STAR CLUSTERS

Lecture 3

Kinematic Properties

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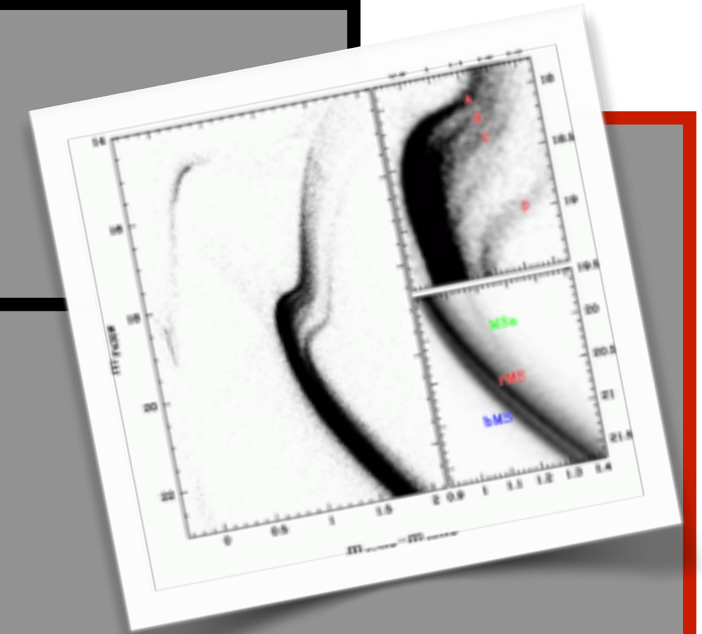
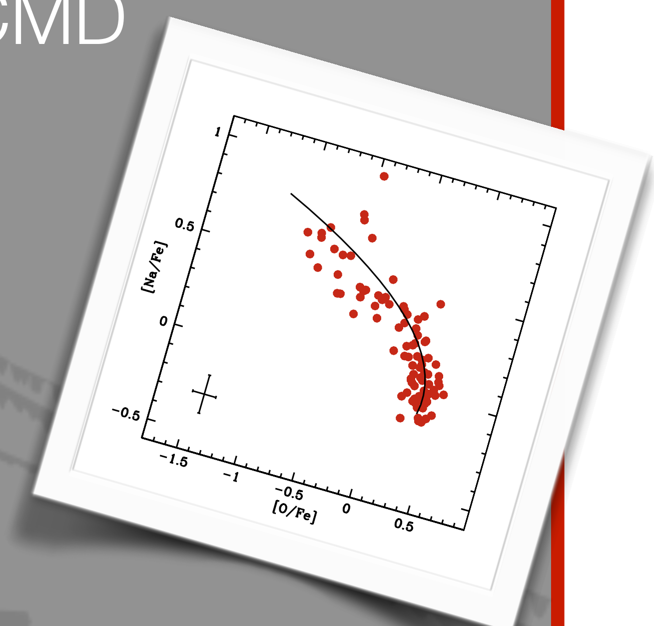
FIRST KATHMANDU ASTROPHYSICS SCHOOL
KAS16

LECTURE 2



1. **Star Formation**
    - from gas clouds, fragmentation
    - Initial mass function (IMF): multiple power laws, changes with time
2. **Multiple Stellar populations**
    - Photometric evidence: Multiple sequences in CMD
    - Spectroscopic evidence: Na-O anti-correlation
    - Explanations:
        1. Polluters + 2nd Generation
        2. Polluters
    - Problems: Mass budget problem (must have lost 90% of their mass??…), and many more…



Outline

1. The Gravitational N-body problem
2. Dynamic Equilibrium
3. Negative Heat Capacity
4. Core Collapse
5. Equipartition of energies
6. Mass Segregation

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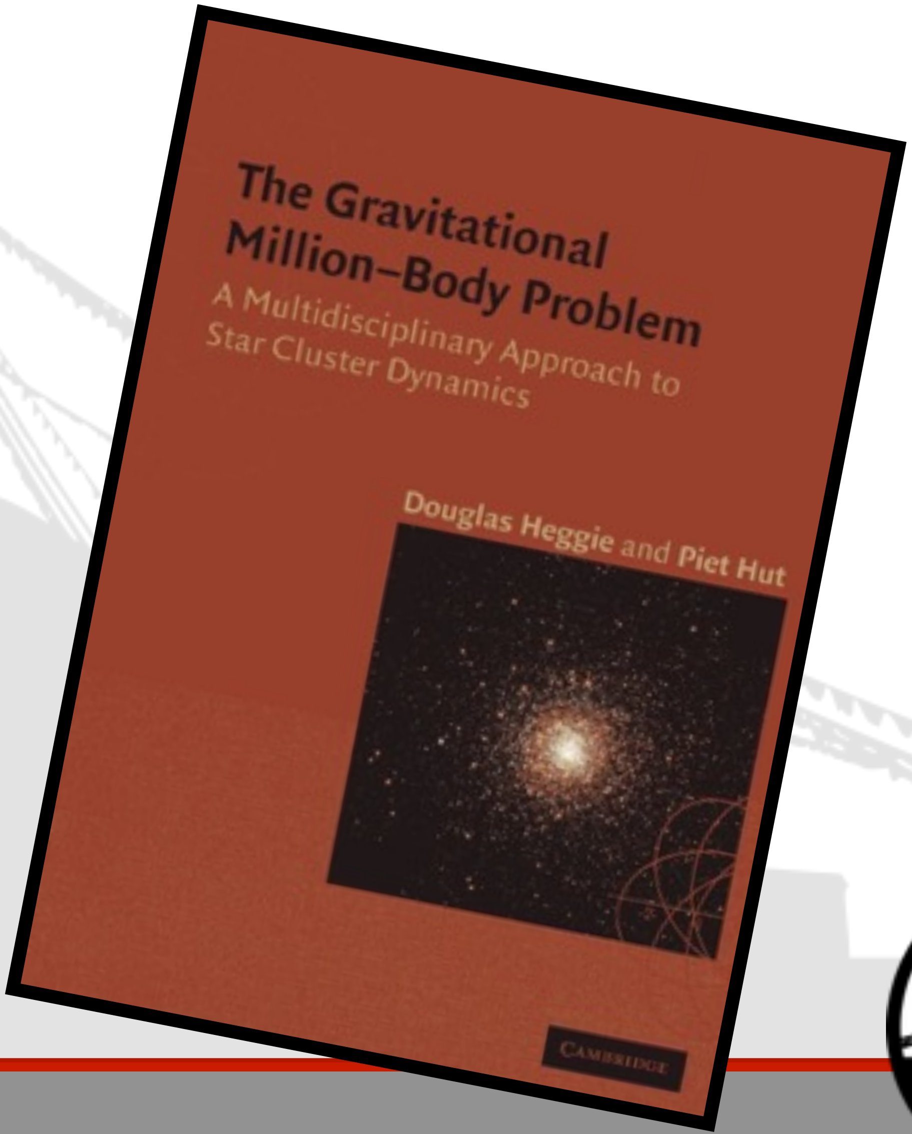
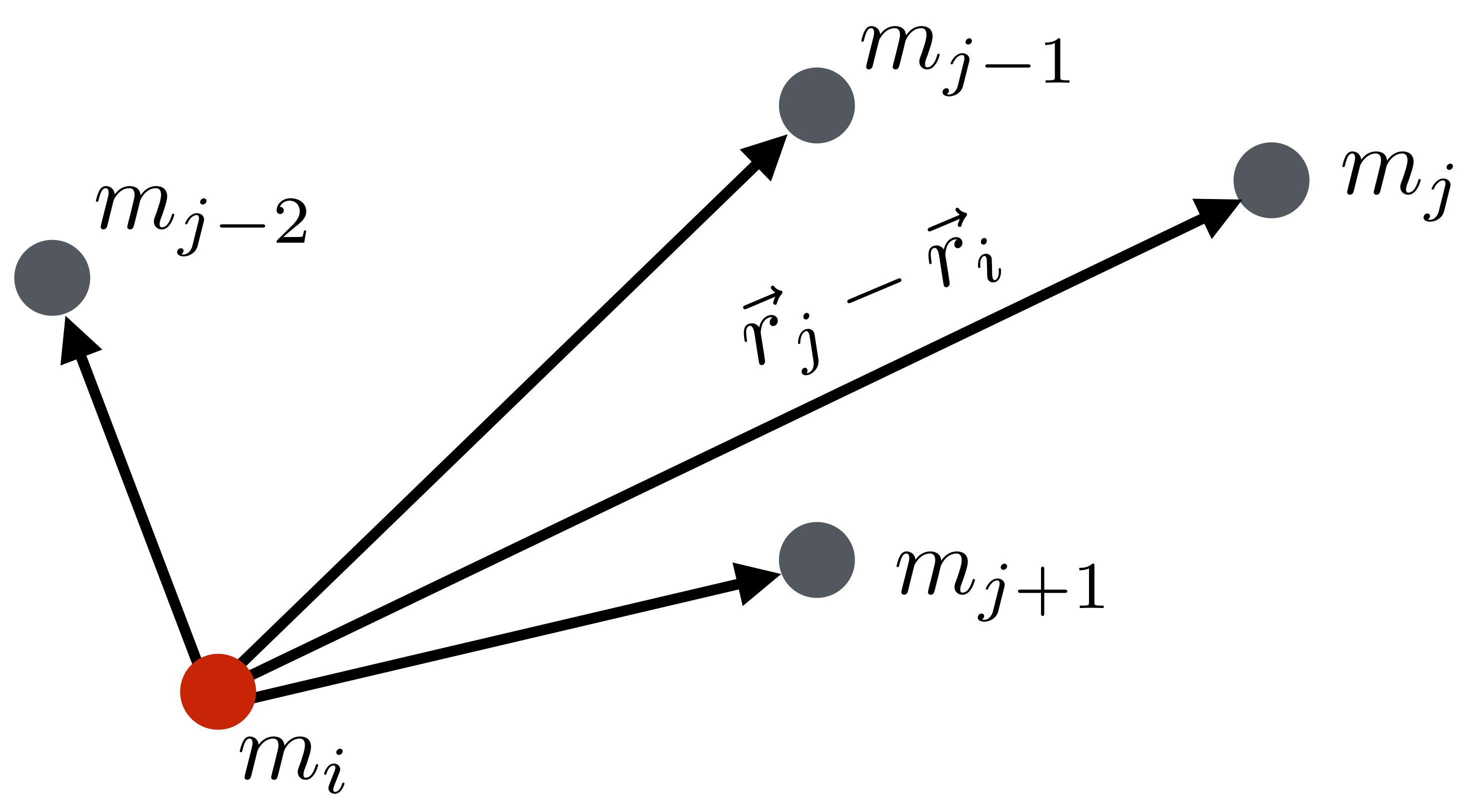


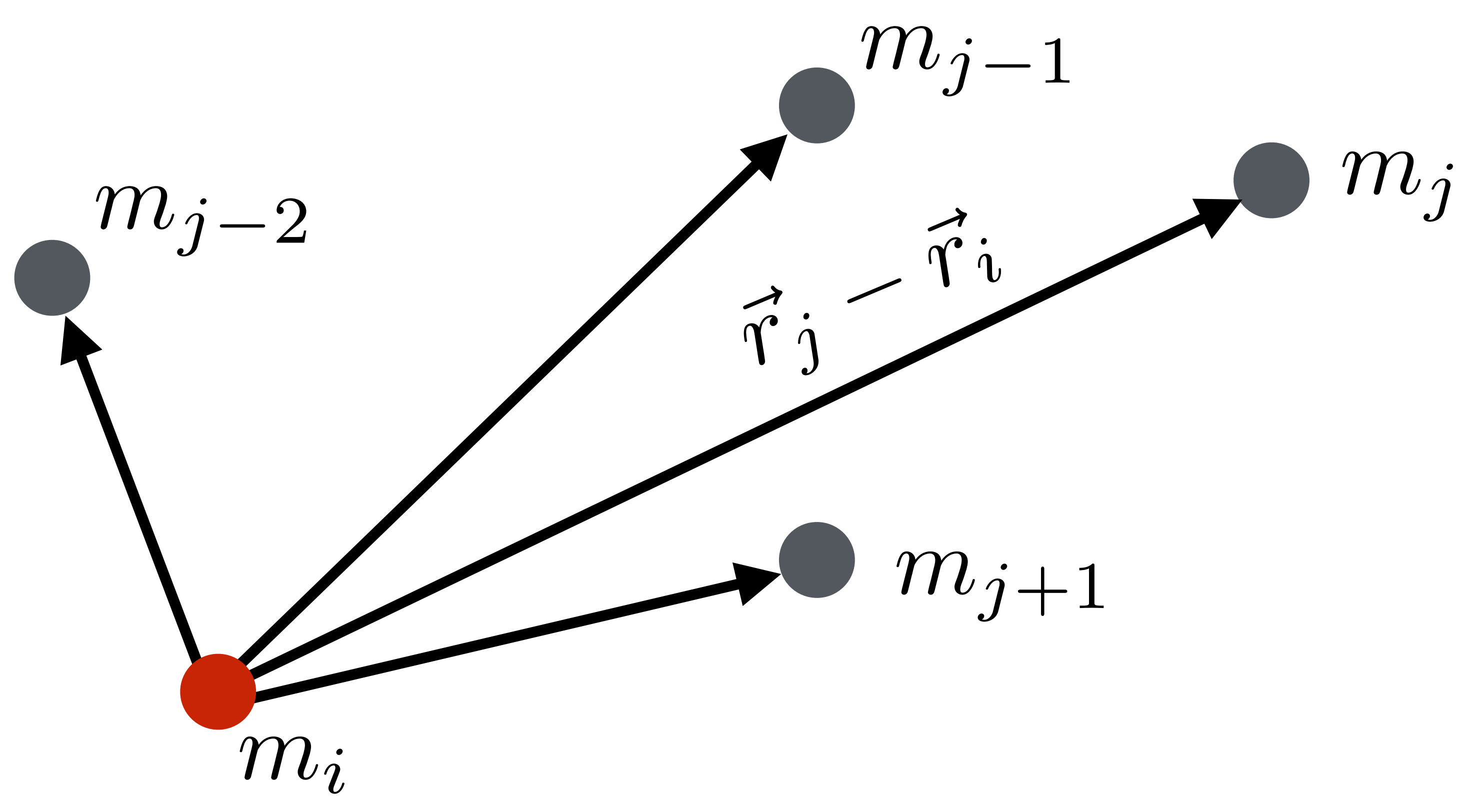
Gravitation

$$\vec{F}_i = \sum_{j=1, j\neq i}^{N} G m_i m_j \frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|^3}$$



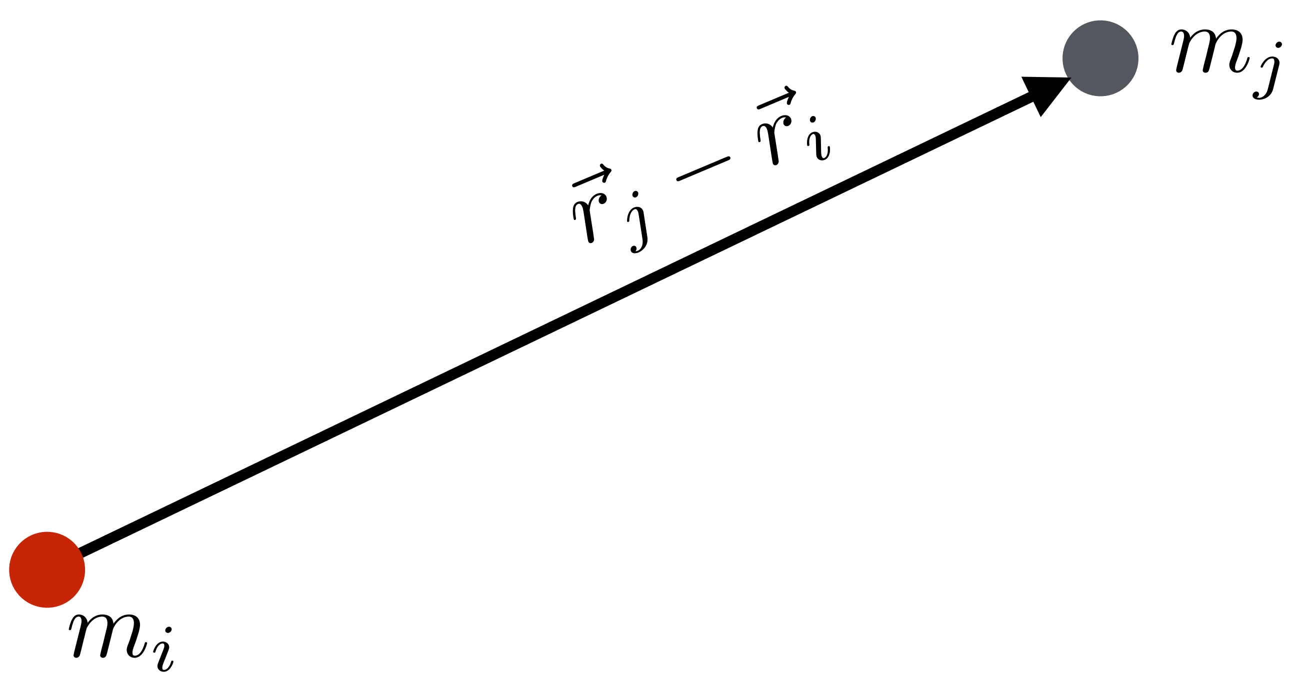
Gravitation

$$\ddot{\vec{r}}_i = -G \sum_{j=1, j\neq i}^{N} m_j \frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|^3}$$



