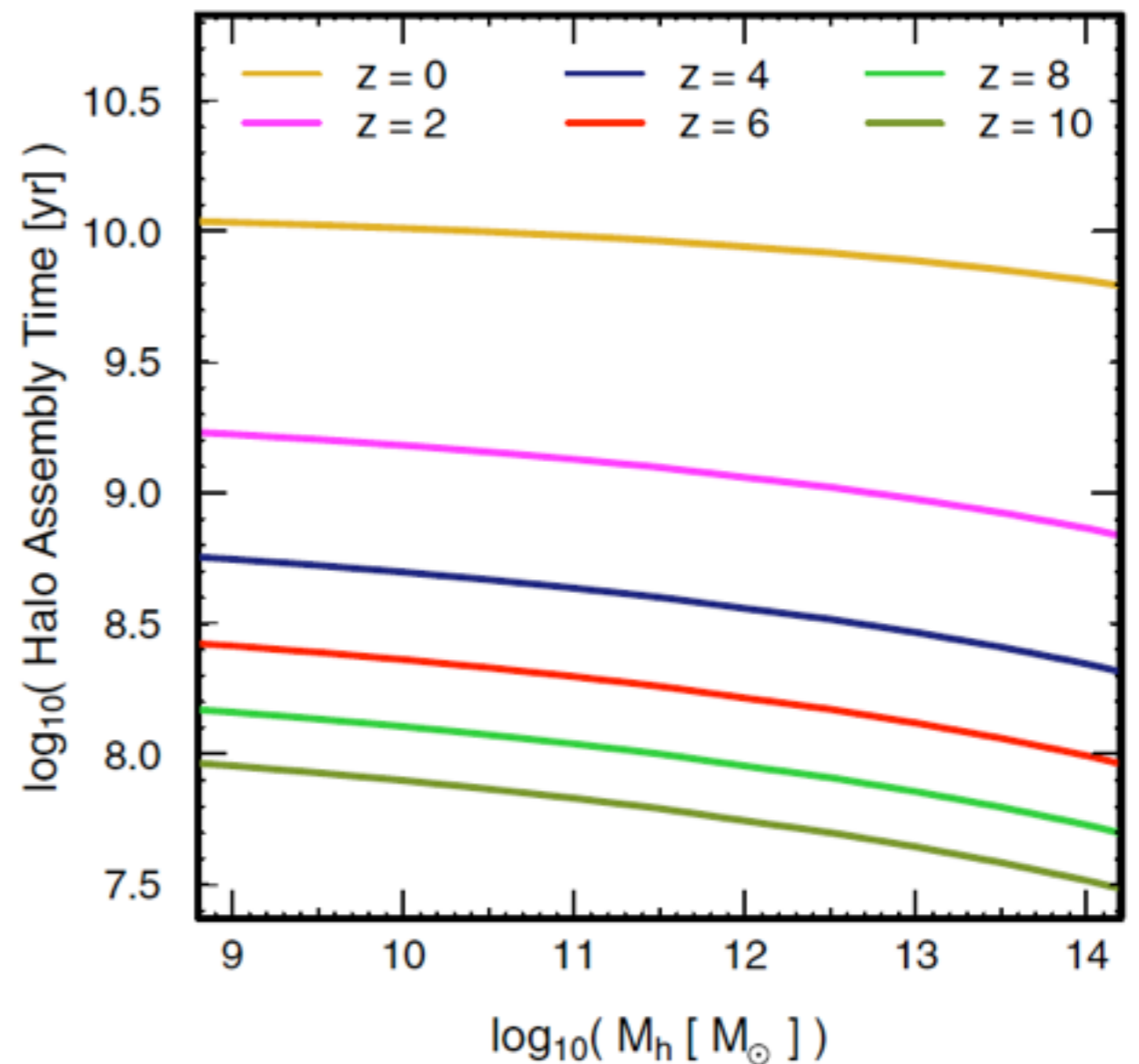
★ Time needed to grow from  $M_h/2$  to  $M_h$

★ On average:

