Atoms and lasers: absorption

- Consider an atom, minding its own business
- The atom has two (or more) discrete (quantised) energy levels, \( g \) (ground) and \( e \) (excited), separated by energy \( \Delta E \)

\[
\Delta E = hf
\]

- Along comes a photon from a laser
- The atom is choosy: it ignores the photon unless the frequency \( f \) is just right:

\[
\Delta E = hf
\]

Photons have angular momentum

- Photons have angular momentum (in the same way that electrons have charge)
- Photon angular momentum = \( \pm \hbar \)
- Completely independent of photon energy and linear momentum
- Often called “spin”. Direction equivalent to polarisation.

Emission by spontaneous decay

- After some time, the atom decays, emitting a photon

\[
\rho_{\text{read}} = \frac{h}{\lambda}
\]

- Spontaneous emission is random, disordered

- Conventional light sources *disordered*
Stimulated emission

- Stimulated emission: photon interacts with atom in excited state, triggers emission of an identical photon
- Same wavelength, direction, phase, polarisation

Lasers: population inversion

- Many atoms excited (more than in ground state) — Population inversion — leads to: Light Amplification by Stimulated Emission of Radiation

Einstein A, B coefficients

- Consider three processes:
  - Absorption
    \[
    \frac{dN_a}{dt} = -B_a N_a u(f)
    \]
  - Spontaneous emission
    \[
    \frac{dN_e}{dt} = -A_e N_e
    \]
  - Stimulated emission
    \[
    \frac{dN_s}{dt} = -B_s N_s u(f)
    \]
- In thermal equilibrium:
  \[
  B_a N_a u(f) = B_s N_s u(f) + A_s N_s
  \]
- \( u(f) \) is the spectral energy density
- \( A, B \) are the Einstein A and B coefficients

Absorption vs Emission

- Two-level atoms in thermal equilibrium with black body radiation, temp \( T \)
  \[
  B_a N_a u(f) = B_s N_s u(f) + A_s N_s
  \] (1)
- Planck’s radiation law
  \[
  u(f) = \frac{8\pi\hbar f^3}{c^2} \frac{1}{e^{\frac{\hbar f}{kT}} - 1}
  \] (2)
- Maxwell-Boltzmann distribution gives \( N_{\text{excited}} \) vs \( N_{\text{ground}} \)
  \[
  N_e = N_g e^{-\frac{\hbar f}{kT}}
  \] (3)
- Solve (1) for \( u(f) \) and substitute (3)
  \[
  u(f) = \frac{A_s}{B_a} \frac{8\pi\hbar f^3}{c^2} e^{-\frac{\hbar f}{kT}} \left[ -1 \right]
  \] (4)
- Equate to (2), i.e. Planck radiation, and find
  \[
  \left[ \frac{A_s}{B_a} \frac{8\pi\hbar f^3}{c^2} \right] e^{-\frac{\hbar f}{kT}} = 0
  \]
  
  \[
  \left[ \frac{A_s}{B_a} \frac{8\pi\hbar f^3}{c^2} \right] e^{-\frac{\hbar f}{kT}} = 0
  \]
- Must be true for all \( T \) so \[ \] terms must be zero
  \[
  B_a = B_s \quad \text{probability of stimulated emission} = \text{probability of stimulated absorption}
  \]
- Thus
  \[
  \frac{A}{B} = \frac{8\pi\hbar f^3}{c^2} e^{-\frac{\hbar f}{kT}}
  \]
- Not negative
- Probability of spontaneous emission is proportional to probability of absorption
- \( [N_g - N_e] u(f) = \frac{A}{B} N_g \frac{8\pi\hbar f^3}{c^2} N_e \) so how do we get population inversion???
Three-level system (e.g. ruby)

- First laser, T.H. Maiman, 1960: flashlamp-pumped ruby
- Ruby = Al₂O₃ + chromium ions (Cr³⁺)

Energy (eV)

<table>
<thead>
<tr>
<th>Photon</th>
<th>694.3nm</th>
<th>Green</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relaxation</td>
<td></td>
<td>Stimulated emission</td>
</tr>
<tr>
<td>Ground state</td>
<td>Blue</td>
<td>Absorption</td>
</tr>
<tr>
<td>Metastable states</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Four-level system (e.g. Ar⁺)

- Improve population inversion with 4-level system
- Excitation to E₂: E₃ always = empty
- Relaxation to E₁: E₂ metastable
  - Normal lifetimes = 10⁻⁸s; metastable = 10⁻³s
  - Population "piles up" in E₂
- Photon at E₂-E₁ can be absorbed or stimulate emission
  - Population inversion ⇒ N₂ > N₁
  - Rate of absorption: BₑNₑ(ν) = BₑNₑ(ν(f))
  - Rate of stimulated emission: BᵣNᵣ(ν) = BᵣNᵣ(ν(f))
  - E₁ quickly depopulates so atom can play again

Laser gain

- Consider plane wave, frequency f, passing through lasing medium in +z direction

In vacuum:
- Rate EM energy passes through cross-section A at z is I(z)A
- At adjacent plane z+Δz rate is I(z+Δz)A & diff is 
  \[ \partial I(z)A/\partial z = \frac{\partial}{\partial z} [I(z)A] \]
- This is rate at which EM energy leaves the volume AΔz
  \[ \frac{1}{c} \partial \Phi \sigma(\nu) A dz = I(z-\Delta z) A \frac{\partial}{\partial z} [I(z)A] \]
  \[ \partial I(z)A/\partial z = \frac{1}{c} I(z-\Delta z) A \partial \Phi \sigma(\nu) \partial z \]