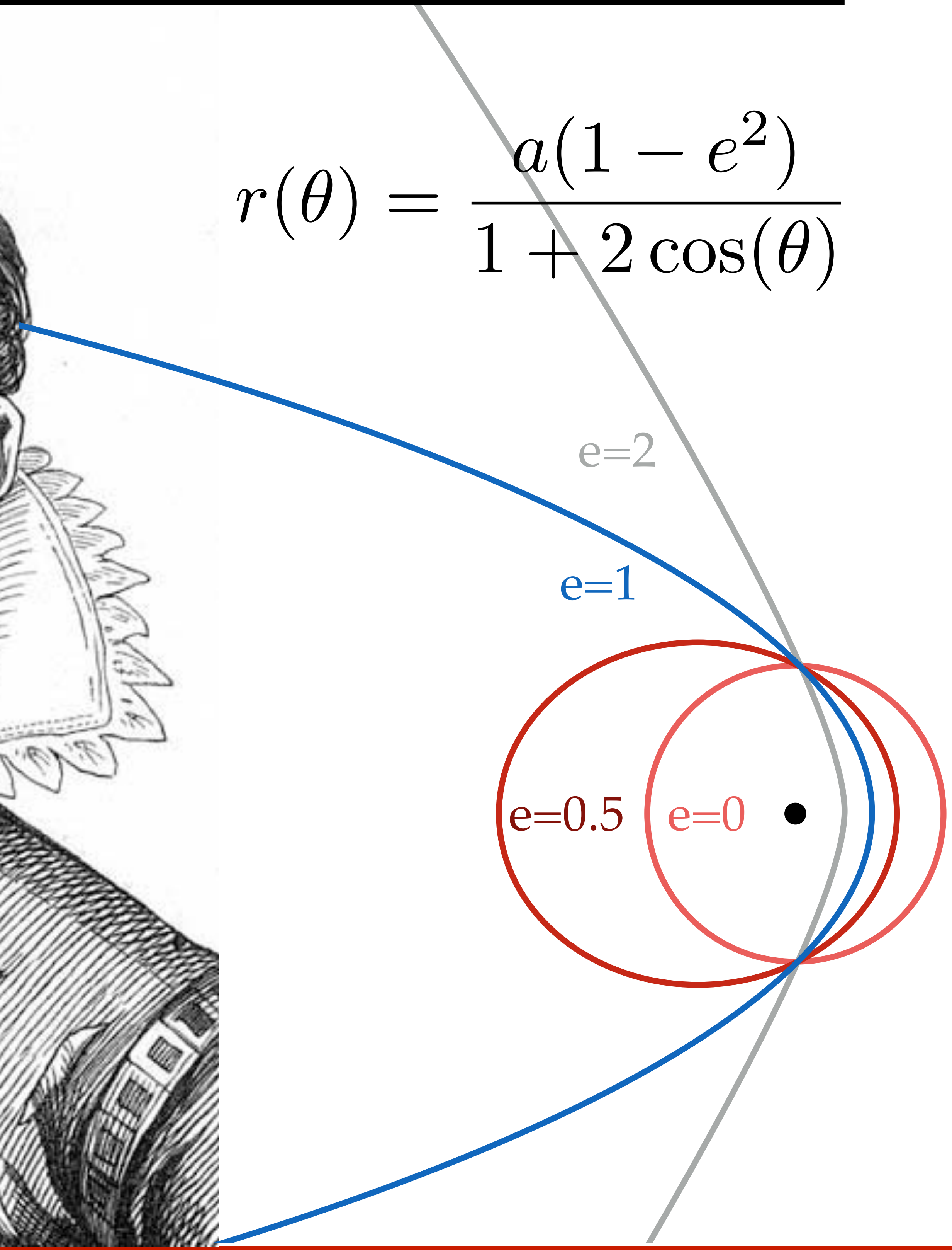
Gravitation - N = 2

$$\ddot{\vec{r}}_i = -G \sum_{j=1, j\neq i}^{2} m_j \frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|^3}$$

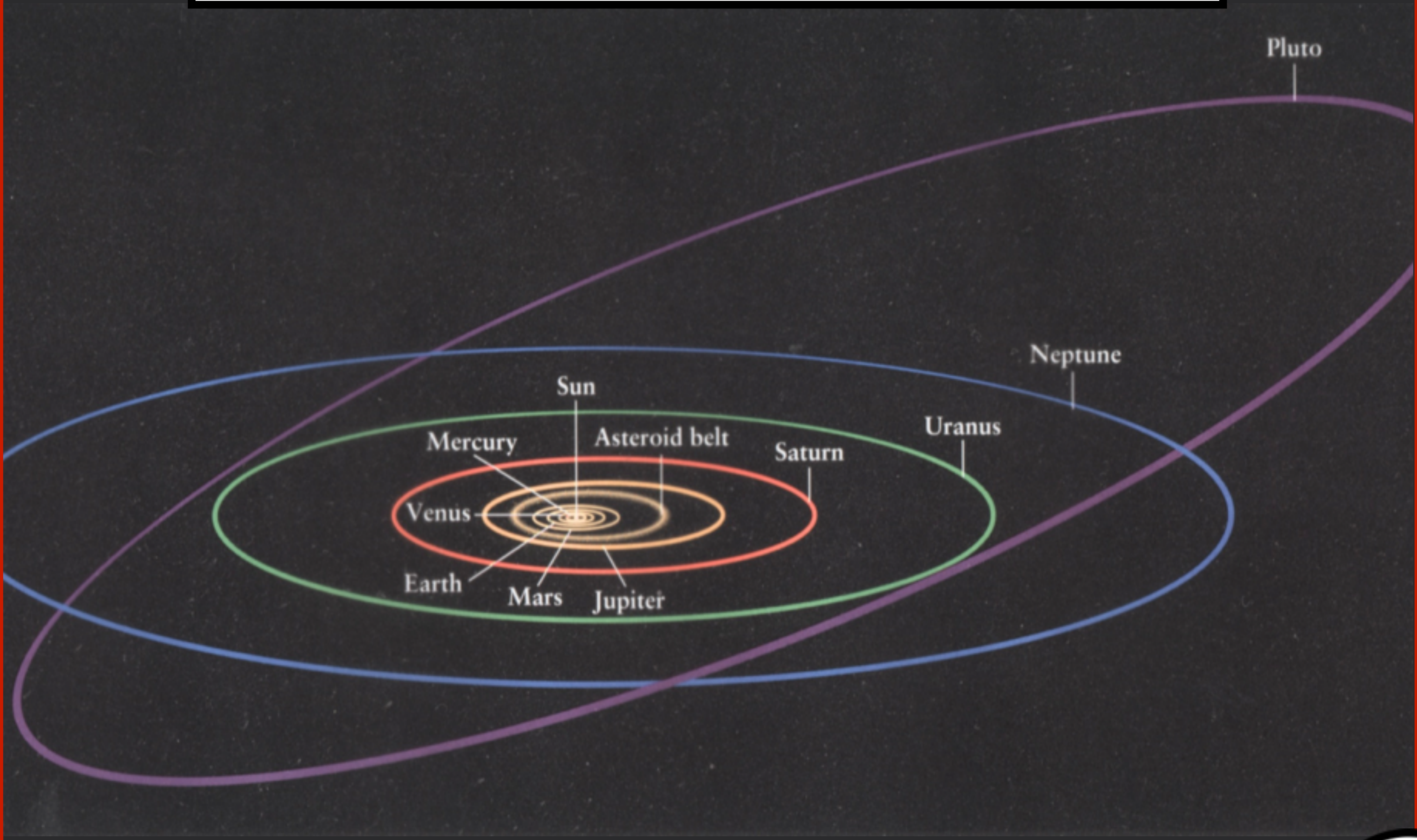


Gravitation - N = 2

$$r(\theta) = \frac{a(1-e^2)}{1+2\cos(\theta)}$$



Gravitation - N = 2



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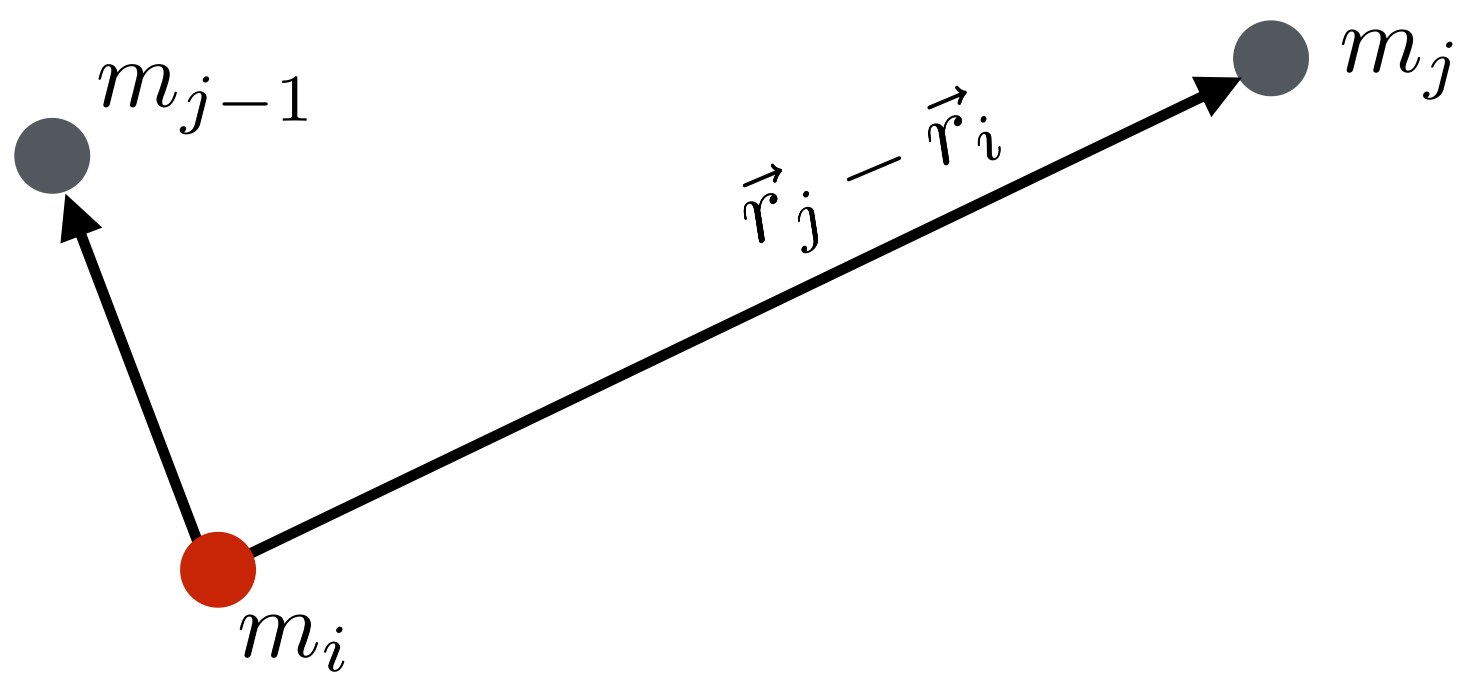
Gravitation - N = 3

$$\ddot{\vec{r}}_i = -G \sum_{j=1, j\neq i}^{3} m_j \frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|^3}$$



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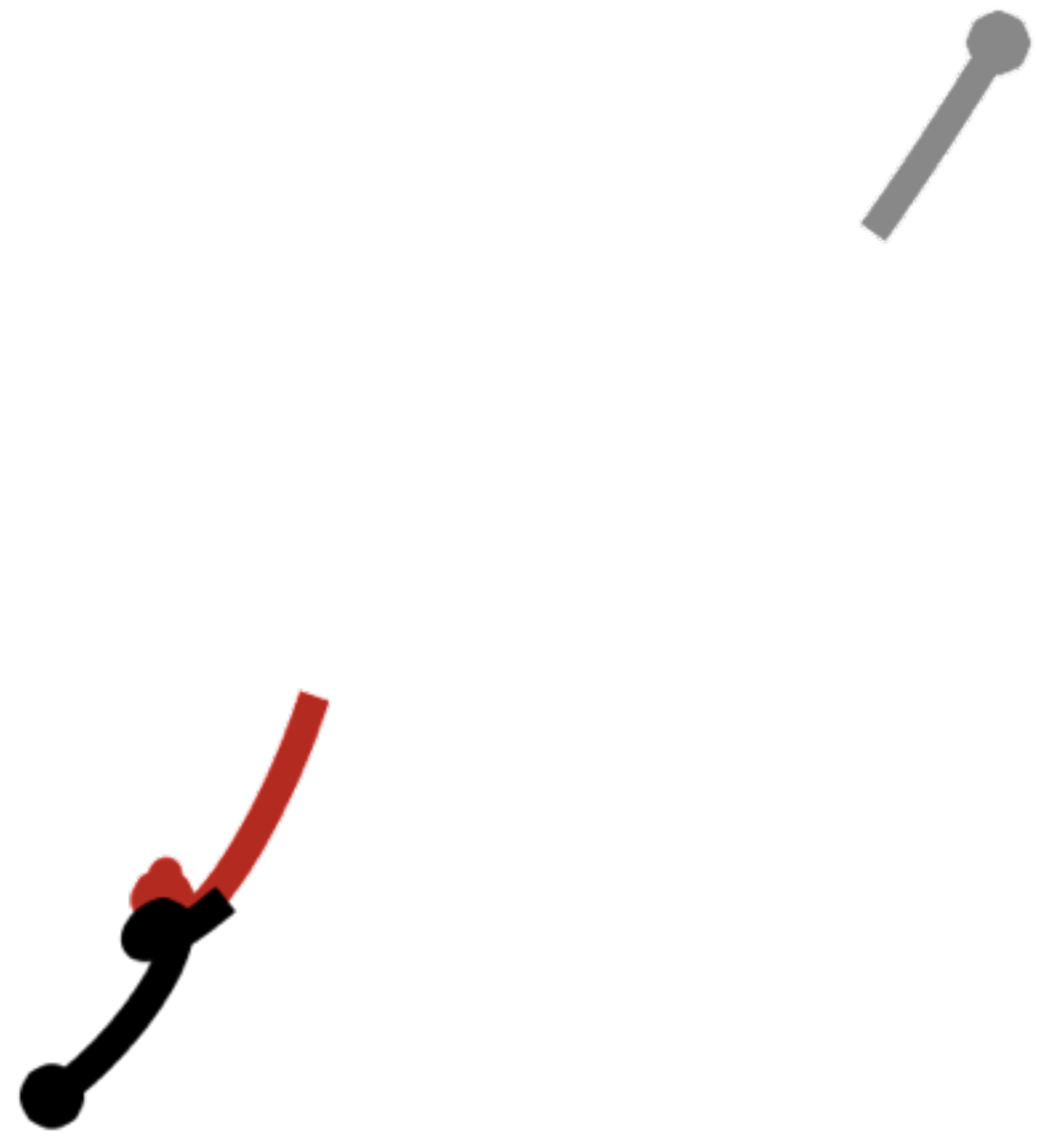
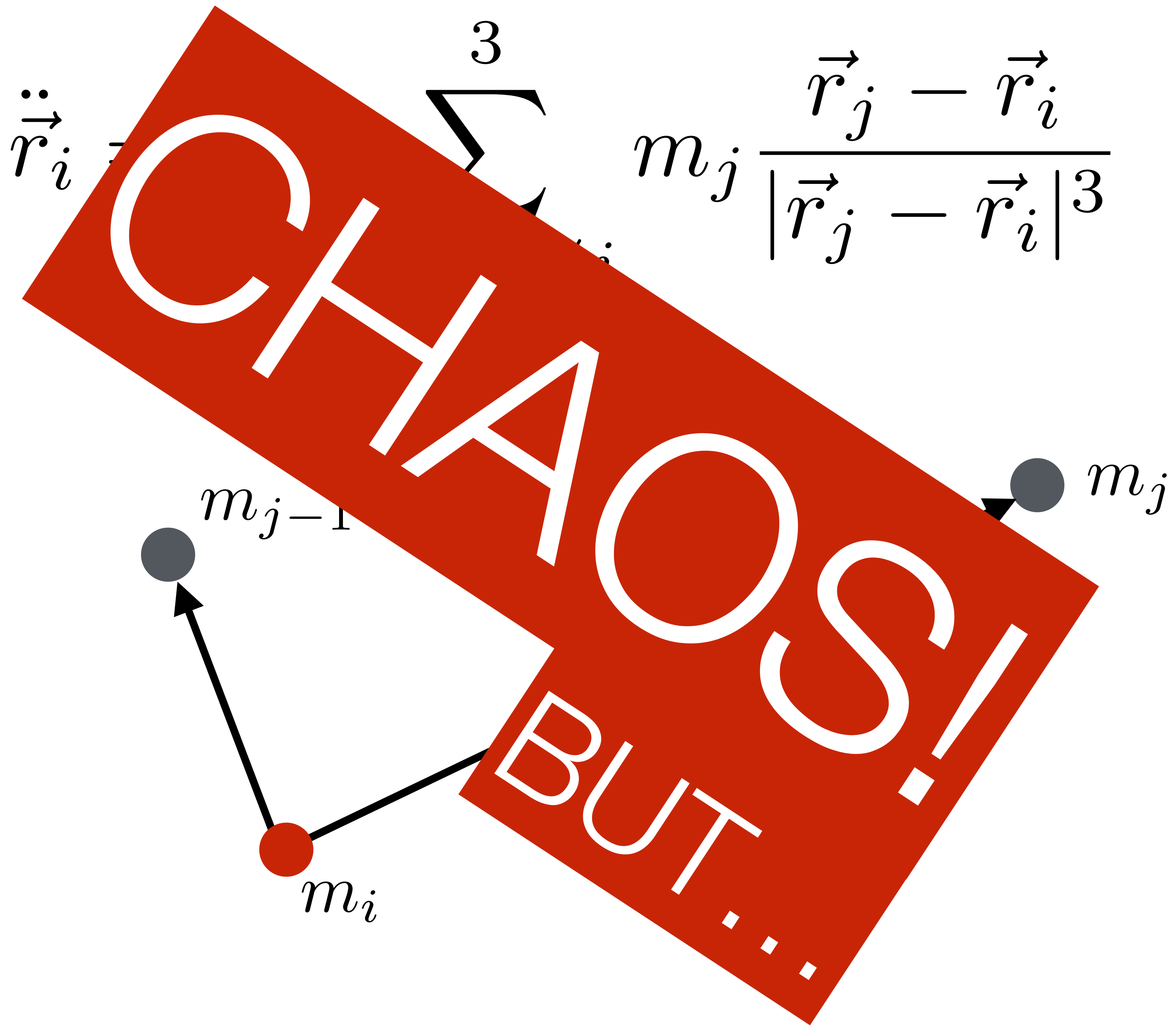