★ High- $z$  halos grow fast

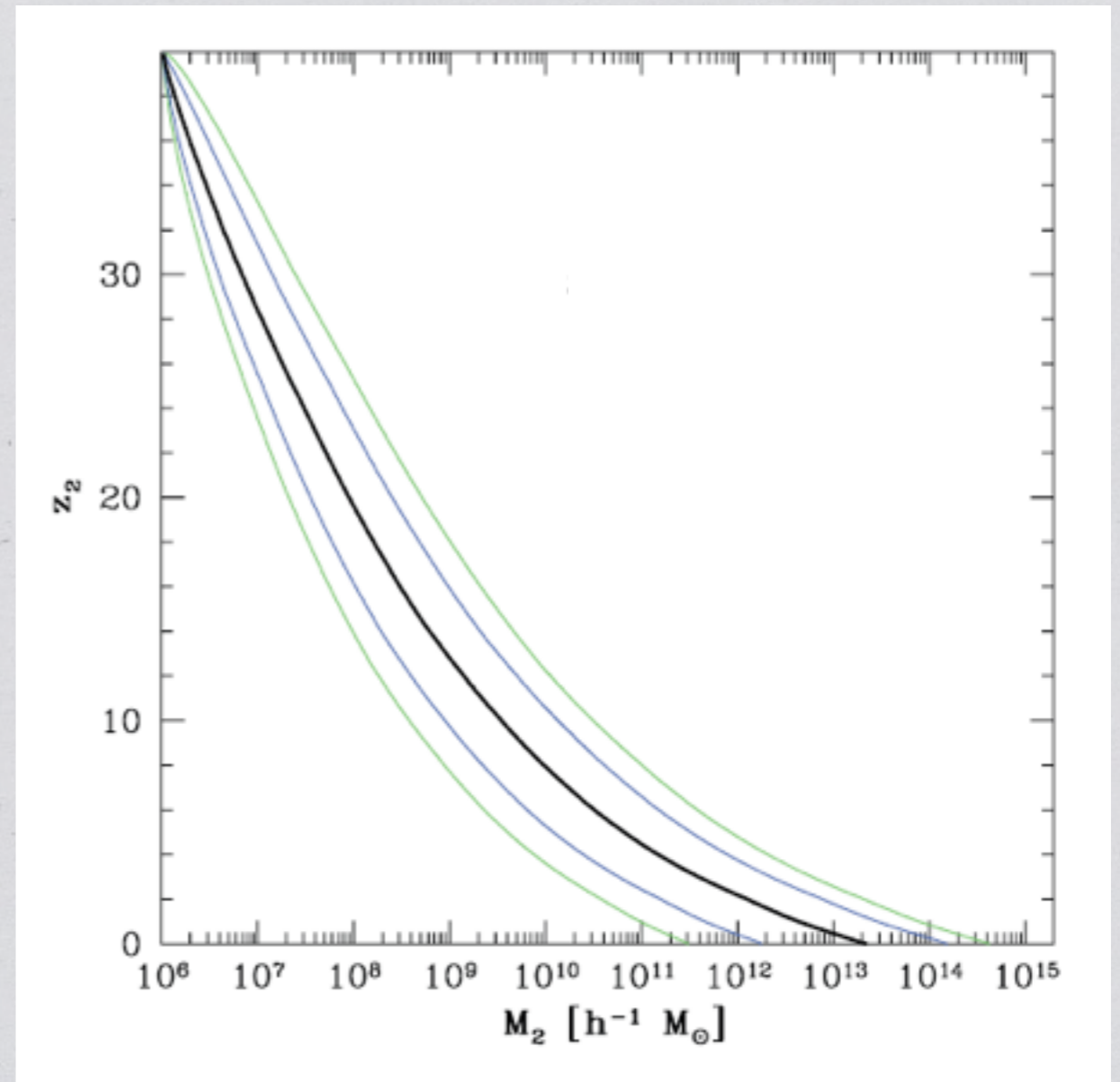
★ Weak dependence on halo mass

**But don't forget it is a stochastic process!**



# Halo growth with time

- ★ Halo growth is a stochastic process
- ★ Can be modeled with "Extended Press-Schechter" formalism
- ★ Growth by  $\sim 10^7$  for rare halo from  $z \sim 40$  to  $z \sim 0$
- ★ Growth by  $\sim 100x$  from  $z=6$  to  $z=0$  for Milky Way like halo

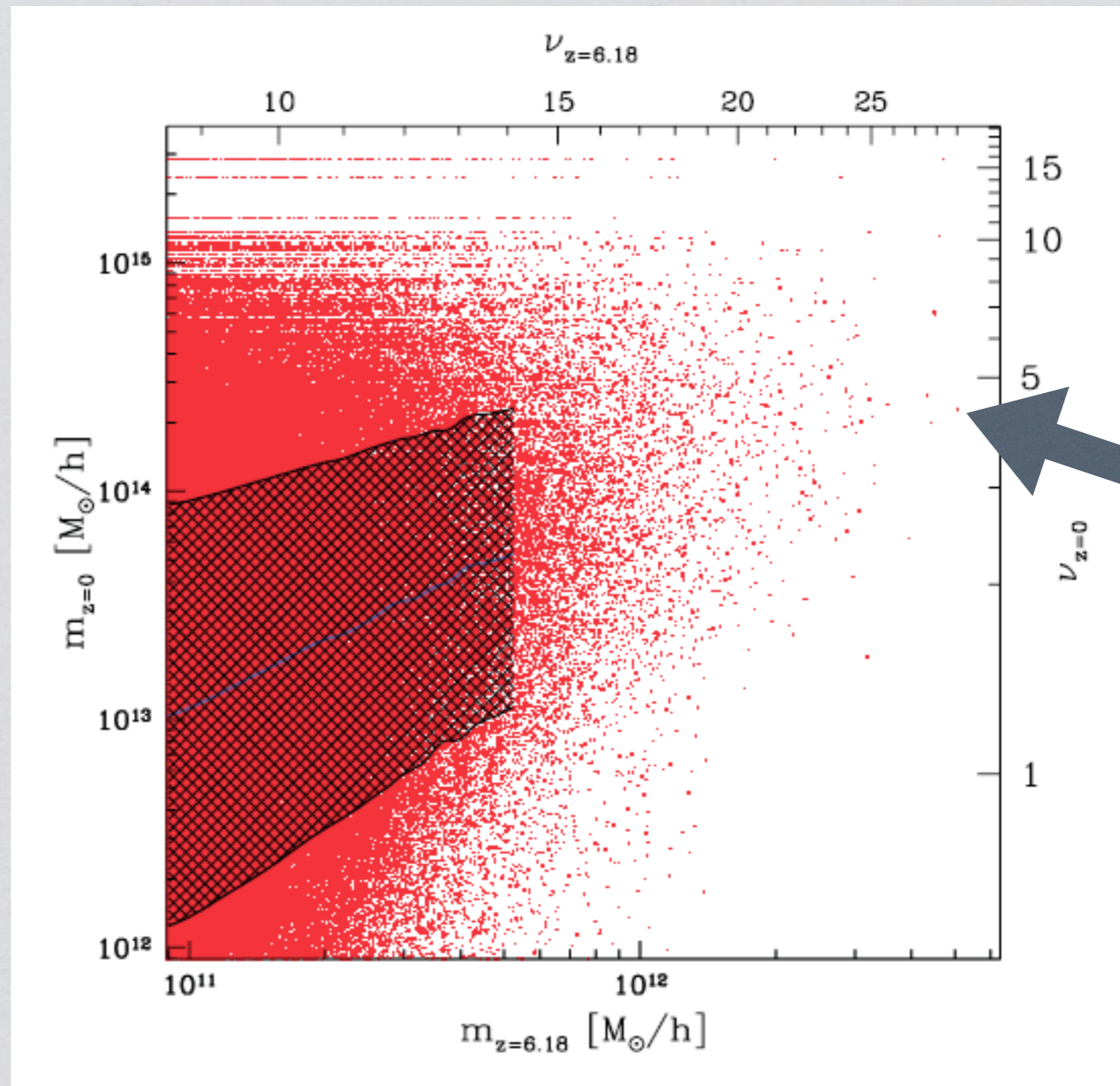


# Merger and growth of halos

- ★ Are the most massive dark matter halos at high redshift evolving into the most massive halos at lower redshift?
- ★ Given the most massive dark matter halos, are their progenitors the most massive at earlier times?
- ★ We can use simulations, or theory, to address the question, but what do you expect?