Gravitation - N = 3



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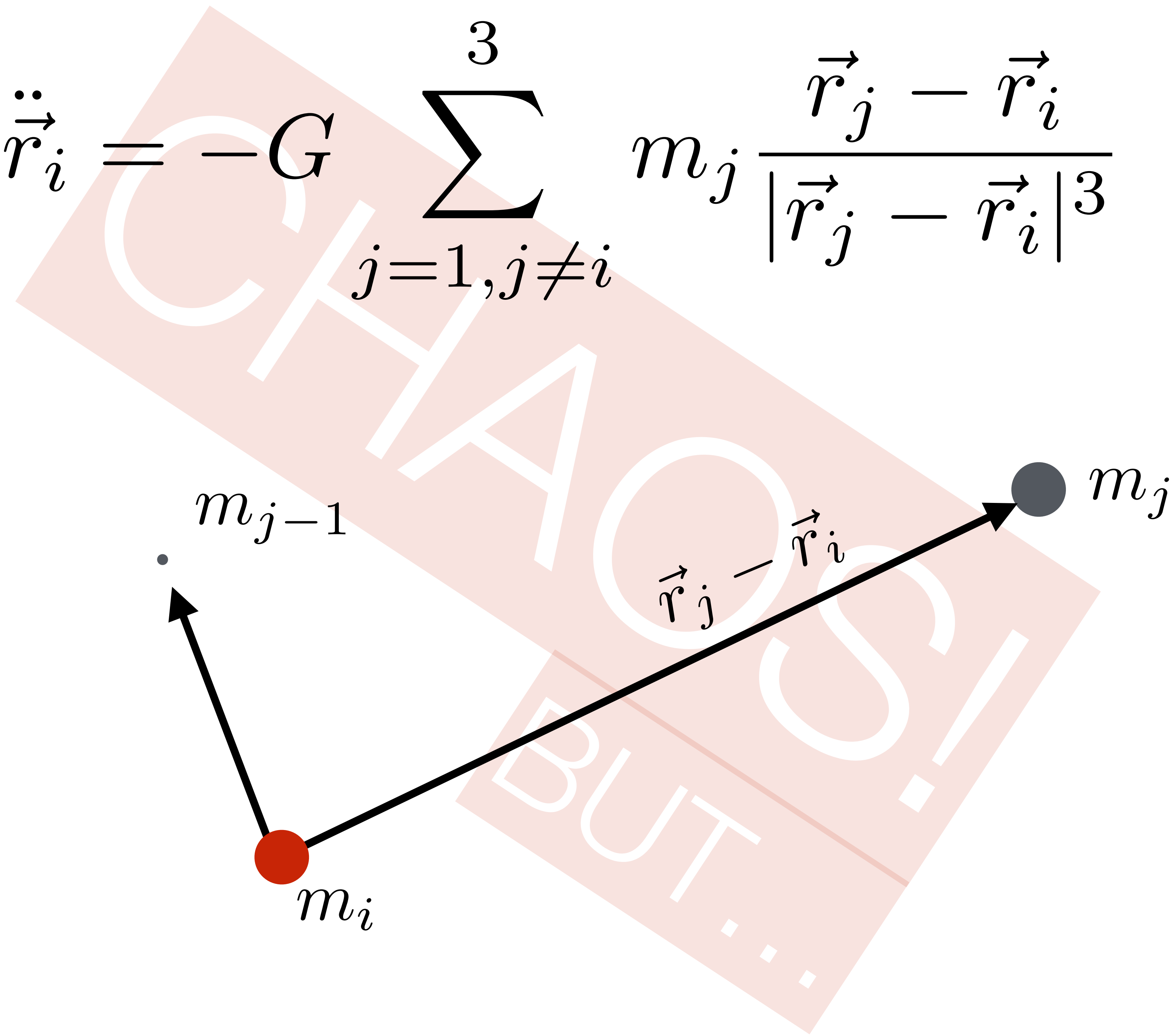
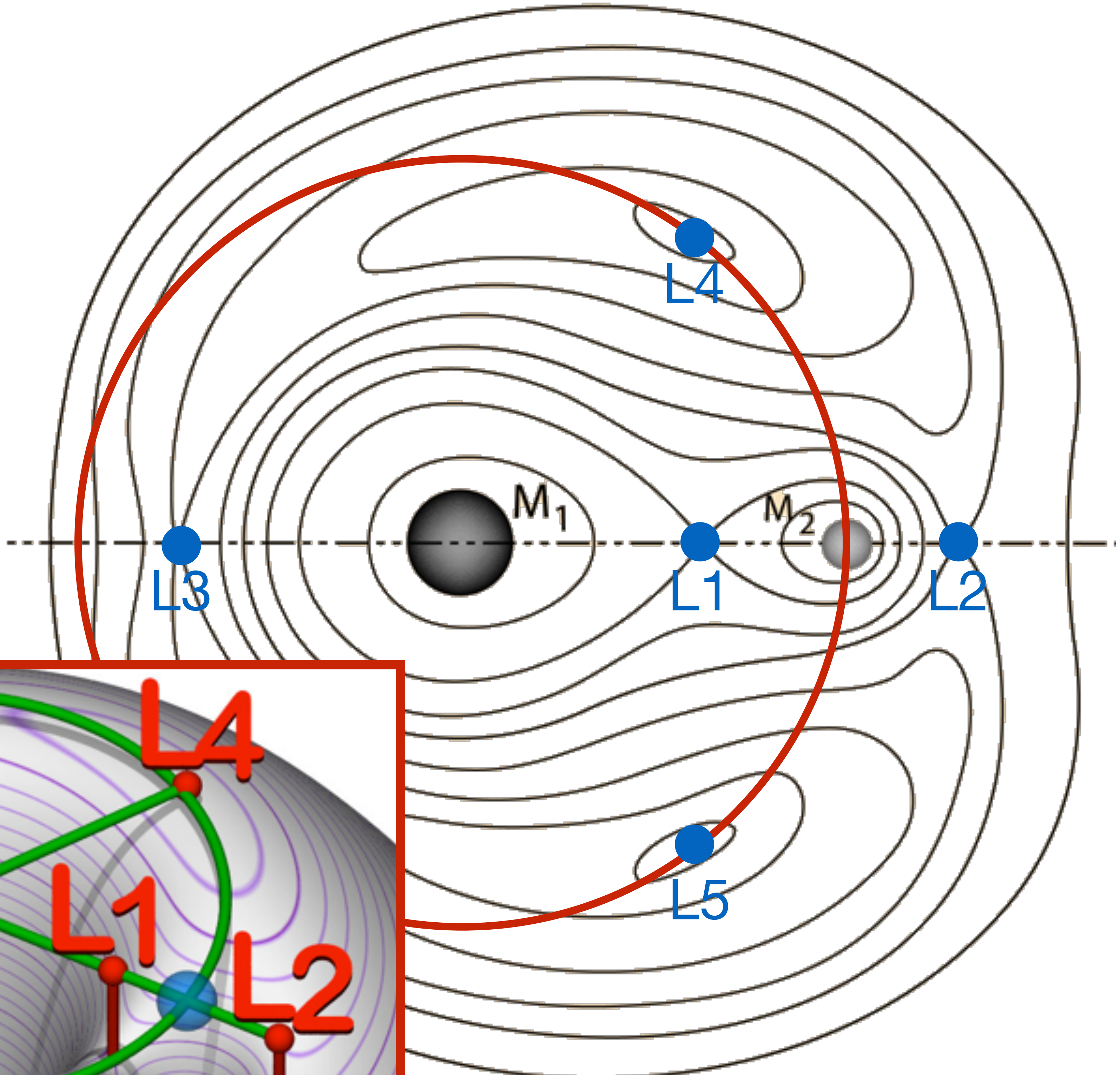
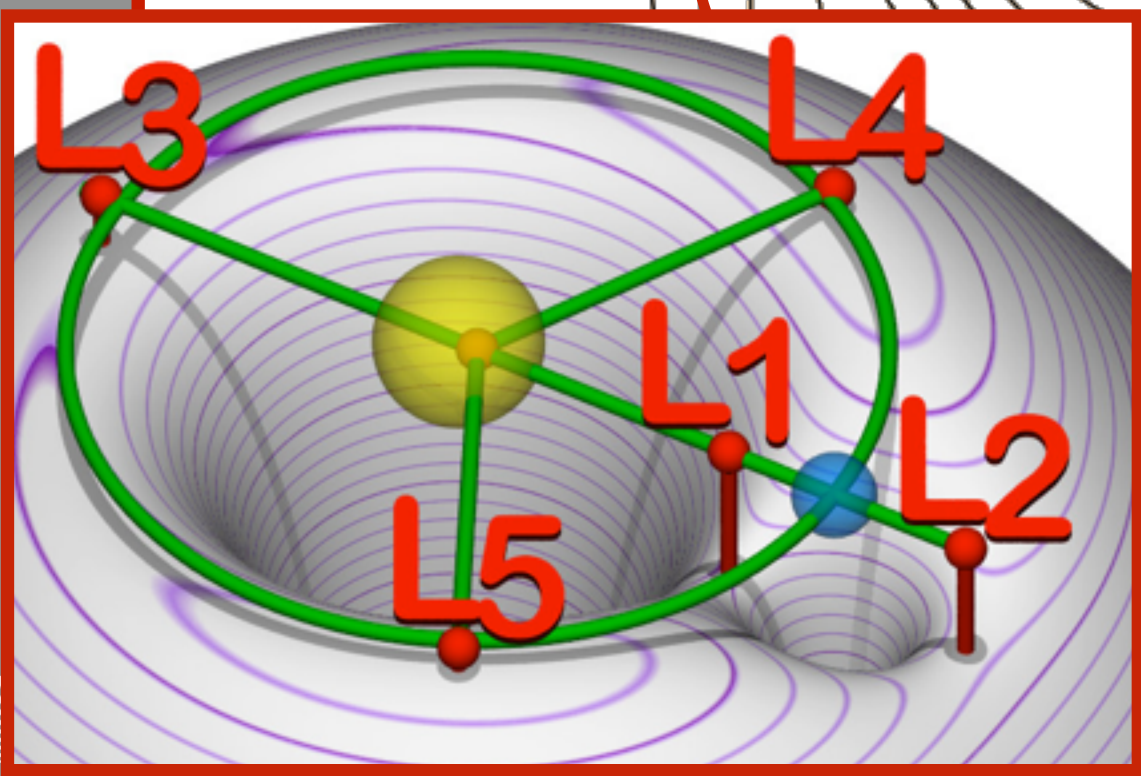


Gravitation - N = 3

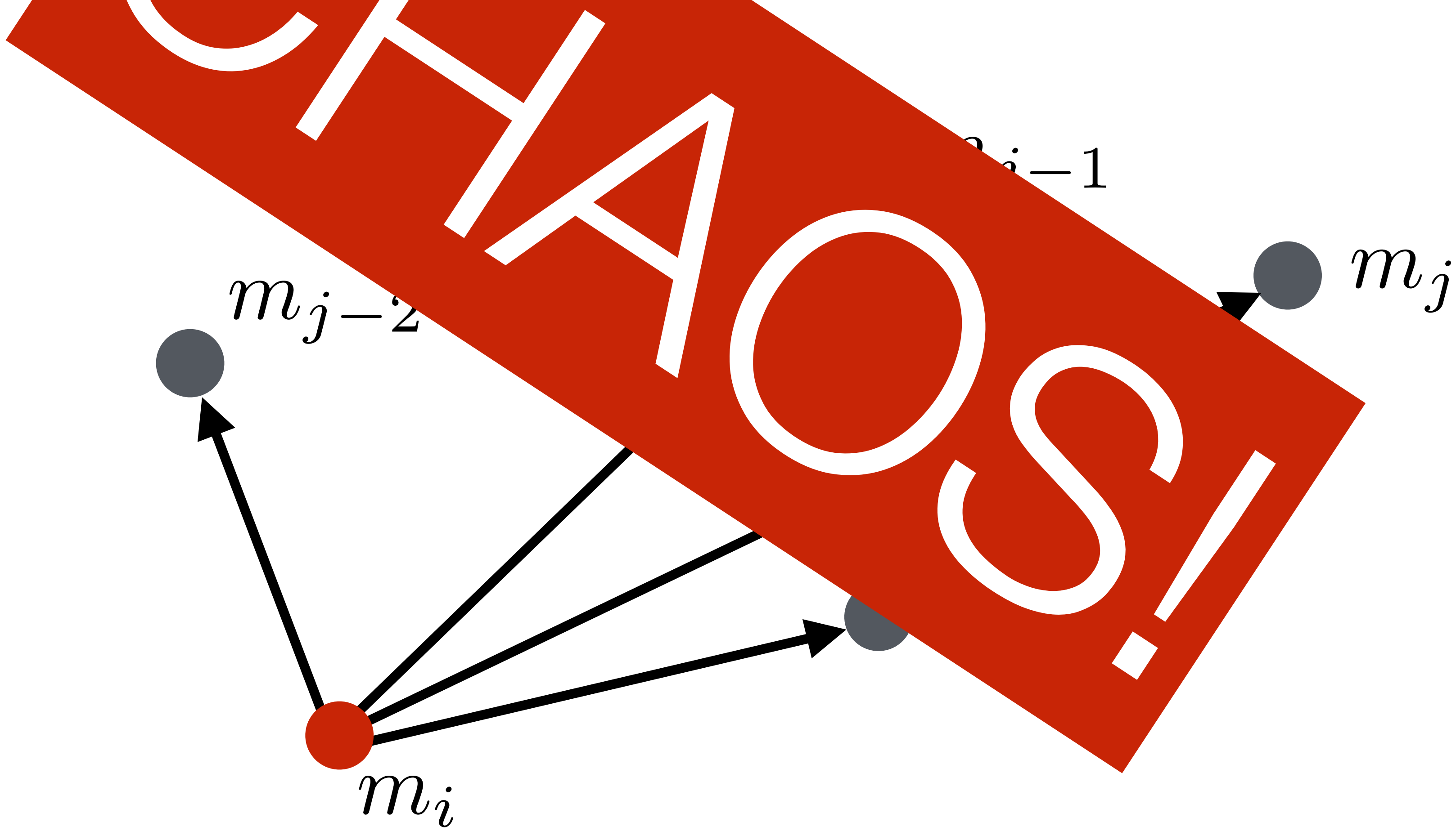




Explanations - Problems

Gravitation - N > 3

$$\ddot{\vec{r}}_i = G \sum_{j \neq i}^{N} m_j \frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|^3}$$



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Outline

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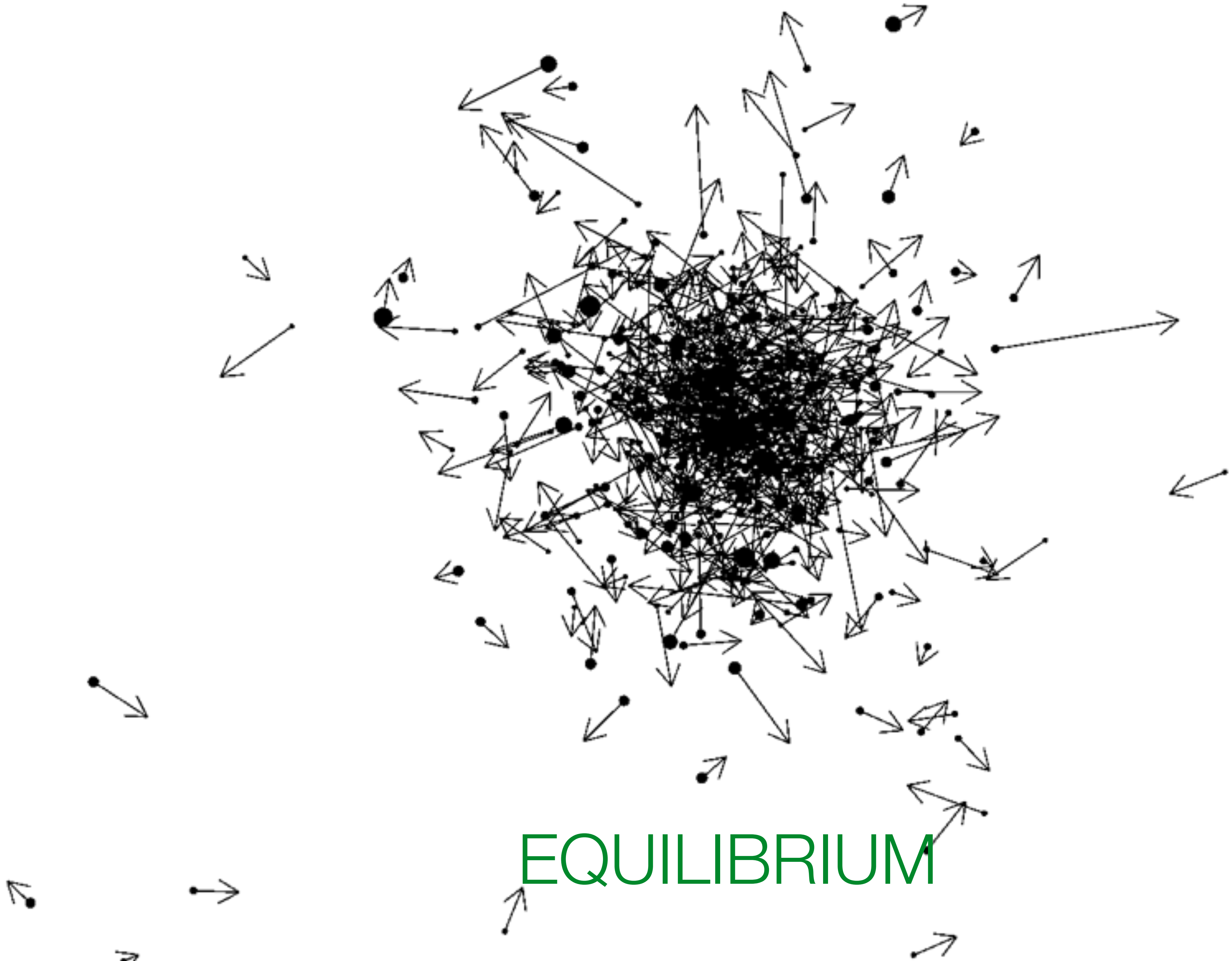
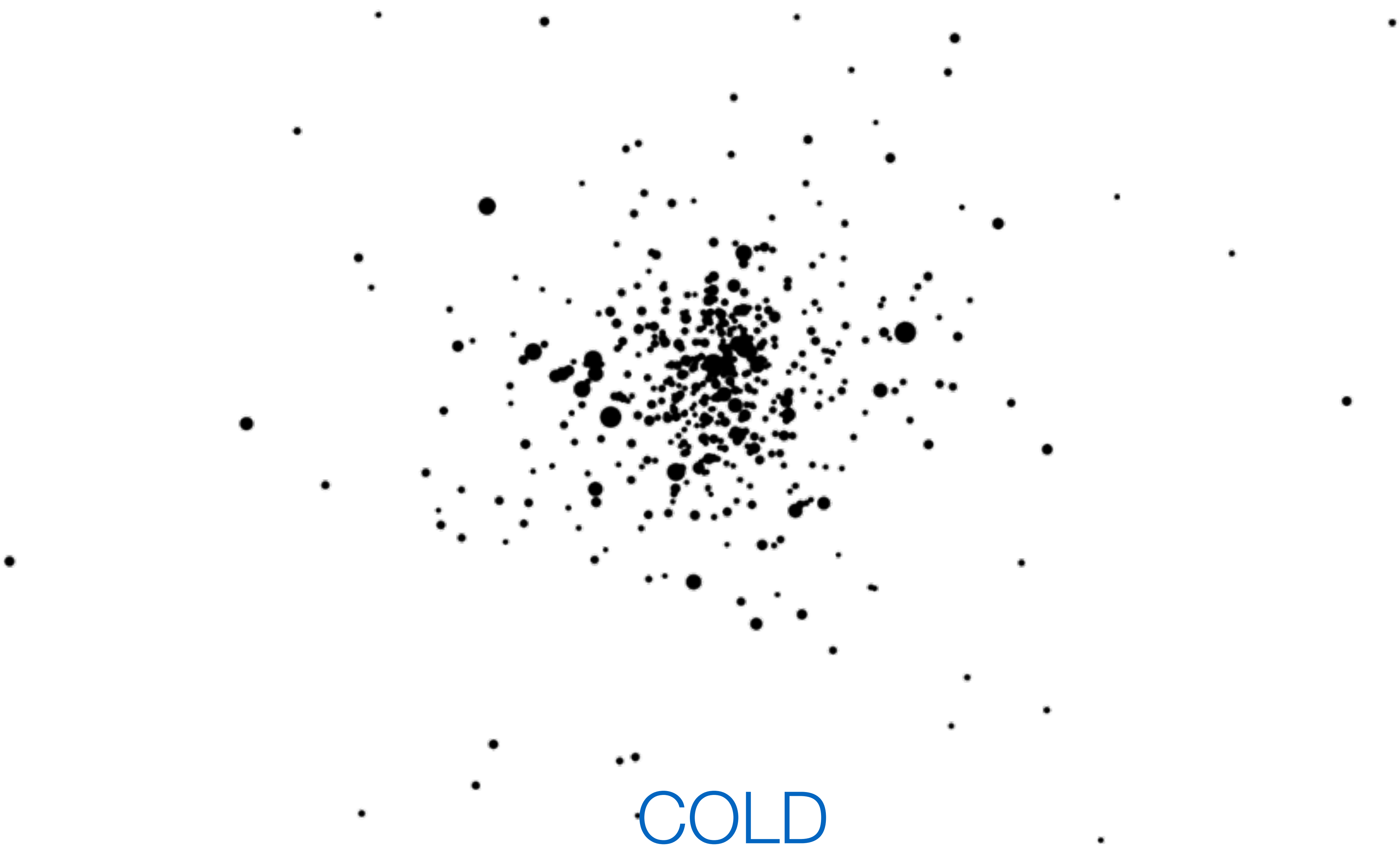
Dynamic Equilibrium



EQUILIBRIUM

Dynamic Equilibrium



COLD

(v = small or 0)

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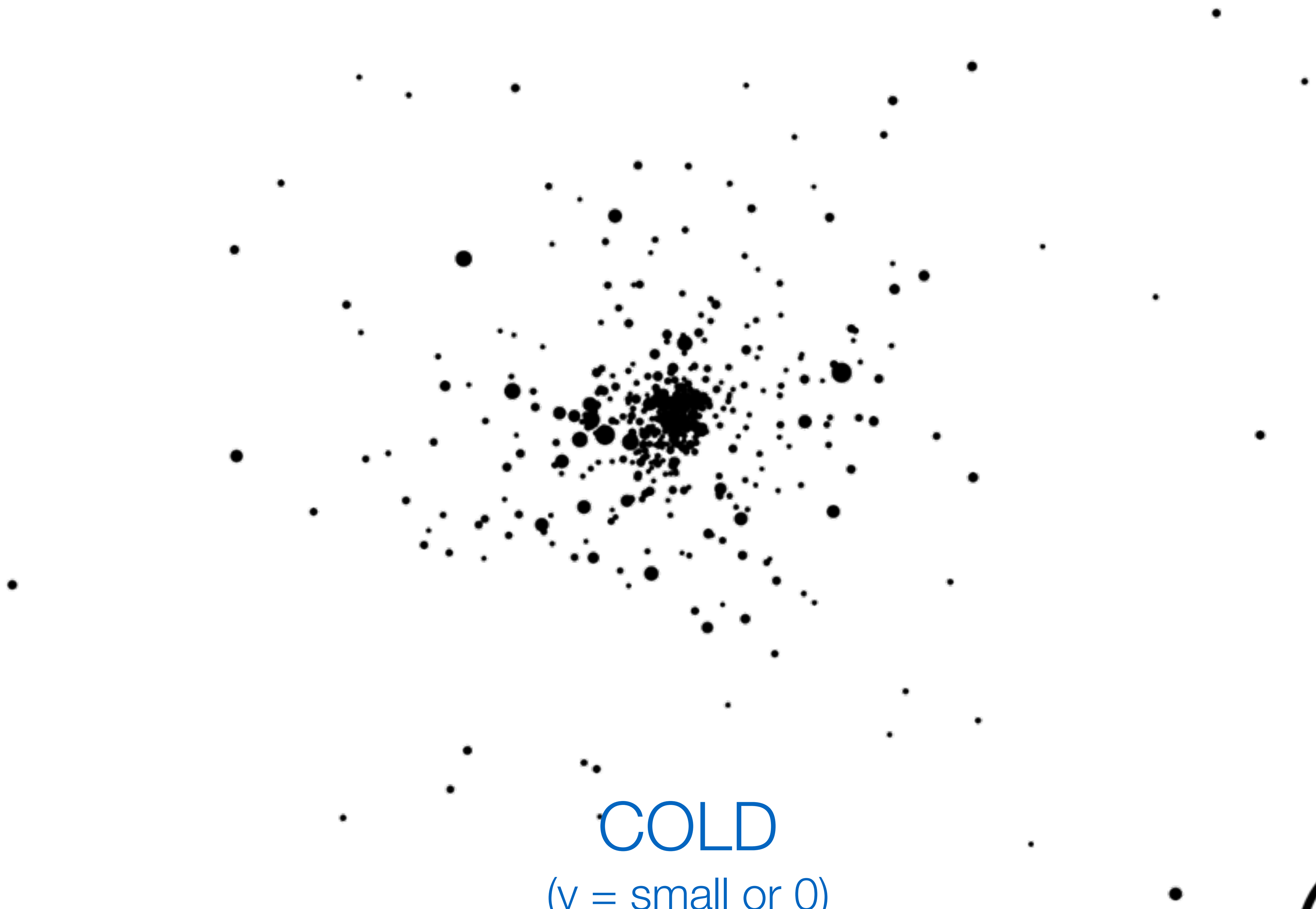
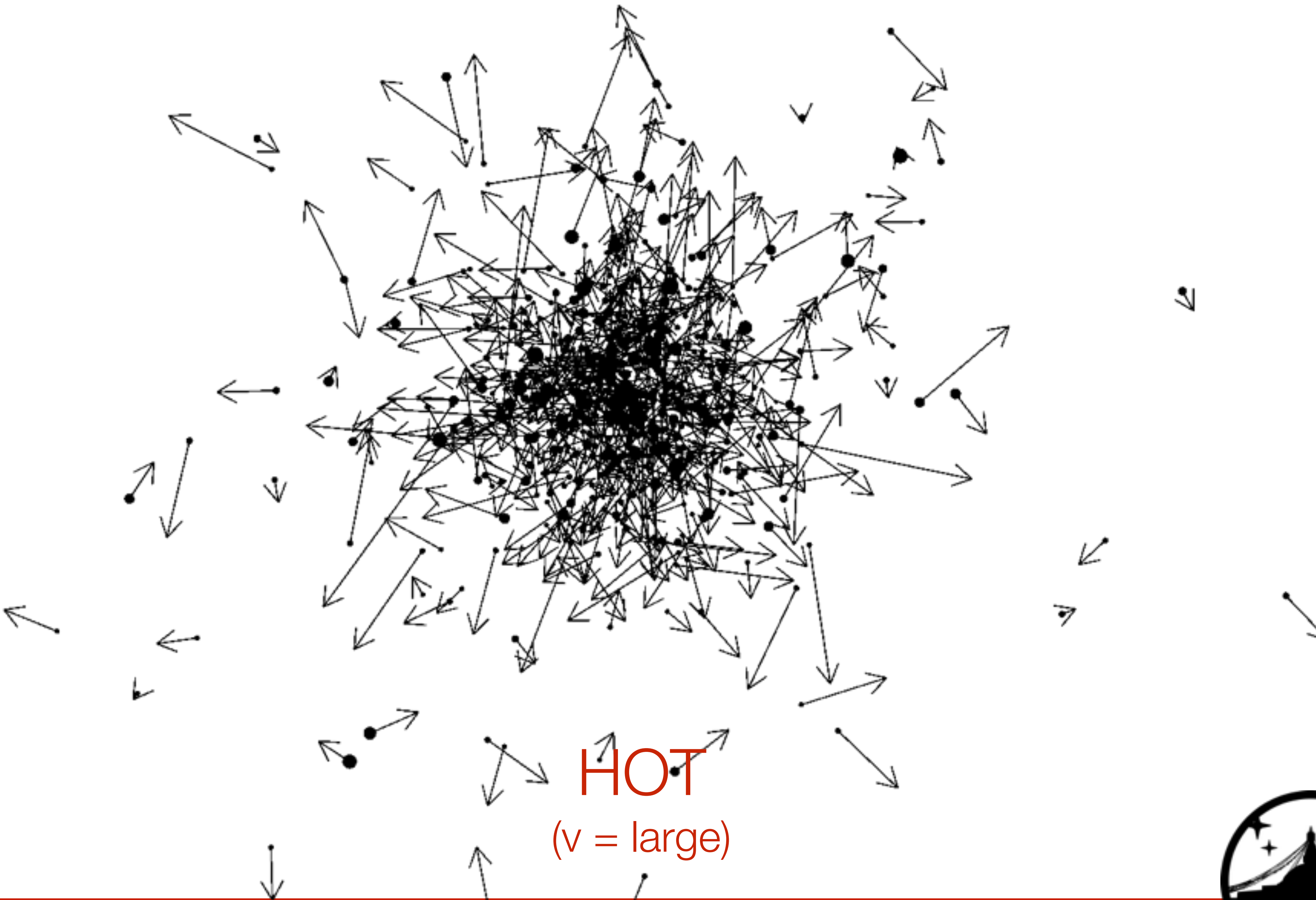
Dynamic Equilibrium



COLD

(v = small or 0)

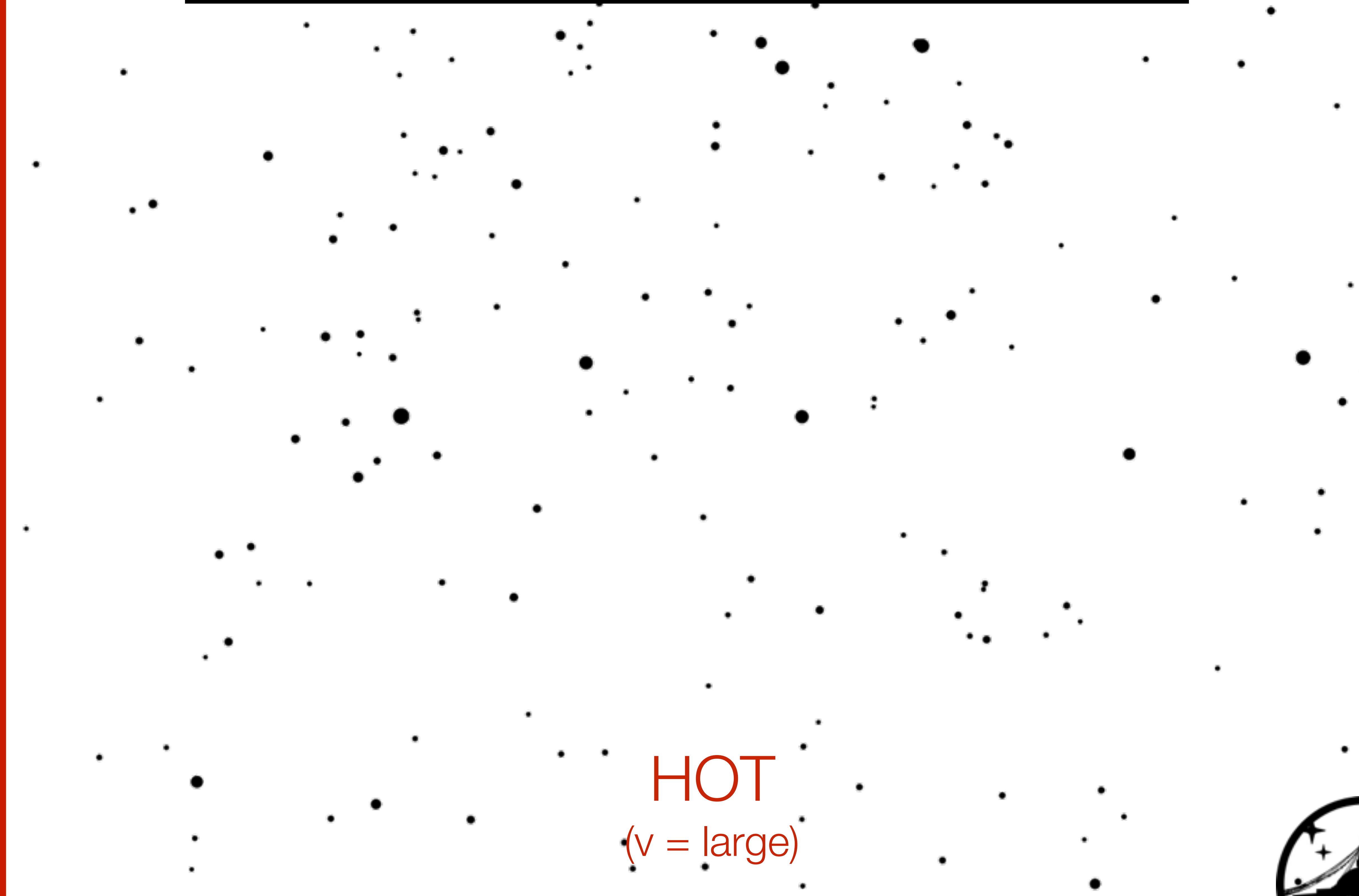
Dynamic Equilibrium



HOT

(v = large)

Dynamic Equilibrium



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Dynamic Equilibrium - Definition

EQUILIBRIUM:

- No EXPANSION, and no CONTRACTION, even though all particles are in MOTION

Virial Theorem

KINETIC ENERGY $$K = \frac{1}{2}\sum_{i=1}^{N} m_i \vec{v}_i^2$$

POTENTIAL ENERGY $$W = -G\frac{1}{2}\sum_{i=1}^{N}\sum_{i\neq j}^{N}\frac{m_i m_j}{|\vec{r}_i - \vec{r}_j|}$$

CONSERVATION OF ENERGY

$$E = W + K = const.$$

VIRIAL THEOREM

$$W = -2K$$

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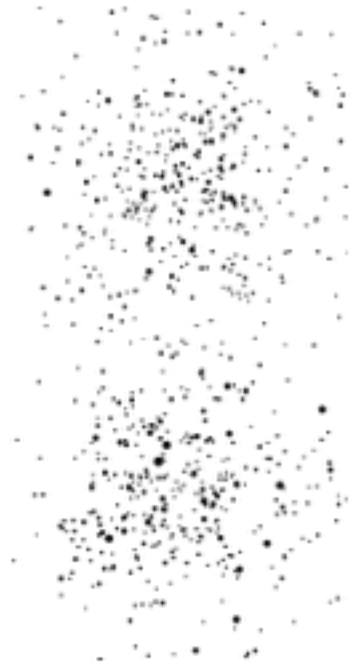
"Temperature"

Like in a gas:

- Particles move fast → system is HOT →
- Particles move slow → system is COLD →



$$\frac{1}{2}m\bar{v}^2 = \frac{3}{2}k_B T$$

↓

$$K = \frac{3}{2}N k_B \bar{T}$$

Heat Capacity

$$K = \frac{3}{2} N k_B \bar{T}$$

$$E = W + K$$

VIRIAL THEOREM

$$W = -2K$$

$$= -K$$

$$= -\frac{3}{2} N k_B \bar{T}$$

$$C \equiv \frac{dE}{d\bar{T}} = -\frac{3}{2} N k_B$$

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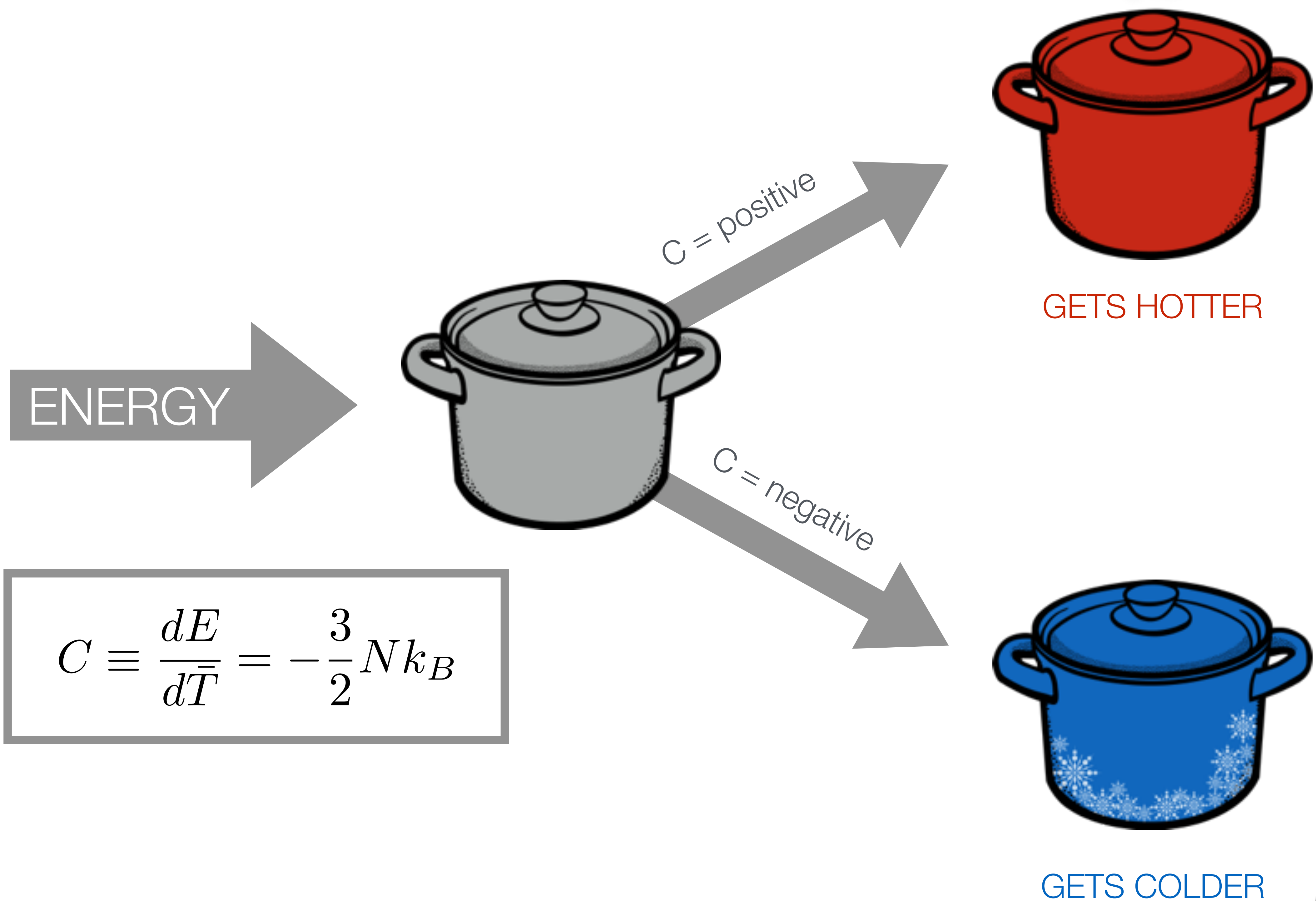
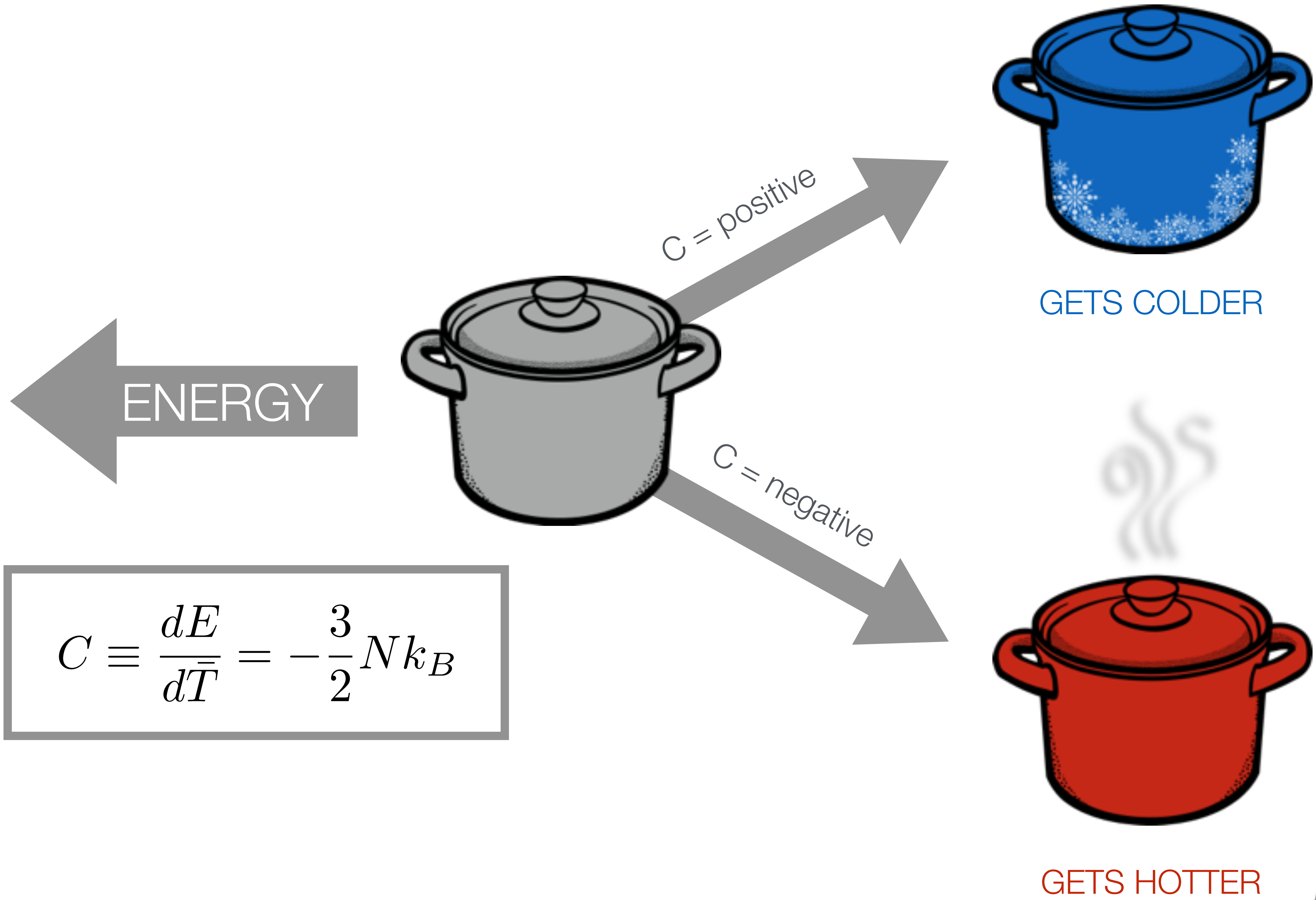
Heat Capacity