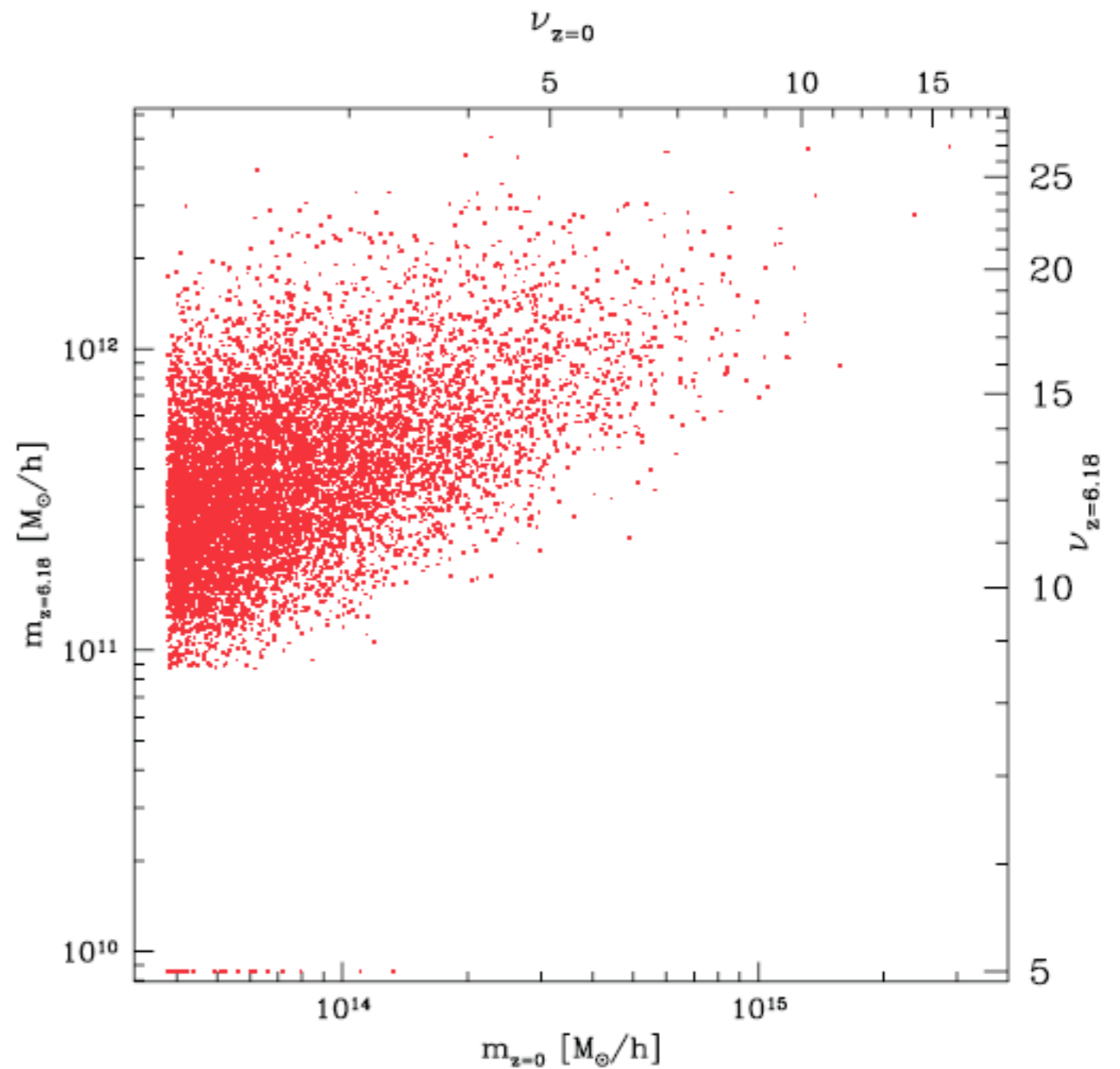
# Merger and growth of halos

- ★ In the Millennium Run, the most massive  $z \sim 6$  halo evolves into an unimpressive  $z=0$  cluster



# Distribution of progenitor halos

- ★ Most massive halo at  $z_1 < z_2$  does not correspond to most massive progenitor at  $z_2$



Trenti, Santos & Stiavelli (2008)

# A random walk interpretation

★ Rarest walks are regressing toward the mean as time passes

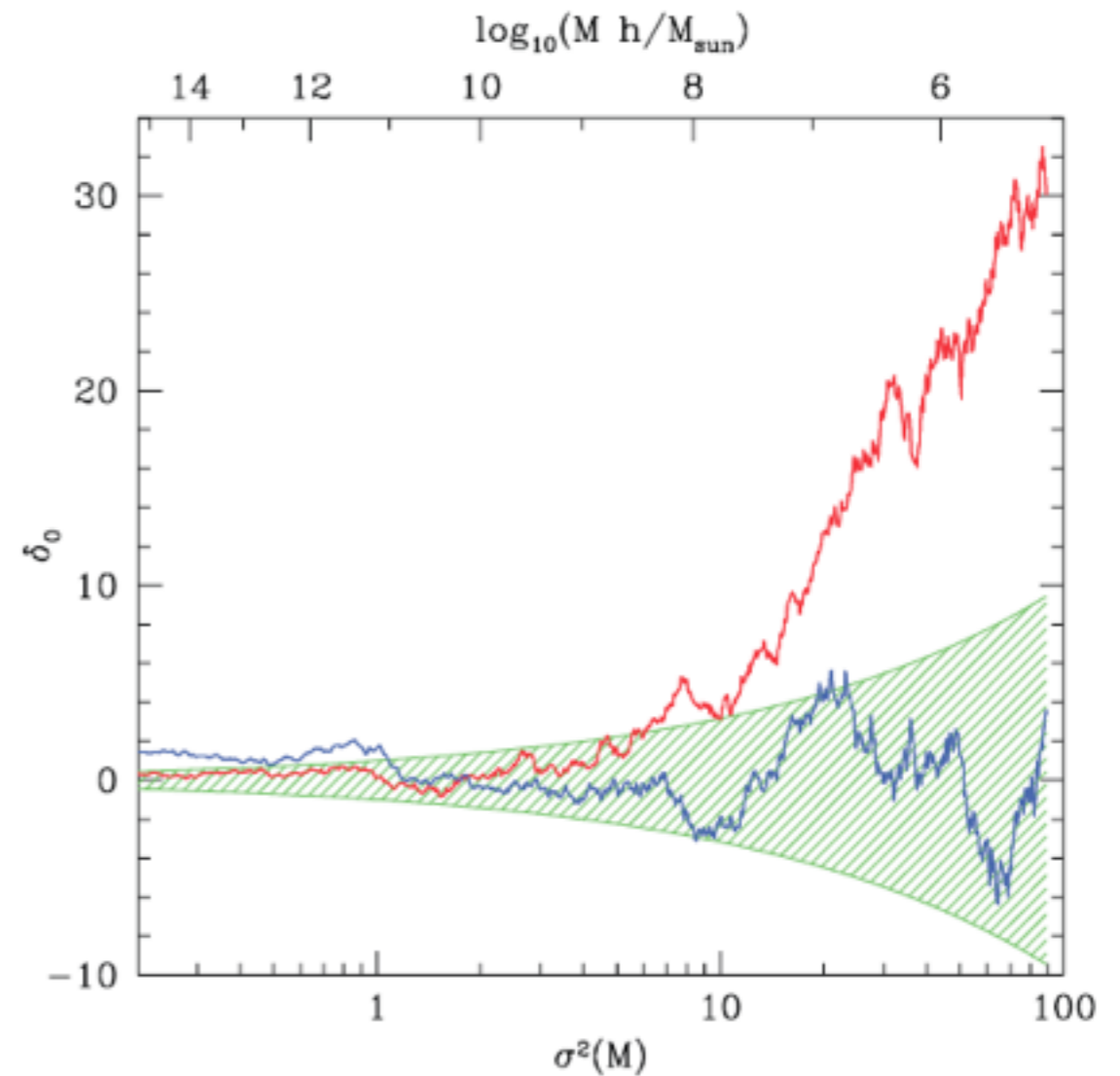


FIG. 3.—Random walks generated using a Millennium run cosmology. The green shaded area represents for any value of  $\sigma^2(M)$ —evaluated at  $z = 0$ —the area enclosing 68% of the walks and thus has amplitude  $\pm\sigma(M)$ . The red and blue walks have been selected from  $10^3$  random realizations as (1) the walk with the largest overdensity at  $M = 3 \times 10^5 h^{-1} M_{\odot}$  (red) and (2) the walk with the largest overdensity at  $M = 2 \times 10^{15} h^{-1} M_{\odot}$  (blue).

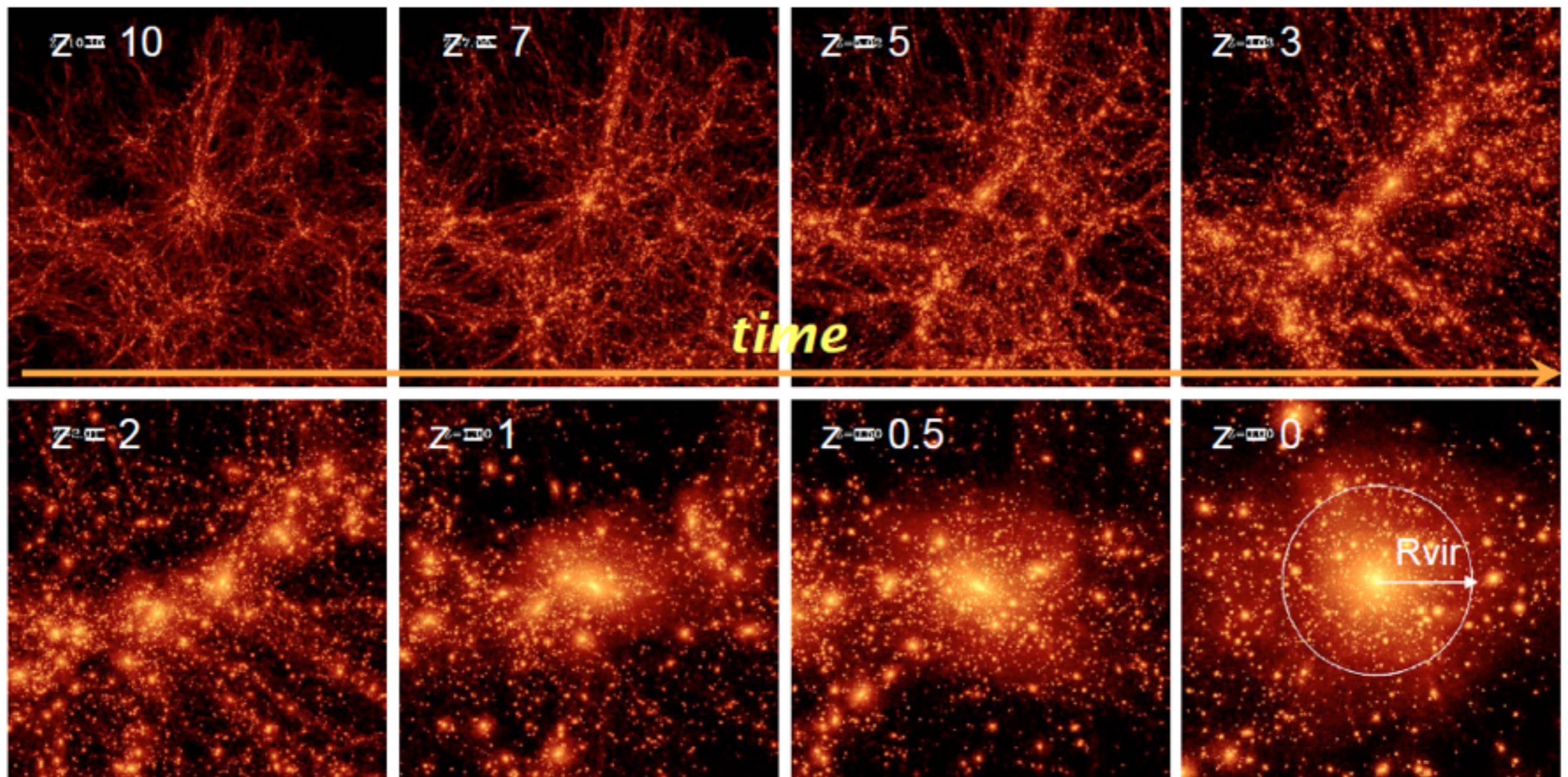
Trenti, Santos & Stiavelli (2008)

# Substructure

- ★ Halos have internal sub-structure
- ★ Let's go beyond the spherical cow approximation!