$$C \equiv \frac{dE}{d\bar{T}} = -\frac{3}{2} N k_B$$

Heat Capacity



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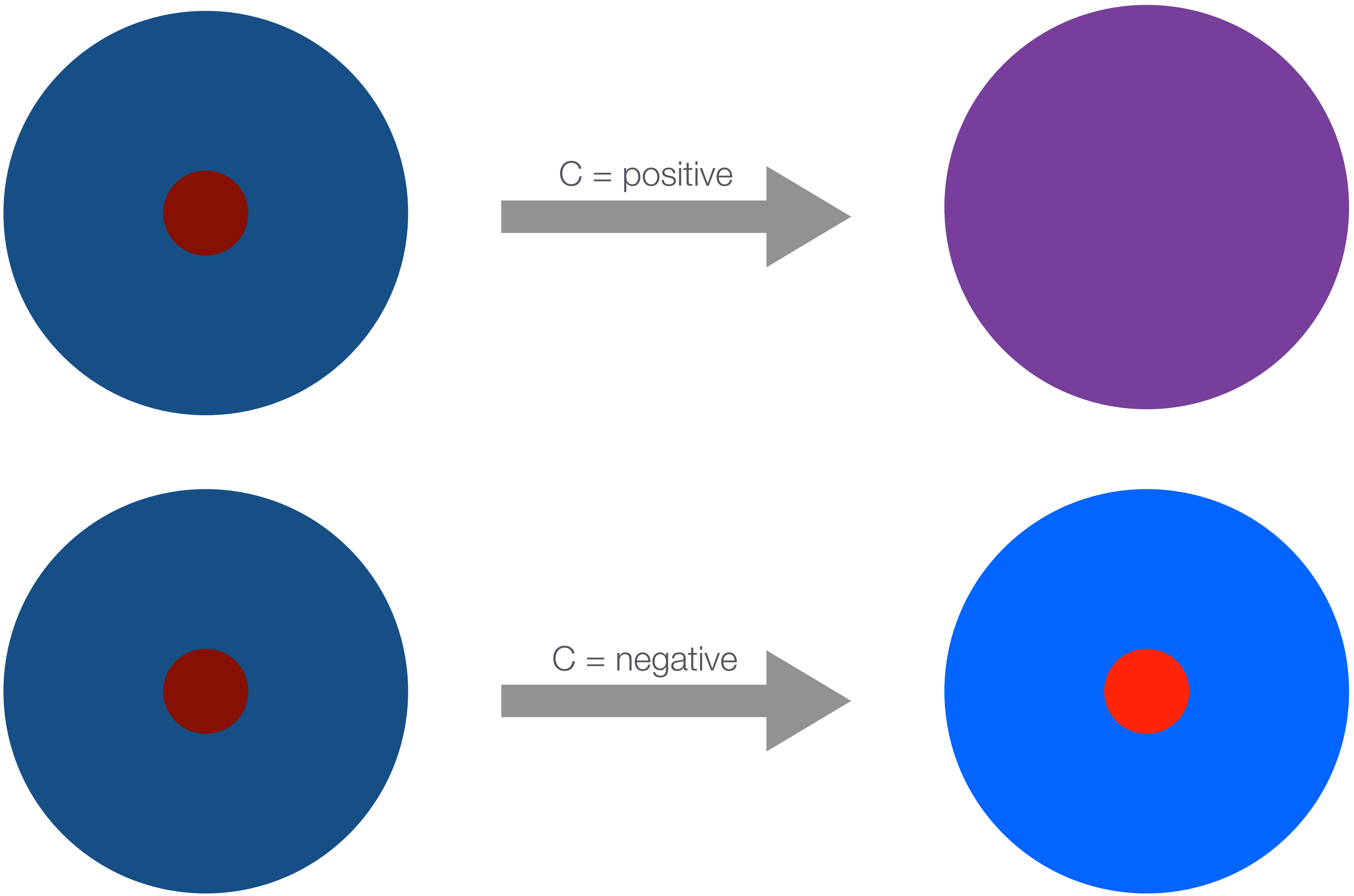
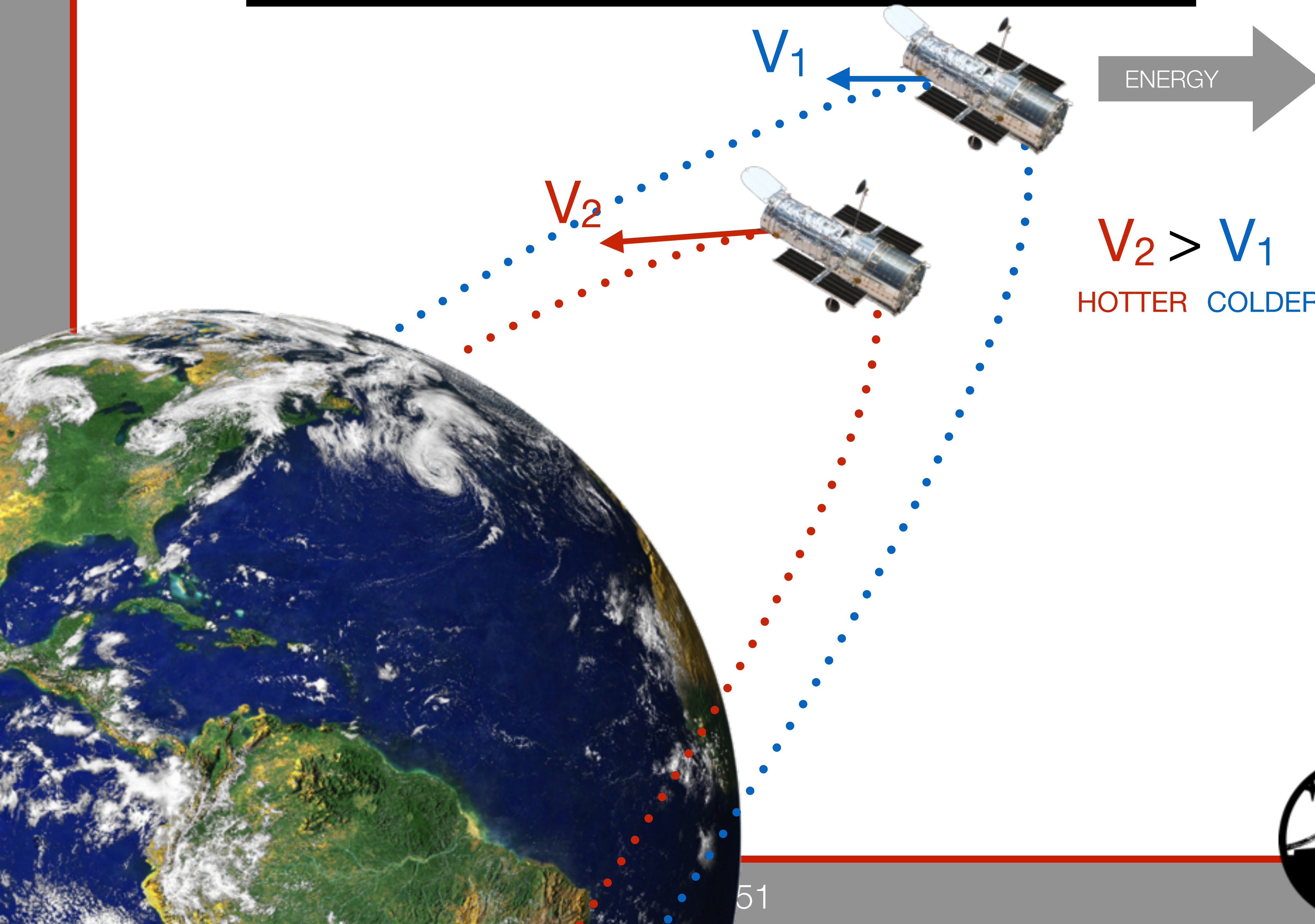
Heat Capacity



Heat Capacity



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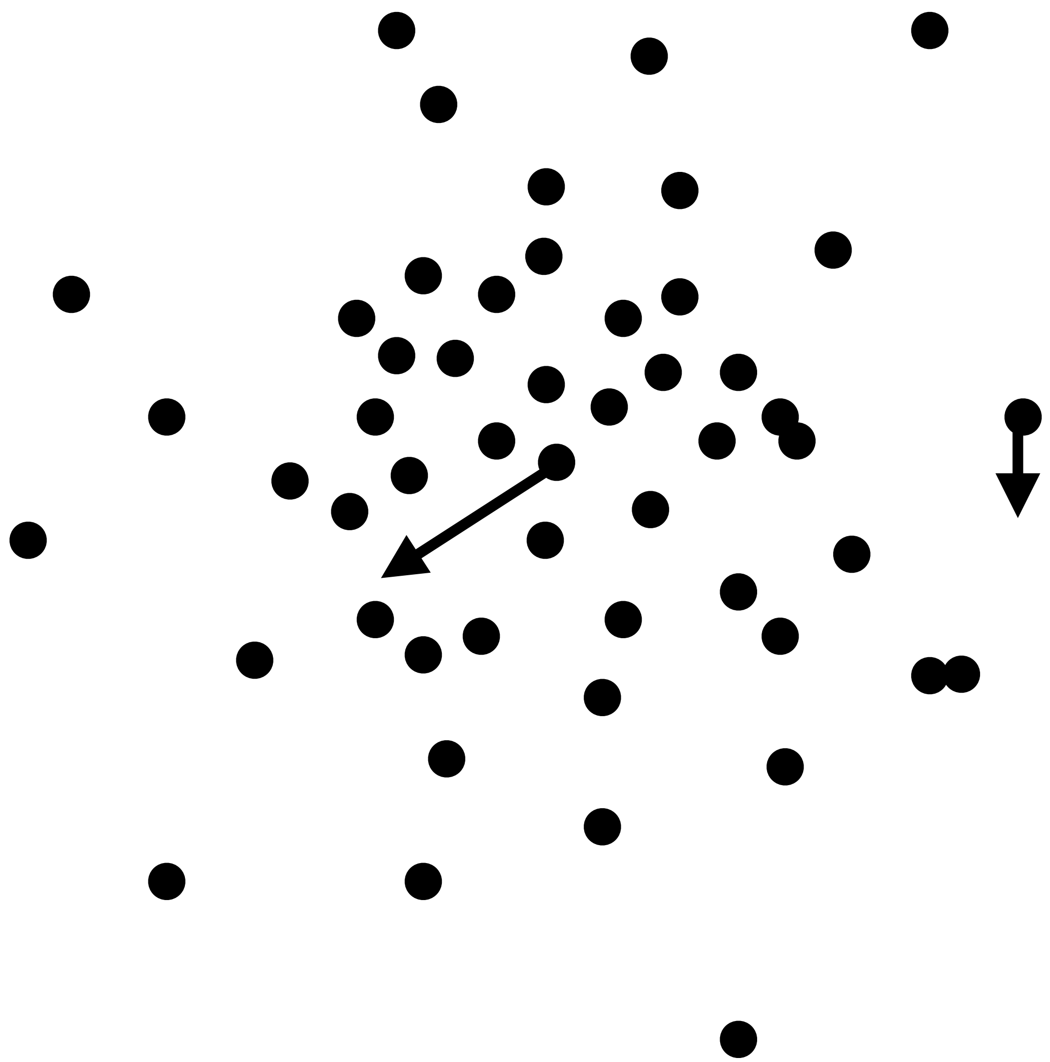
Core Collapse

Cluster of stars with equal mass:



Stars deeper in the potential move faster (hot)

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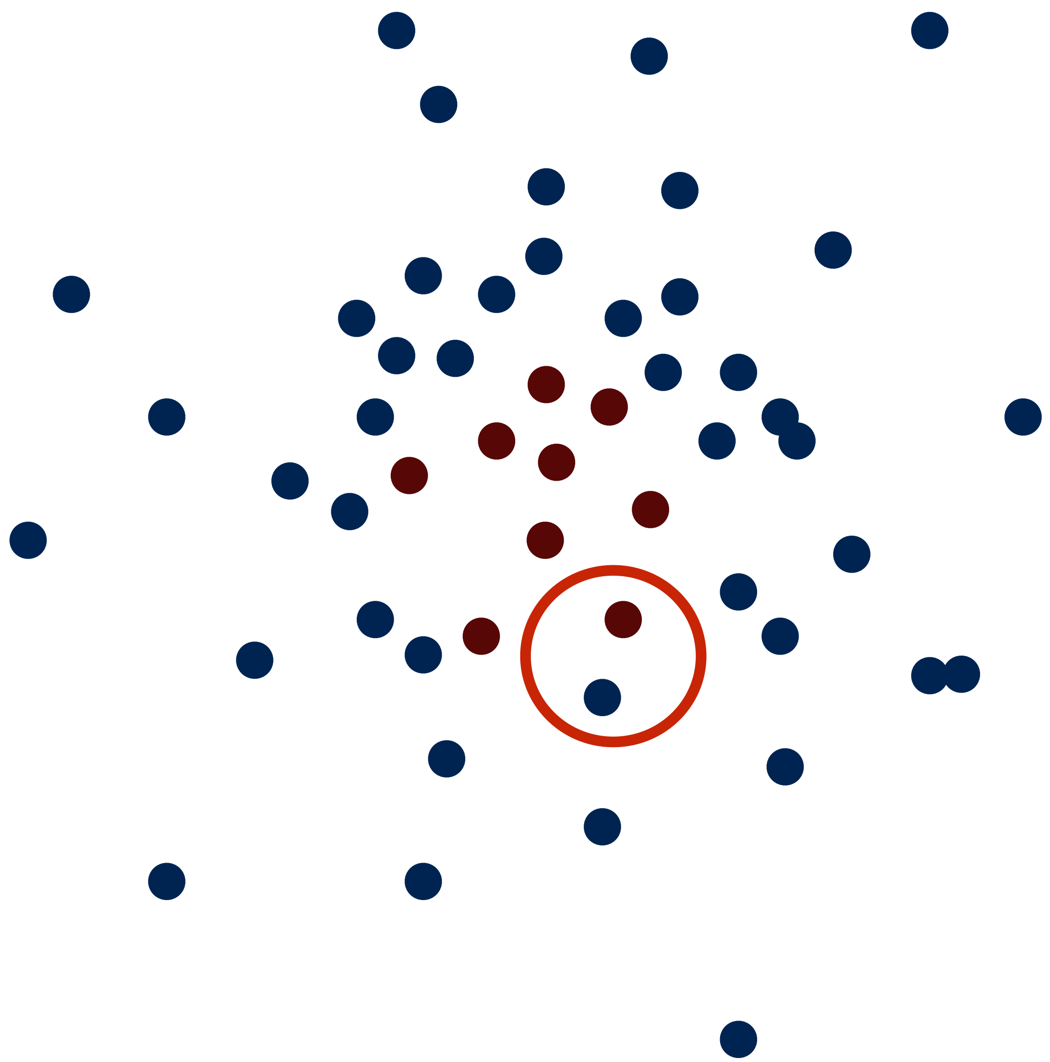
Core Collapse

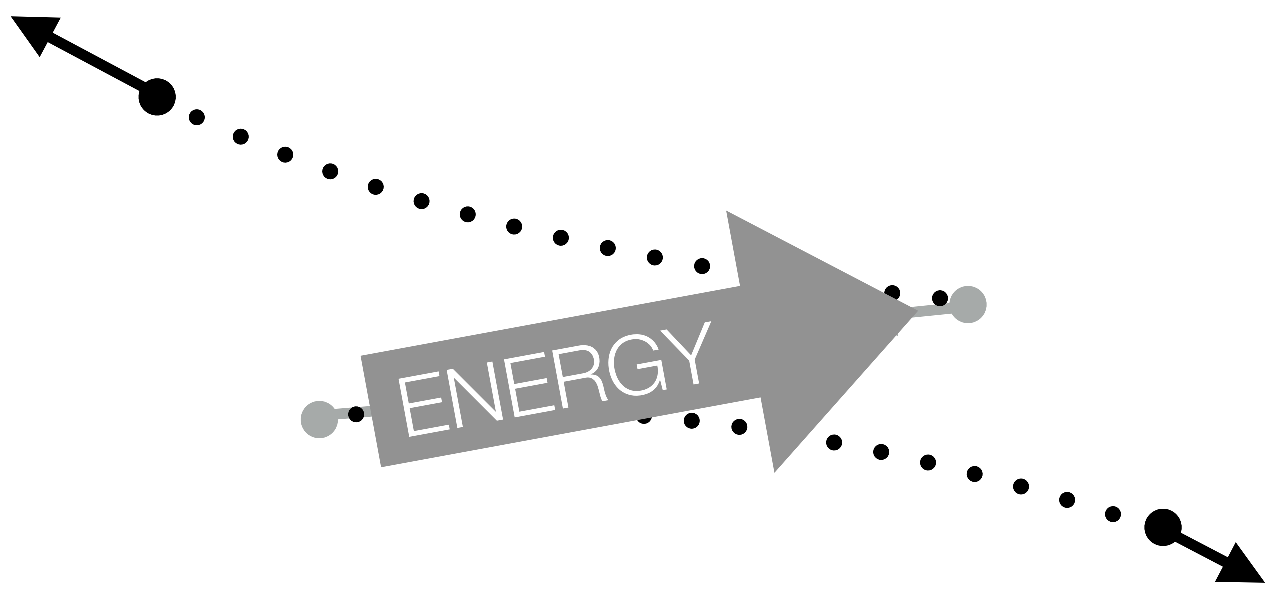
Cluster of stars with equal mass:



Stars deeper in the potential move faster (hot)

Core Collapse

Encounters of fast and slow stars:

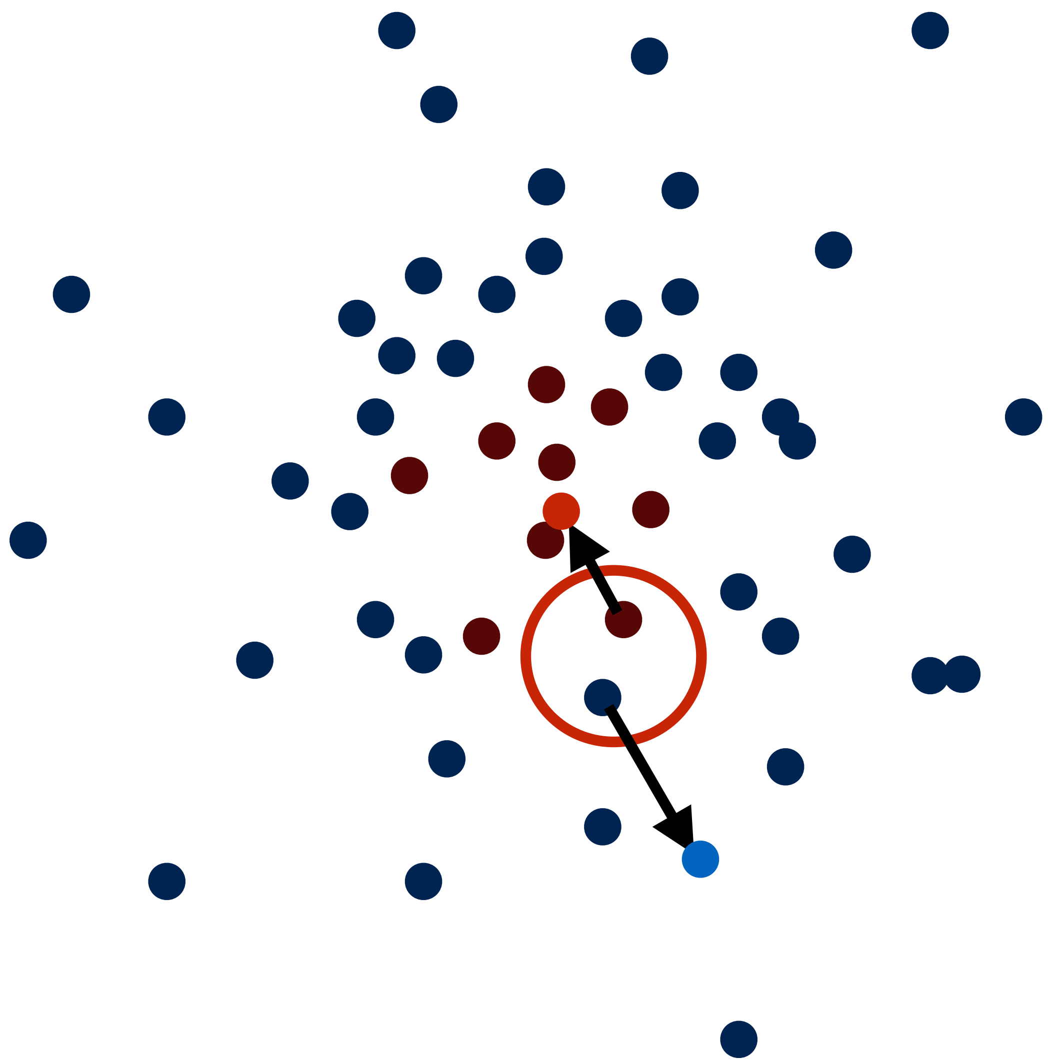


$$\vec{P} = M_1 \cdot \vec{v}_1 + M_2 \cdot \vec{v}_2 = const.$$

➡ Slow star gets faster, fast star gets slower

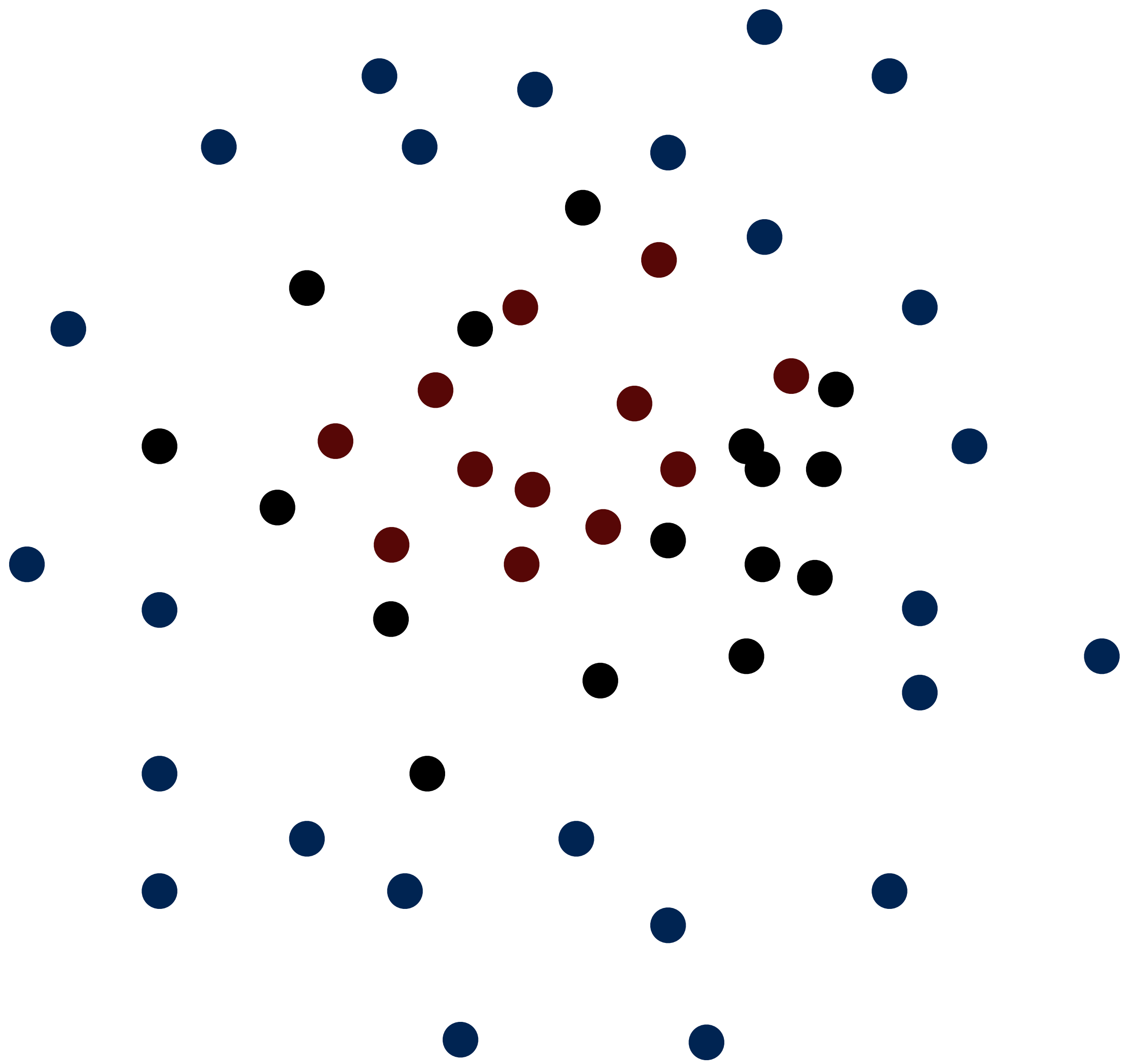
Core Collapse

Fast star: looses energy ⇒ sinks deeper in the potential well ⇒ gains speed ⇒ becomes **even faster (hotter)**



Slow star: gains energy ⇒ climbs out of the potential well, ⇒ looses speed ⇒ becomes **even slower (colder)**

Core Collapse



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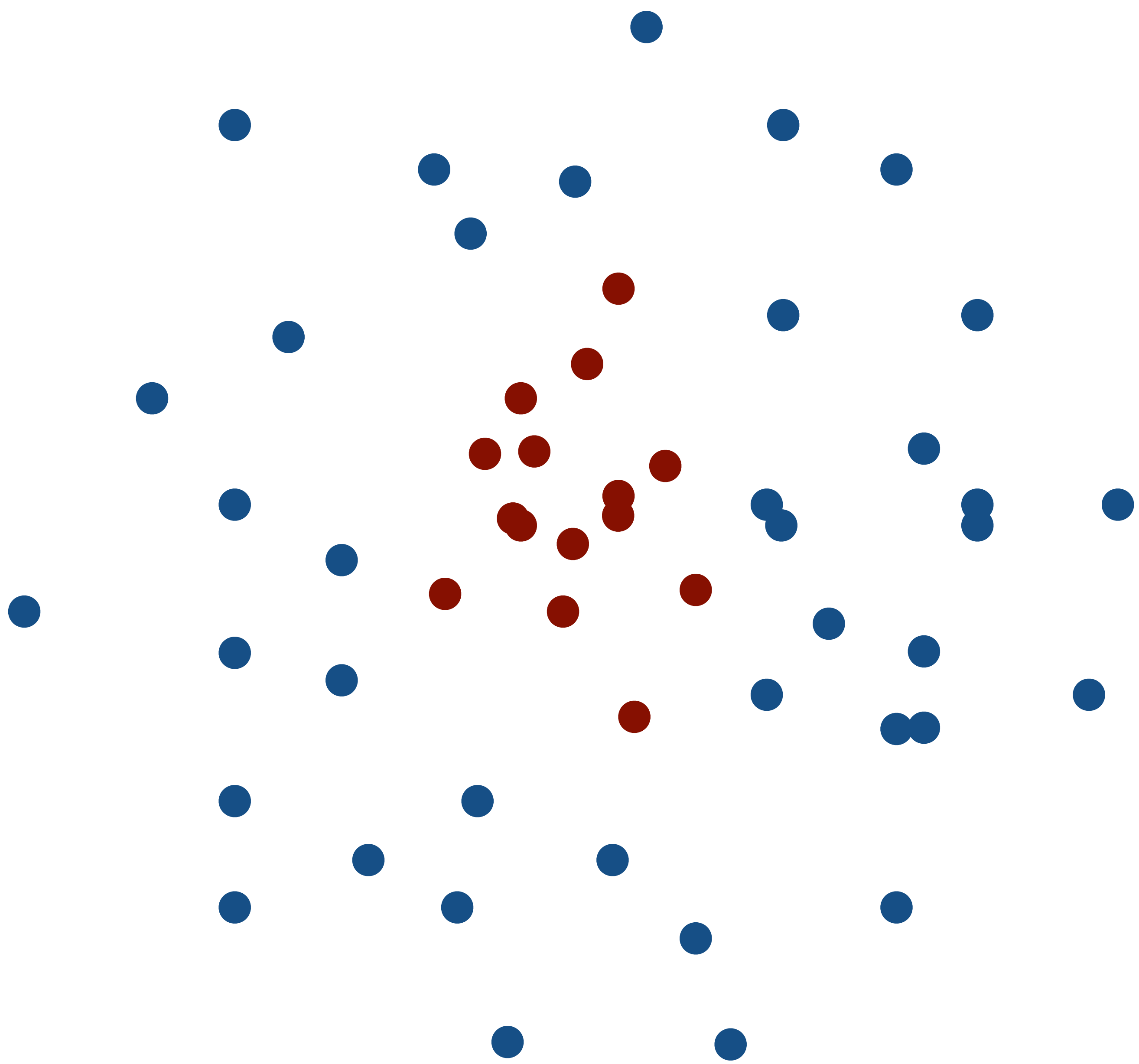

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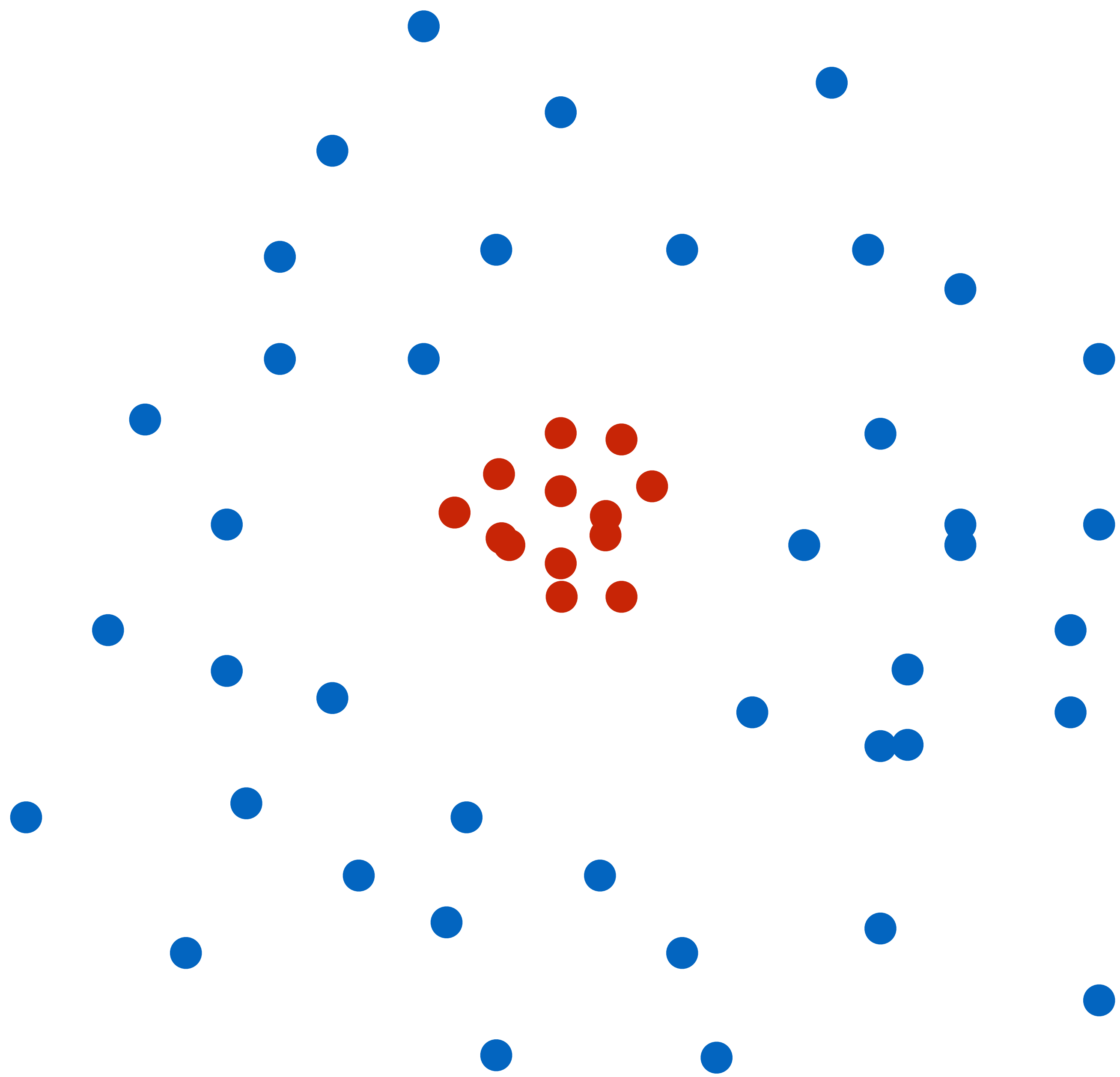

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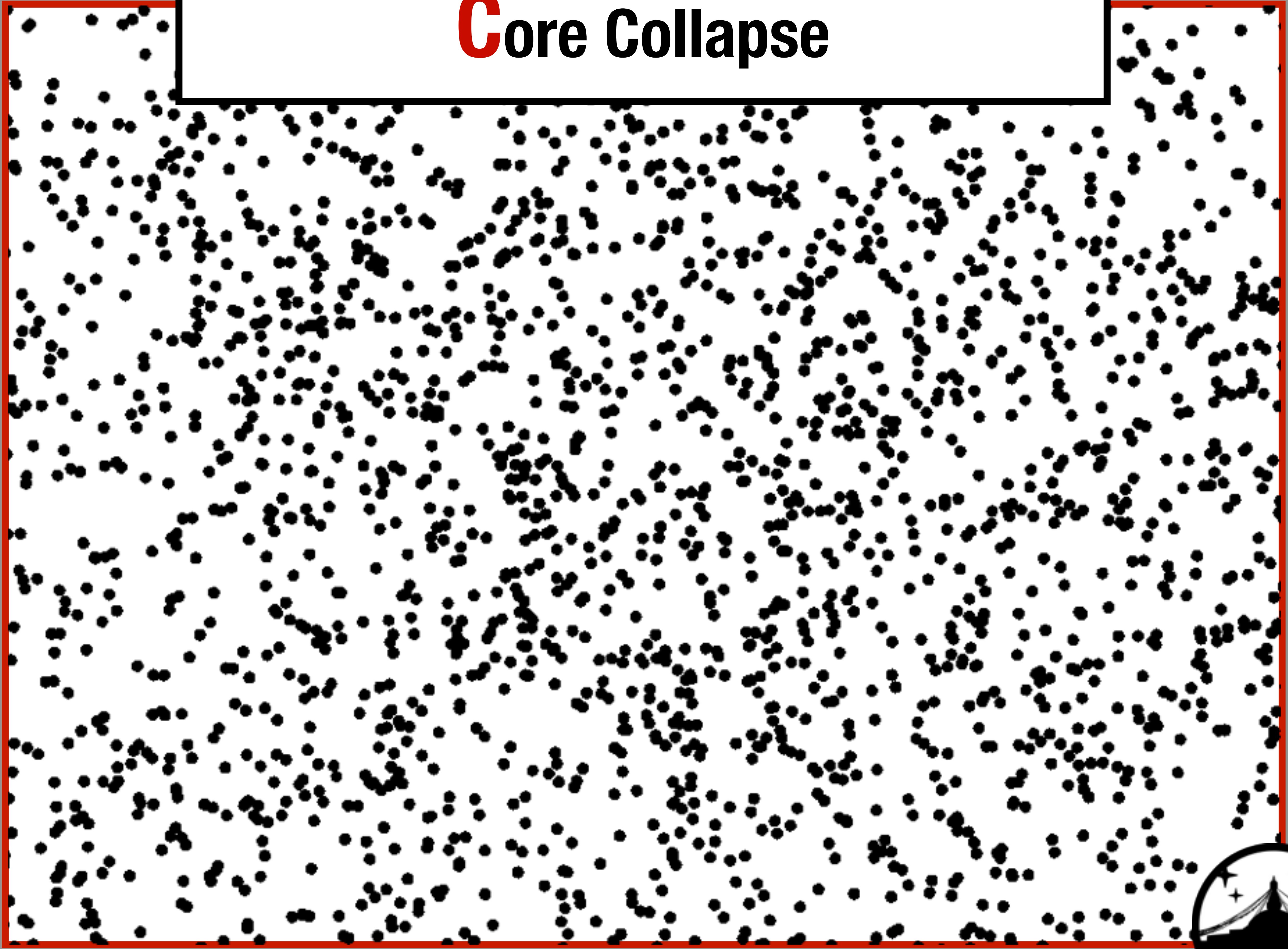