# Substructure: An overview

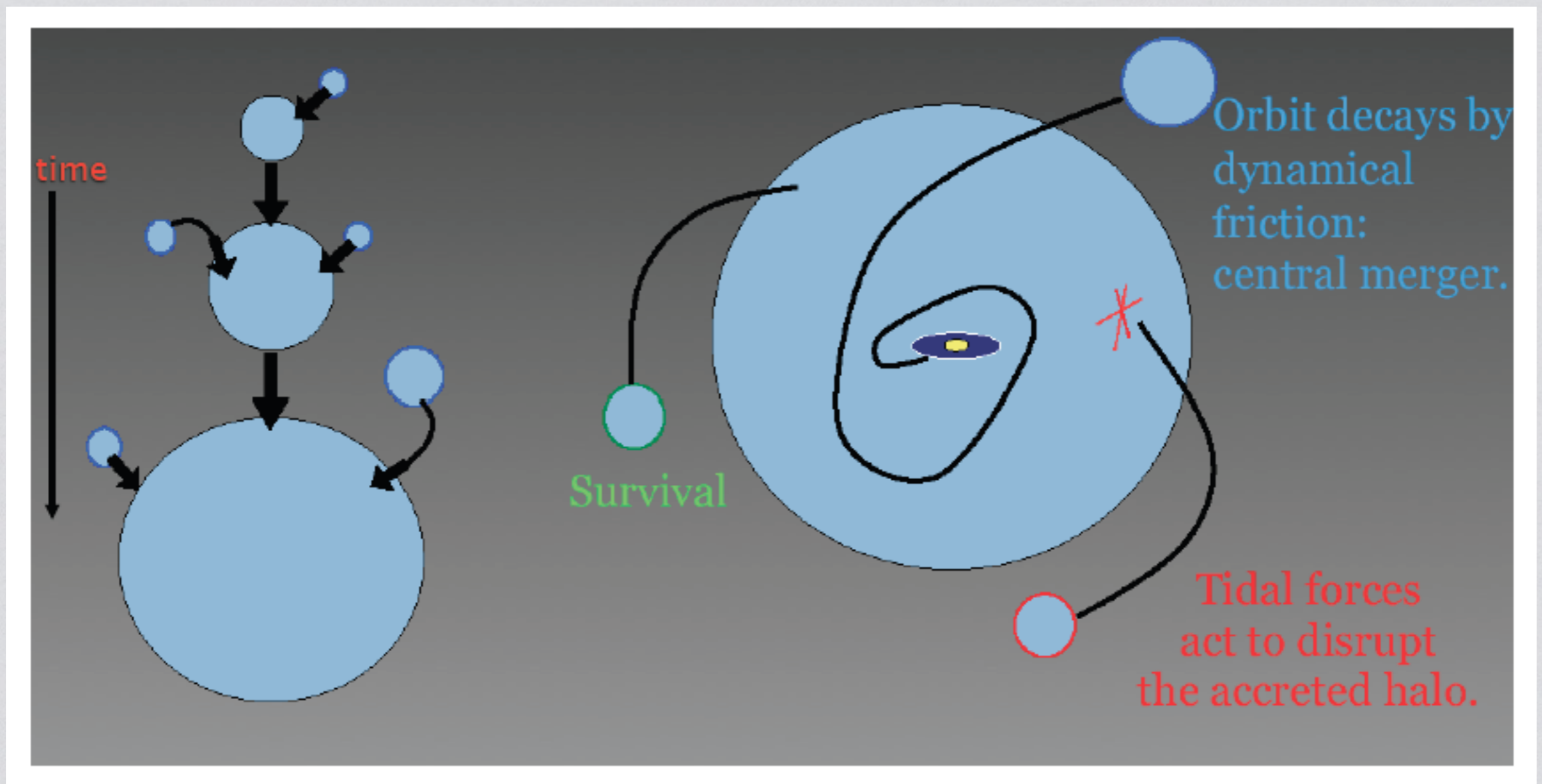


Formation of a galaxy-sized halo in LCDM,  $M_{\text{vir}}=3 \times 10^{12} h^{-1} M_{\text{sun}}$ ;  $R_{\text{vir}}=293 h^{-1} \text{ kpc}$ ;

credit: R. Wechsler

# Substructure: the drivers

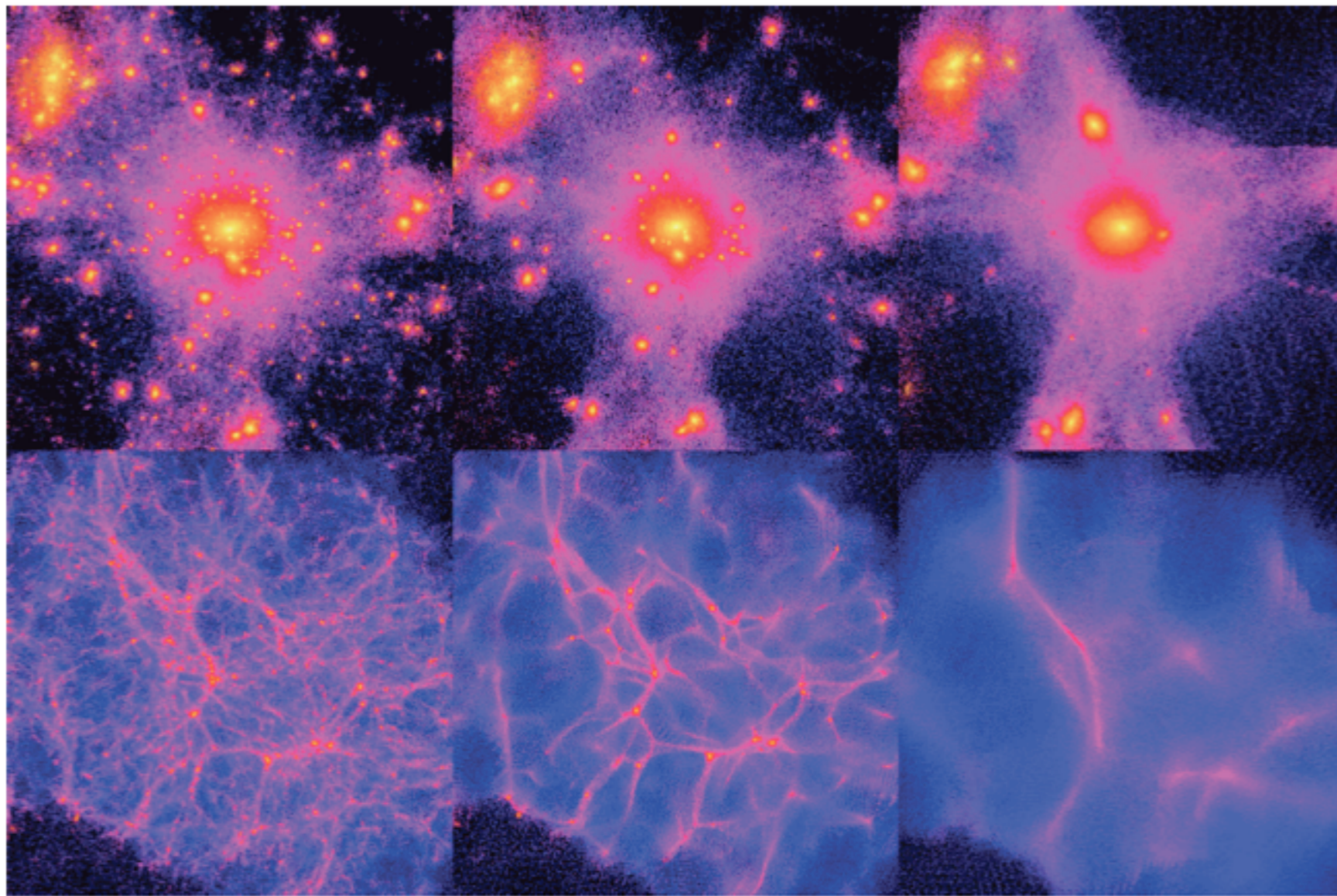
- ★ Competition between accretion and disruption of subhalos



credit: R. Wechsler

# The warmness of dark matter

★ Substructure dramatically affected



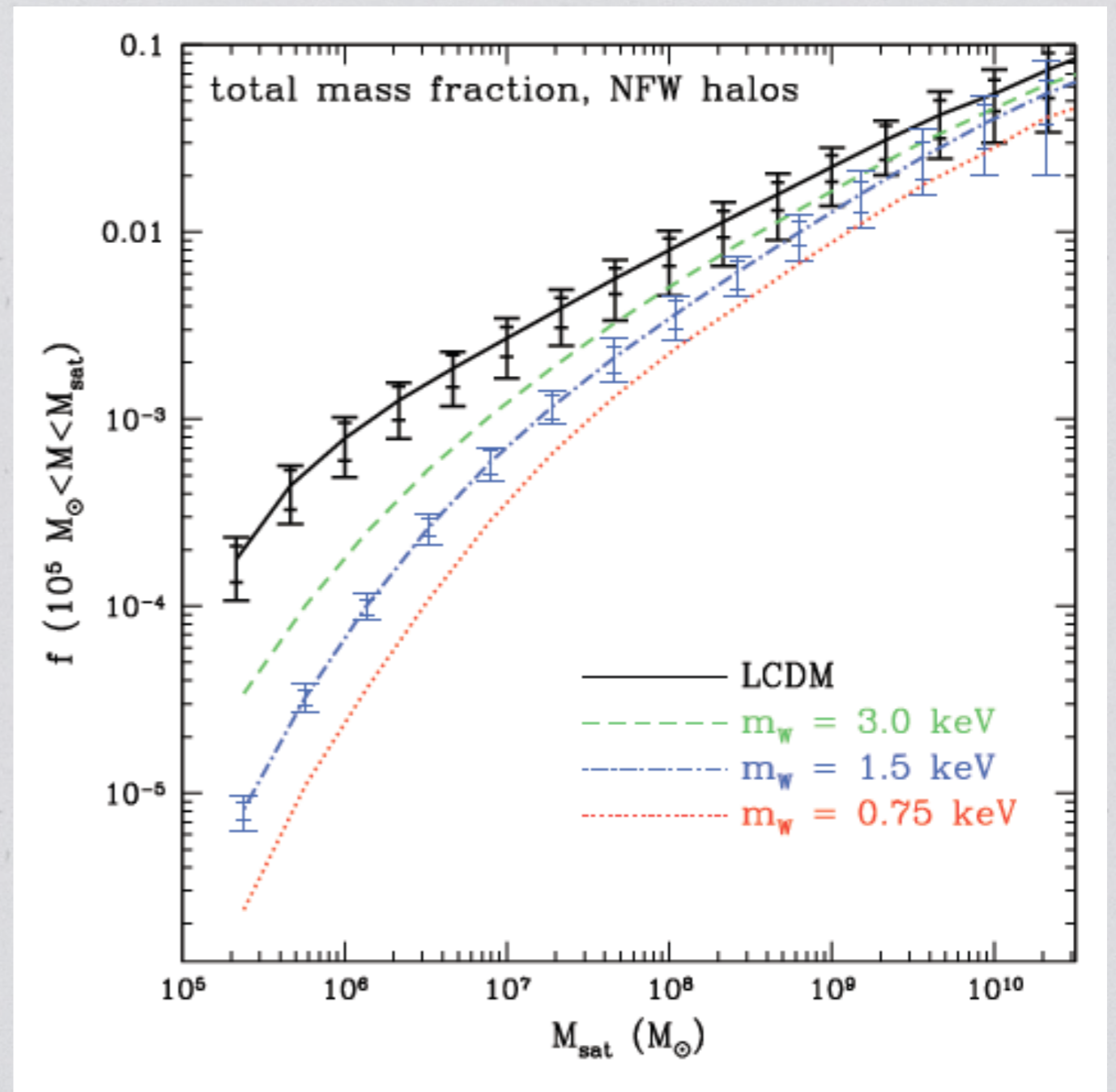
cold

warm

hot

# The warmness of dark matter

- ★ Fraction of mass in substructure depends on DM properties
- ★ We can use halo structure to learn about fundamental physics