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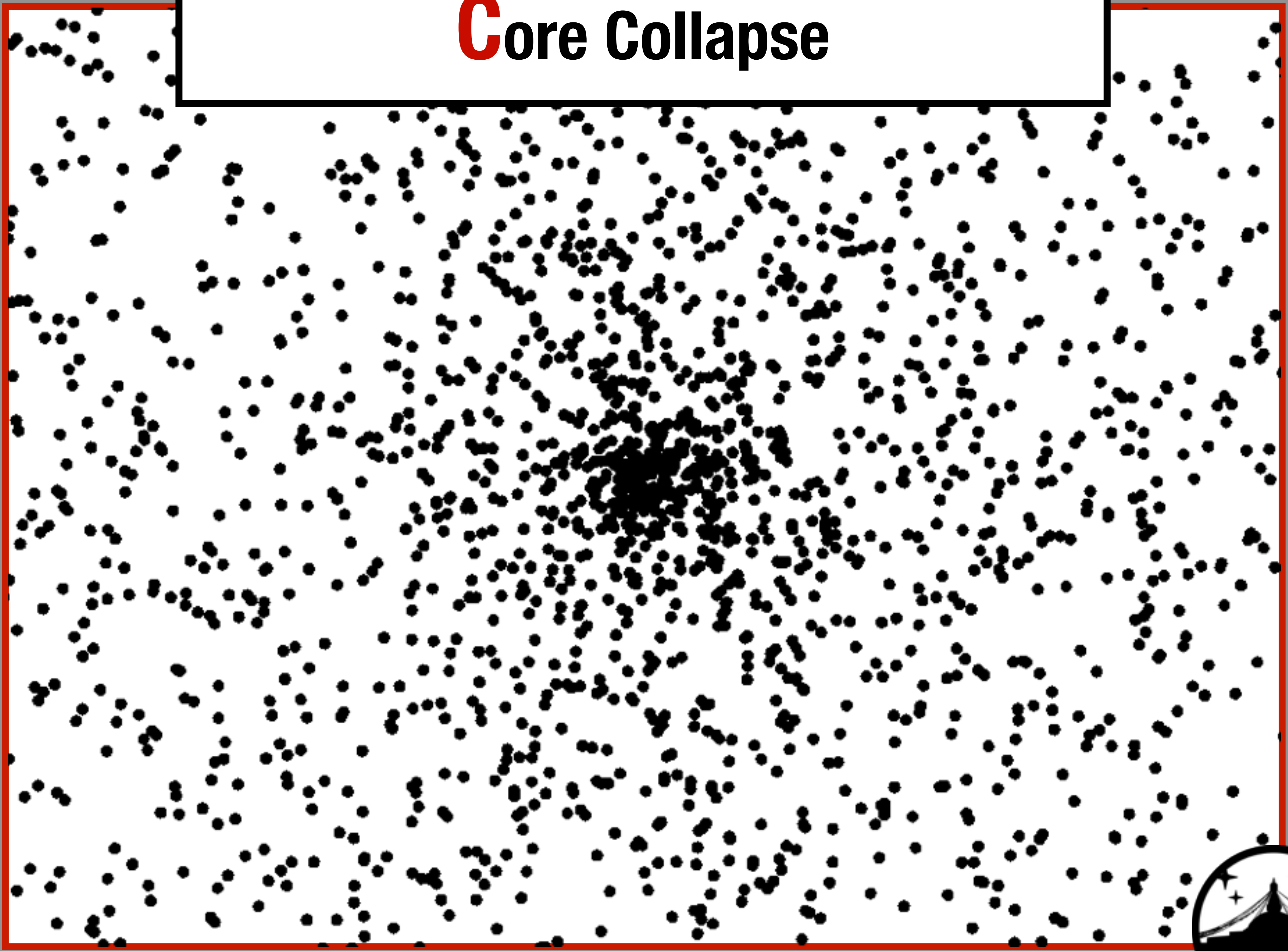

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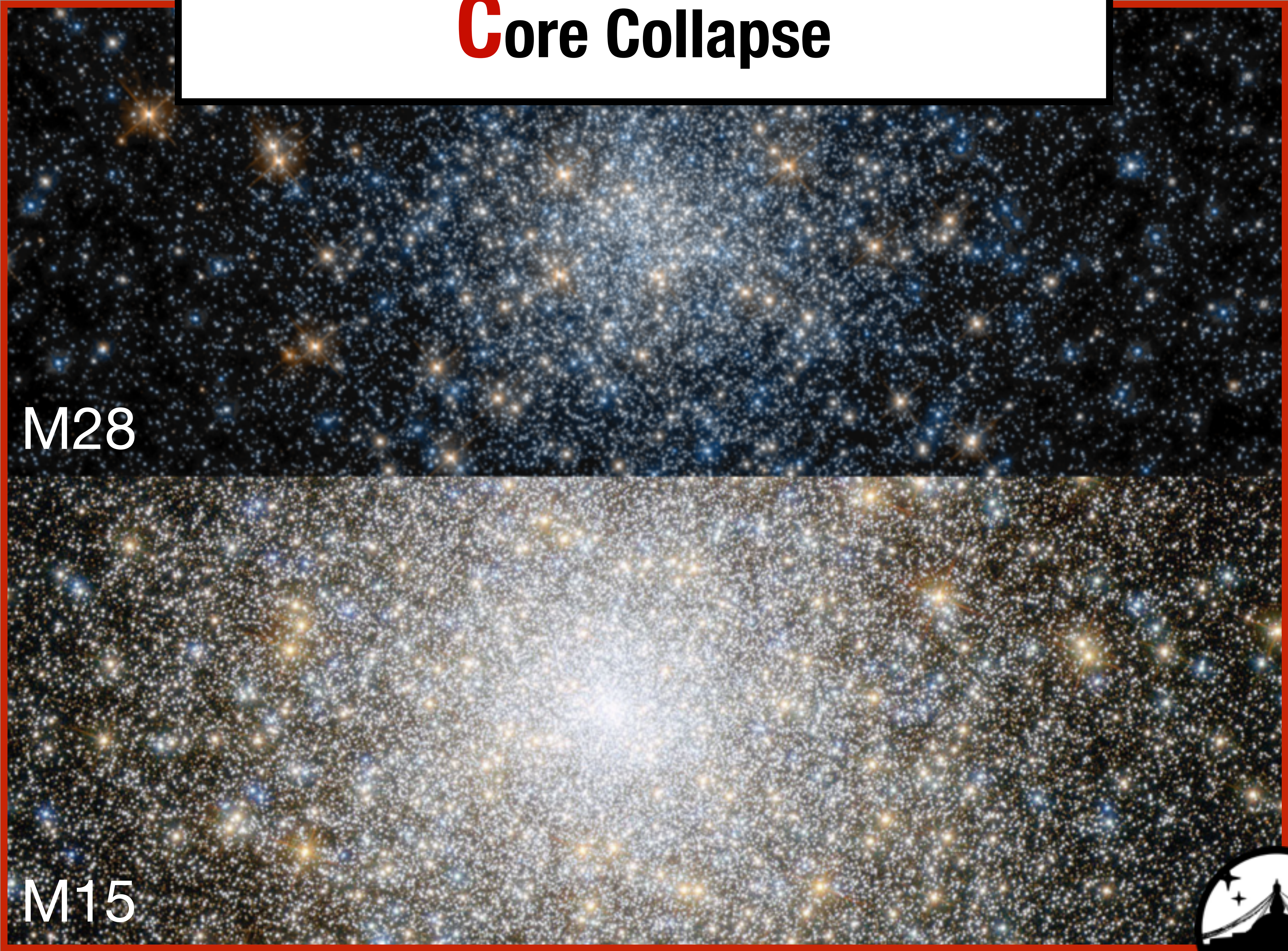

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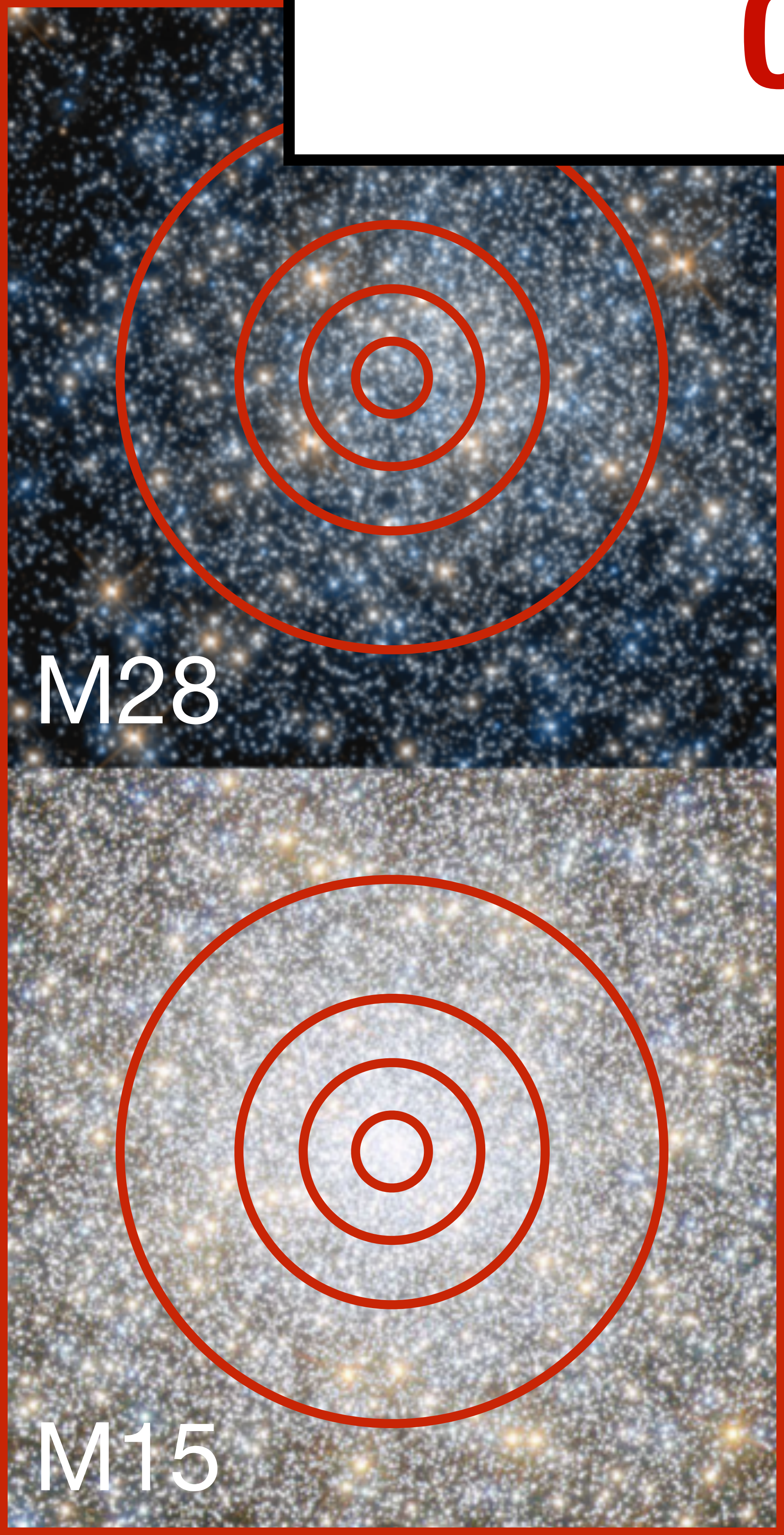