# The warmness of dark matter

- ★ How can we observationally measure clumpiness (substructure) of DM halos?



# The warmness of dark matter

★ How can we measure clumpiness (substructure) of DM halos?

★ Gravitational Lensing

★ Milky Way: Satellites and Stellar streams

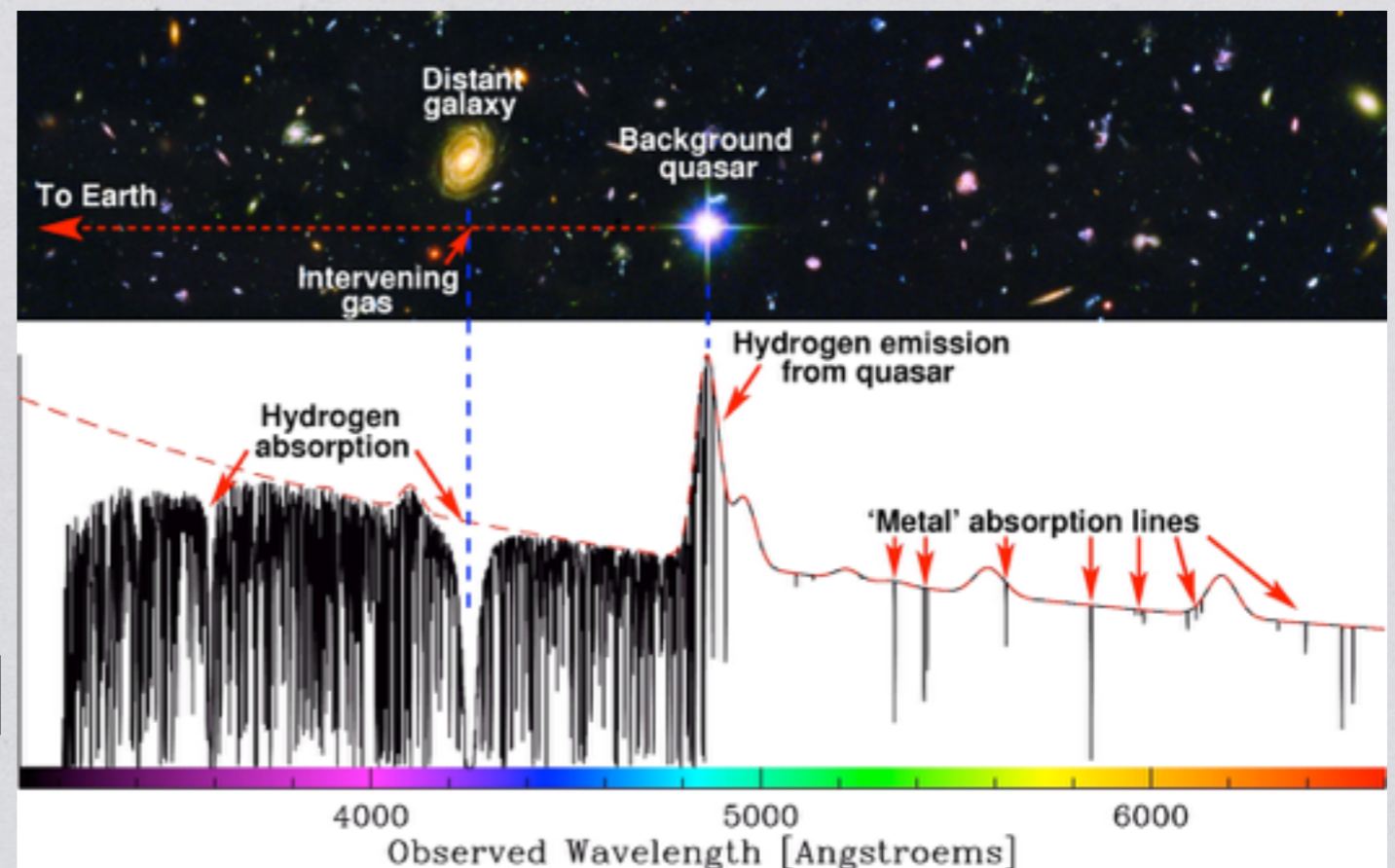
★ Lyman-alpha forest

# Looking at high-z: the Ly $\alpha$ forest

★ Subhalos at  $z \sim 0$  are messy and inference depends on baryonic physics

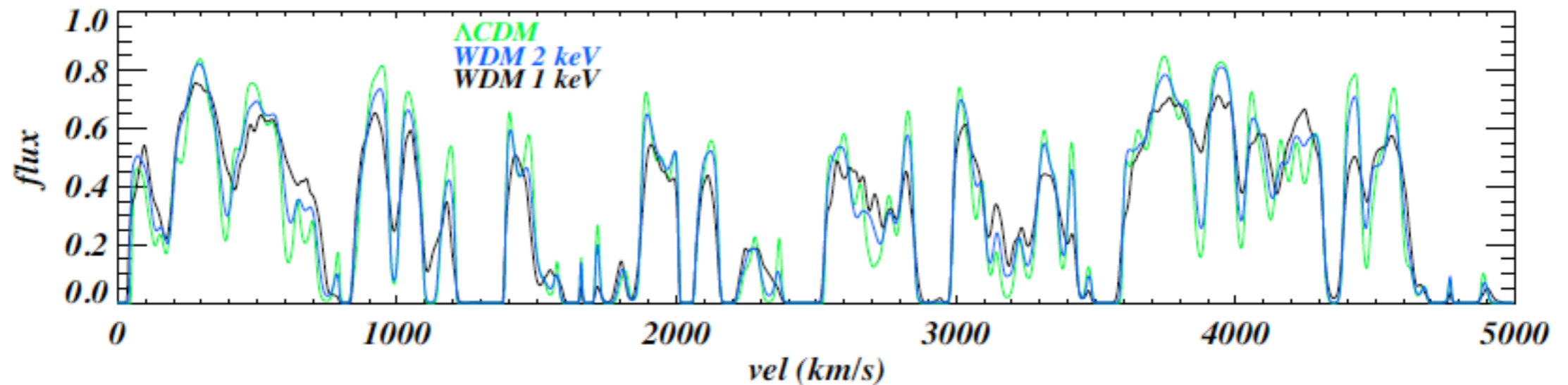
★ Let's go to high-z when small scales have not collapsed yet

★ Look at forest of absorption lines in distant QSOs  
[given by intervening clouds in small-mass halos]



# Looking at high-z: the Ly $\alpha$ forest

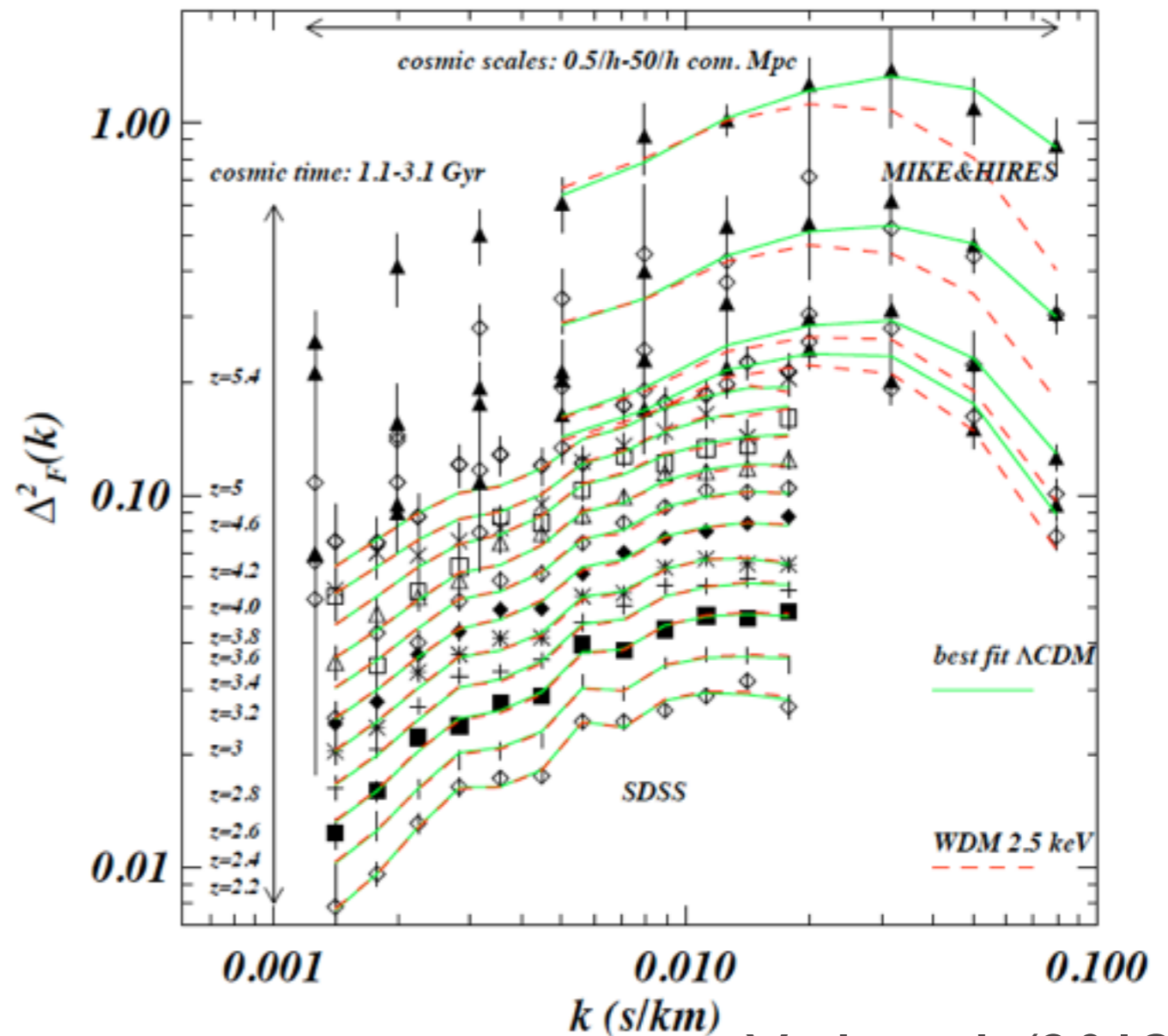
- ★ In simulated forest spectra, WDM leads to smoother profiles (structure is suppressed)



Viel et al. (2013)

# Looking at high-z: the Ly $\alpha$ forest

- ★ Data-model comparison has clear preference for  $\Lambda$ CDM
- ★ Most stringent observational result to date



Viel et al. (2013)

# Summary

- ★ Overview of how DM halos form and evolve
  - ★ Remember key properties for next class
- ★ Substructure in halos
  - ★ Tool to investigate coldness of dark matter:  
Ly-alpha forest rules out warm models
- ★ Tomorrow: Gas collapse and Star formation in halos

# Suggested readings

- ★ "Principles of Physical Cosmology" Peebles,  
Princeton University Press  
[generation and growth of fluctuations]
- ★ "Galaxy Formation and Evolution" Mo, van den  
Bosh & White, Cambridge University Press  
[DM halos]
- ★ "A random walk through models of non-linear  
clustering" Sheth, R.K. 2001
- ★ <http://adsabs.harvard.edu/abs/2001NYASA.927....1S>