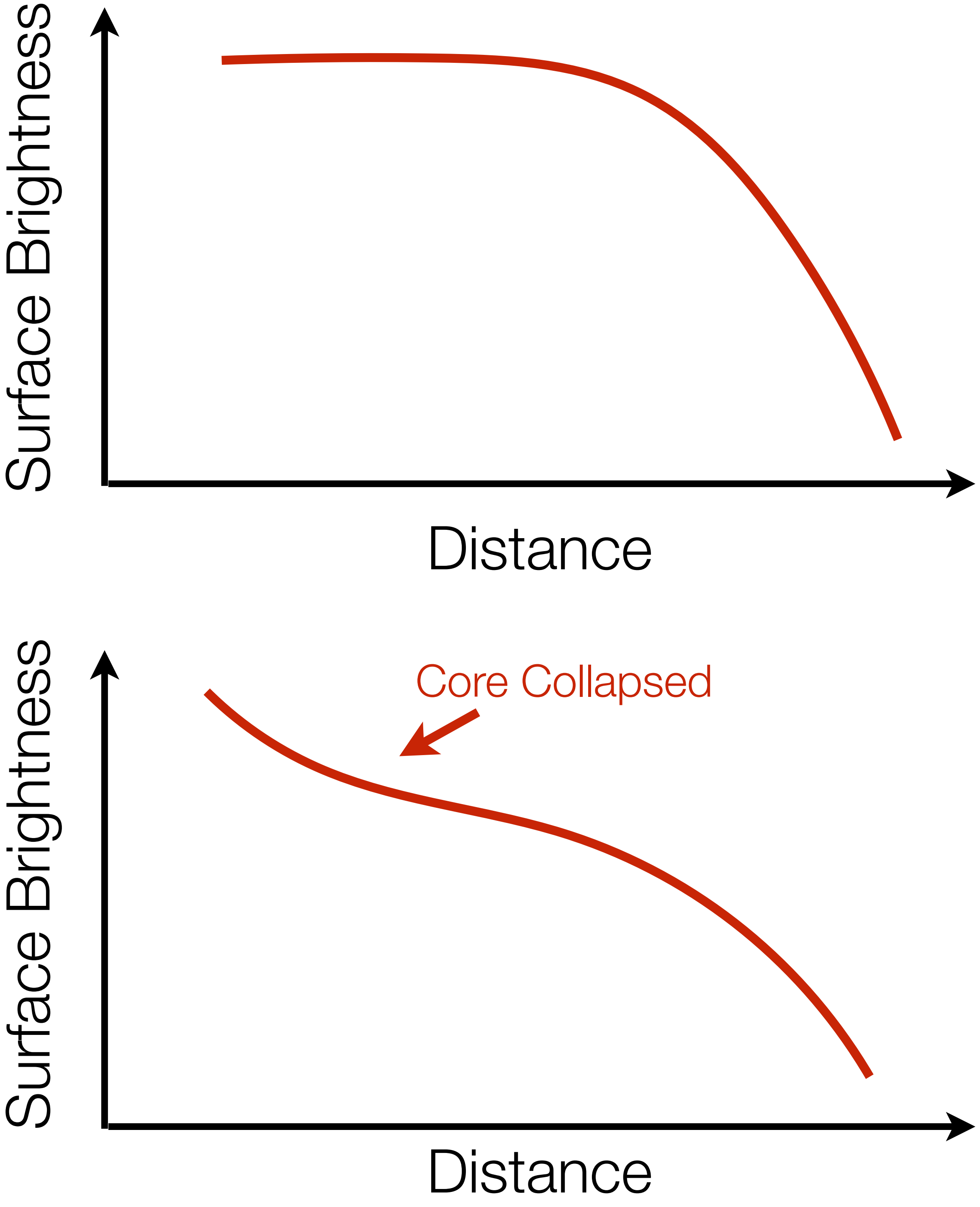

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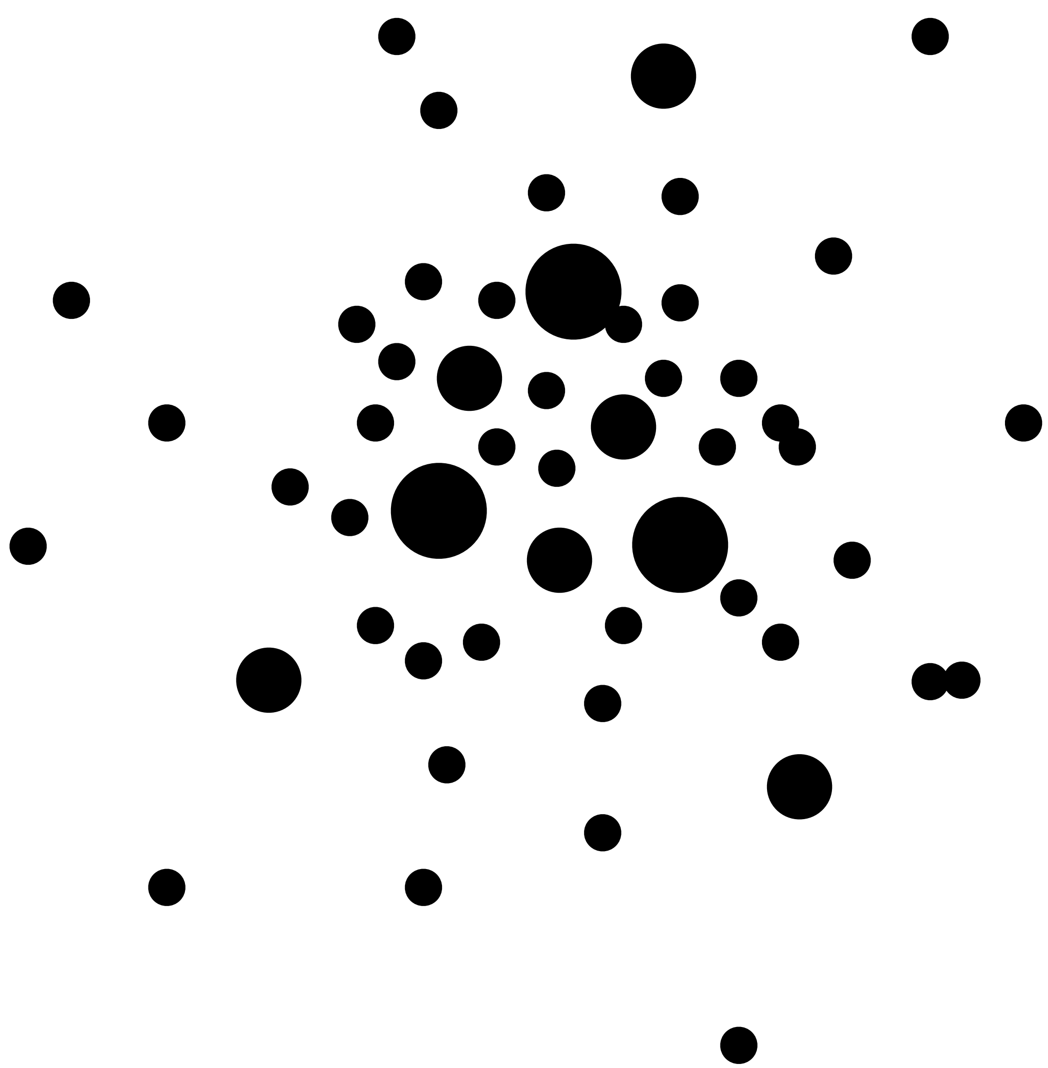
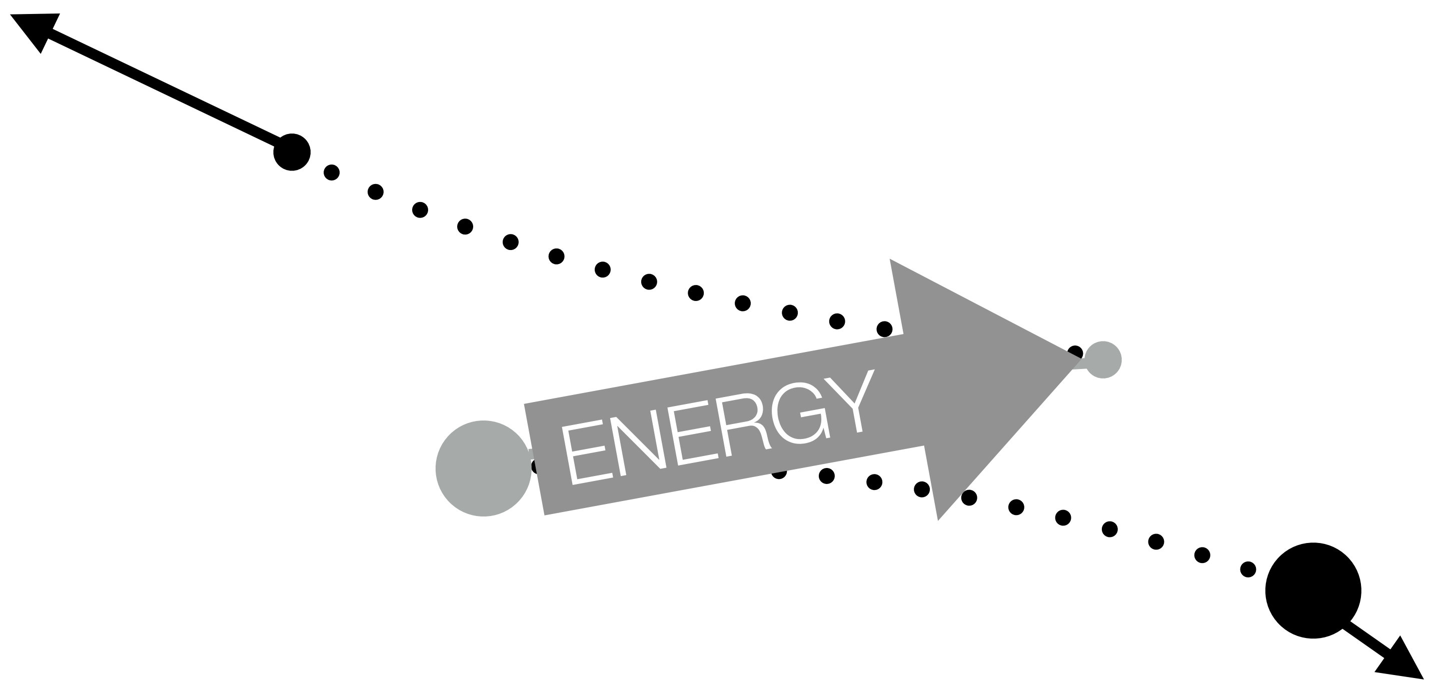
Equipartition of Energies

Cluster of stars with UN - equal mass:



Equipartition of Energies

Encounters of high-mass and low-mass stars:

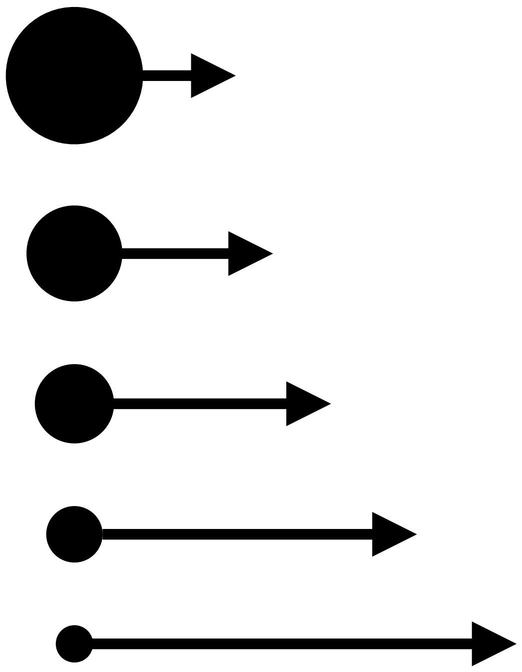


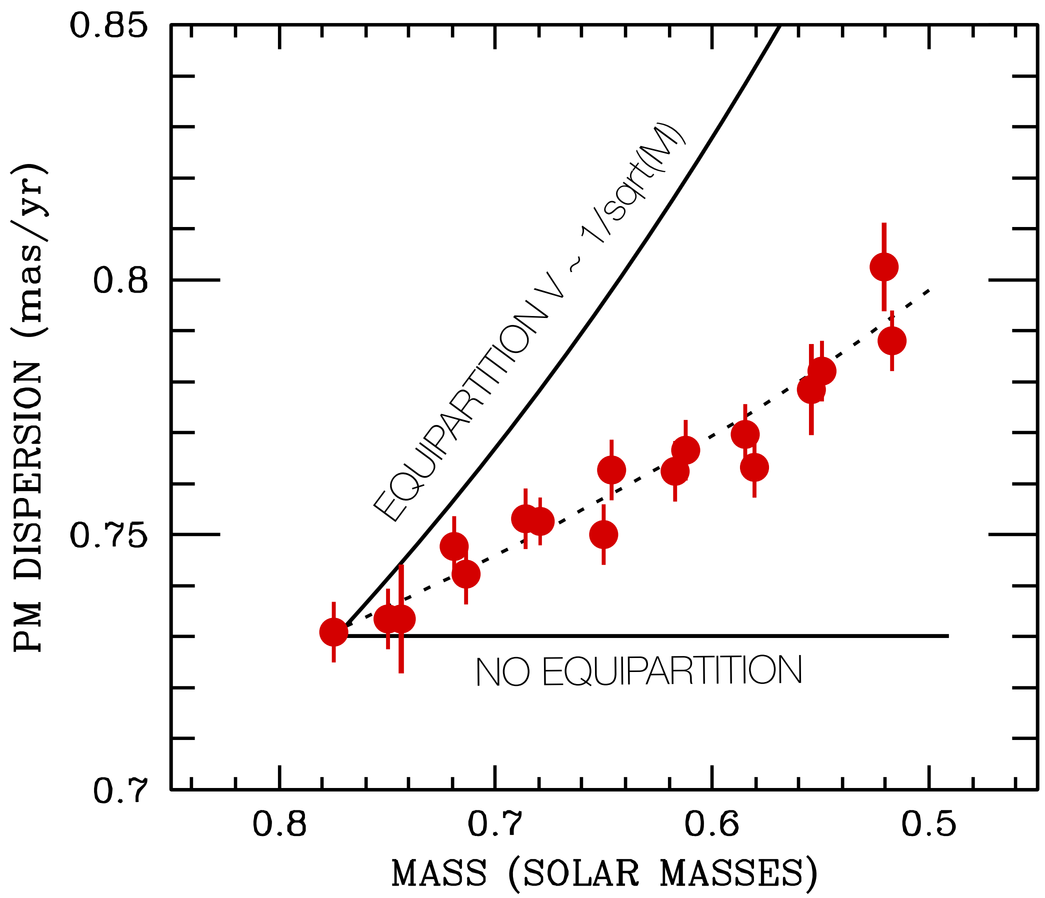
$$\vec{P} = M_1 \cdot \vec{v}_1 + M_2 \cdot \vec{v}_2 = const.$$

- Low-mass star gets faster, high-mass star gets slower
- Kinetic energies become more equal $K_i \sim \frac{M_i}{2} v_i^2$

Equipartition of Energies

- When all stars (at radius R) have the same kinetic energy
- High-mass stars are slow, low-mass stars are fast





Anderson & van der Marel, 2010

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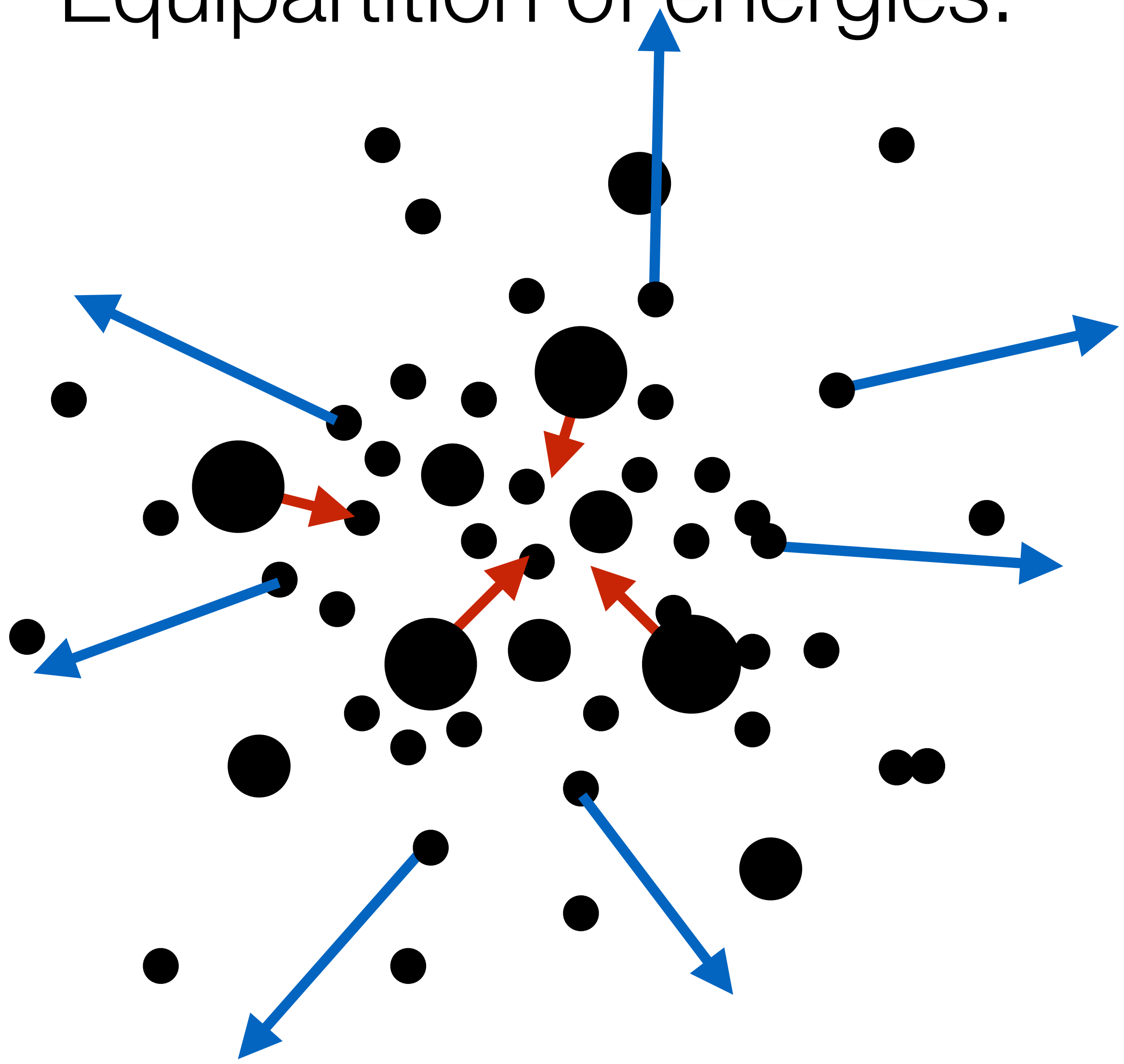
Mass Segregation

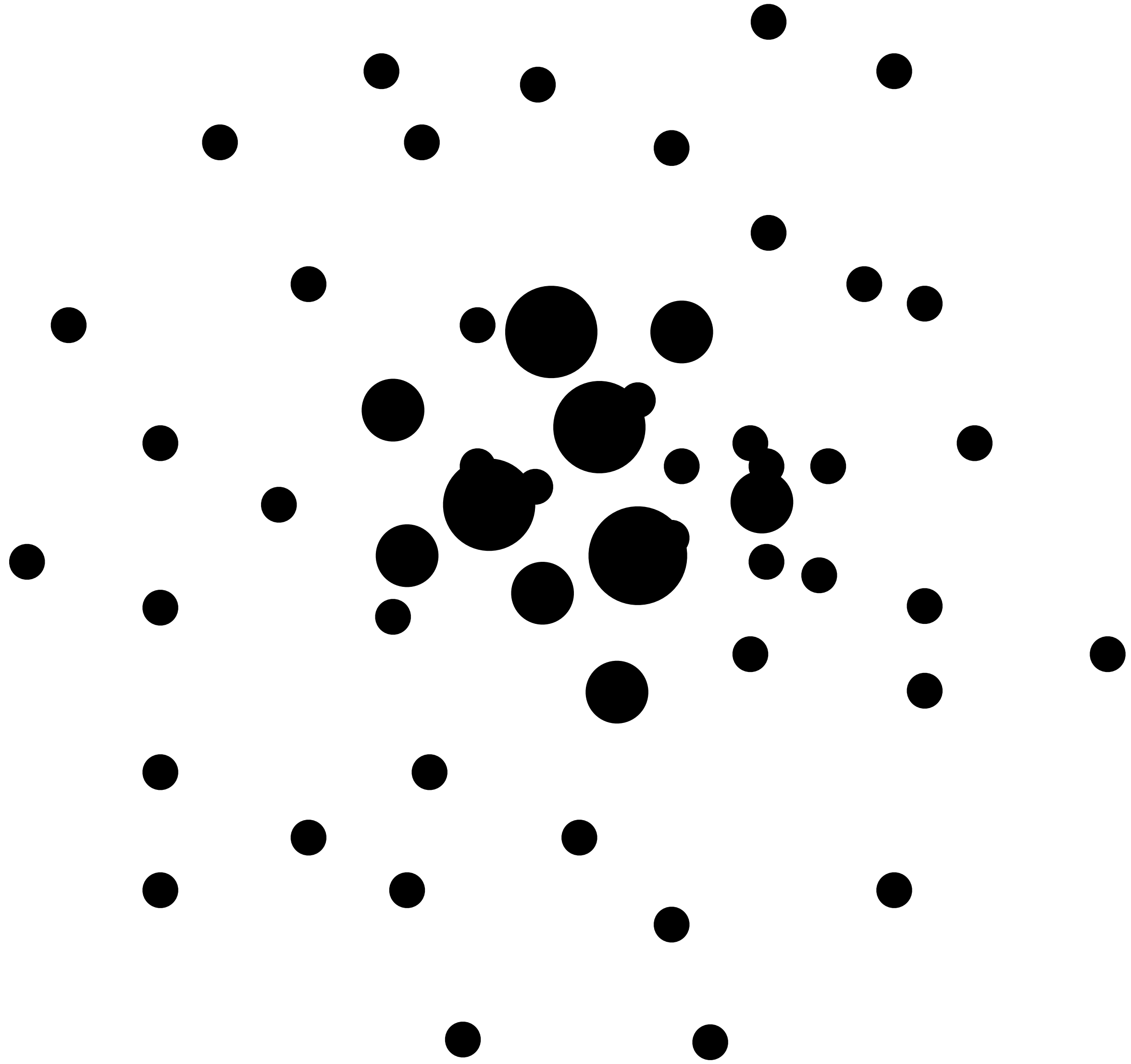
Equipartition of energies:

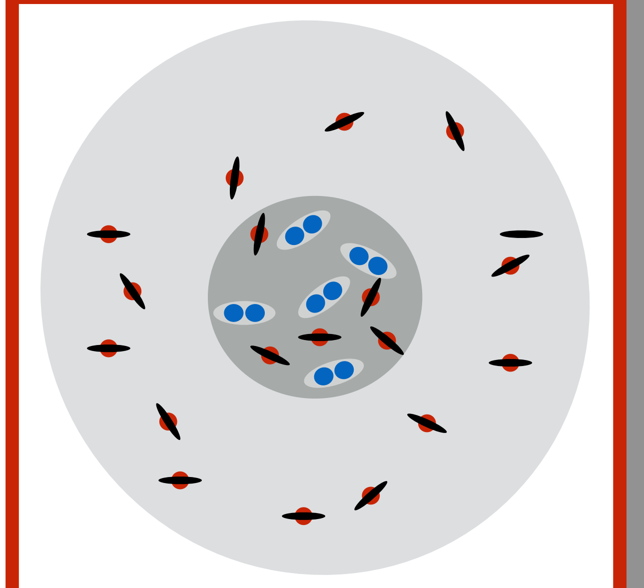


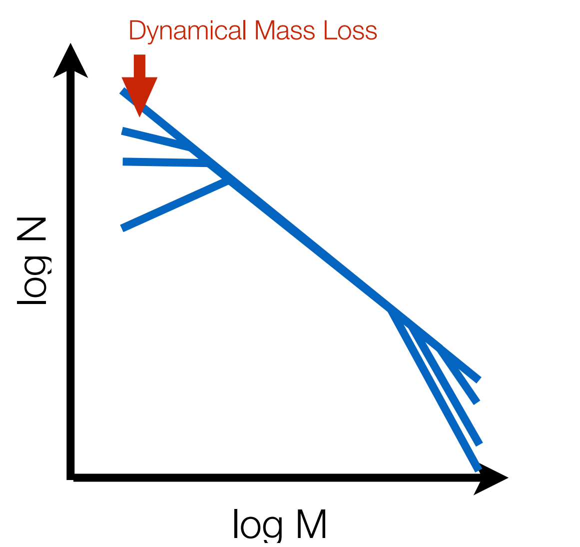
- High-mass stars sink to the center
- Low-mass stars rise to the outskirts

Mass Segregation

Mass gradient from center to the outskirts







Summary - 1

1. **The Gravitational N-body problem**
    - N=2: exactly solvable
    - N=3: approximately solvable
    - N>3: only numerical solvable
2. **Dynamic Equilibrium**
    - No EXPANSION or CONTRACTION of the system
3. **Negative Heat Capacity**
    - Remove energy —> hotter
    - Gain energy —> colder

Summary - 2

4. **Core Collapse**
   - Very condensed core, steep light profile
5. **Equipartition of Energies**
   - All the stars (at radius R) have the same kinetic energy
   - High-mass stars: slow, low-mass stars: fast
6. **Mass Segregation**
   - Mass gradient from center to the outskirts