In laser medium:
- absorption=energy loss from field; stimulated emission=energy gain to field

Absorption rate

- Recall 
  \[ \frac{dN_i}{dt} = -R_{\nu N_i} \nu(f) \]
  \[ \frac{dN_i}{dt} = -N_i \frac{\partial}{\partial t} N_i \]
  \[ \frac{dN_i}{dt} = -B_{\nu N_i} \nu(f) \]

- Rate of stimulated emission/absorption is \( R_{\nu} = \Phi \sigma(\nu) \) where
  - \( \Phi \) is photon flux (photons/m²/s)
  - \( \sigma(\nu) \) is atomic probability of absorption ("cross section")

  \[ \sigma(\nu) \equiv \frac{\text{energy absorption rate per ground state atom}}{\text{incident radiant energy flux}} \]

  Thus we can write 
  \[ \frac{dN_i}{dt} = A N_i + \Phi \sigma(\nu)[N_i - N_i] \]

  Each change in ground state population = one photon energy change, \( \Delta E = h\nu \)

  Thus \( \sigma(\nu) = \frac{hf R_{\nu}}{I_{\nu}} \) and 
  \[ \frac{1}{c} I_{\nu} \frac{\partial}{\partial z} I_{\nu} = \sigma(\nu)[N_i - N_i] \]
Cross-section and lineshape

- On-resonance $\sigma(f_\text{res}) = \frac{\lambda^2}{2\pi}$ but depends on frequency relative to atomic resonance
- Atomic resonance usually Lorentzian with linewidth $\Gamma$: $\sigma(f) = \frac{\Gamma}{(f-f_\text{res})^2 + \Gamma^2}$
- For arbitrary atomic lineshape† $S(f)$: $\sigma(f) = \frac{hf}{I_f} \frac{R_\text{eg}^2}{c u(f)} - \frac{hf}{c} B S(f)$

![Graph of Gaussian and Lorentzian lineshapes]

† Note $S(f)$ as used here is not related to Poynting vector

Gain coefficient

- We have $\left(\frac{1}{c} \frac{\partial}{\partial t} + \frac{1}{c} \frac{\partial}{\partial x}\right) I_f = \sigma(f) [N_e - N_g] I_f$
- Rewrite as $\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) I_f = g(f) I_f$, so the gain coefficient is $g(f) = \sigma(f) \frac{N_e - N_g}{8\pi}$

- In temporal steady state $\frac{d I_f}{dt} = g(f) I_f$, so $g(f)$ gives rate of growth of $I_f$
- Solution $I_f = I_f(0) e^{g(f)t}$: but only in weak field limit
- $g(f)$ is only positive if $N_e > N_g$
- $g(f)$ increases with $f$ but that reduces excited-state population: 3-level needed

- Caveats: modifications are necessary for refractive index of medium, and if there are degenerate excited or ground states:
  
  $g(f) = \frac{\lambda}{8\pi} \left[N_e - \frac{g_e}{g_g} N_g\right] S(f)$

Laser light properties

- Lasers are special:
  - Monochromatic
  - Temporally coherent
  - Spatially coherent
    - Directional
    - Bright
    - Focusable

Monochromatic

- Lasers are almost perfectly pure in colour: monochromatic
- Perfect? Impossible: requires infinitely long wavetrain
- Why are lasers so monochromatic?
- Several things combine:
  1. Narrow linewidth of atomic transitions
  2. Stimulated emission process
  3. Laser cavity frequency selection

![Graph showing laser light properties]

Monochromaticity

- Classical light source (hot filament) $\rightarrow$ continuous black-body spectrum
- Atomic transitions narrow – but do have a finite linewidth, $\Gamma$

![Graph showing stimulated emission process]

- Spontaneous emission
- Fluorescent sources emit light with this bandwidth
  - E.g. fluoro tubes, fluorescent pens, LEDs (light emitting diodes), glow from HeNe tube
- Stimulated emission: new photon has identical frequency, but which frequency is selected?
Optical cavities

- Most of initial photons in laser escape, but some are reflected by Fabry-Perot
- Only these are amplified by spontaneous emission → LASER
- In cavity, only certain frequencies sustained (like sound in organ pipe):
  - Standing wave: integer number \( m \) of half-waves in length \( L \):
    \[ m = \frac{L}{\lambda/2} \]
  - Frequency \( f_n = \frac{mv}{2L} \) where \( v \) is the velocity (e.g. \( c \))
  - Adjacent longitudinal cavity modes separated by constant difference:
    \[ f_{n+1} - f_n = \frac{v}{2L} \]
  - Example: gas laser \( L = 300\text{mm} \) gives \( \Delta f = 500\text{MHz} \)

Multibeam interference

- Consider all the beams
- Simplify: assume \( n_1 = n_2 \)
- Define amplitude reflection, transmission coefficients \( r, r', t, t' \)

\[ \begin{align*}
\text{Incident and subsequent reflected rays:} & \\
E_0 & \rightarrow E_{r}\rightarrow E_{r'} \rightarrow E_{rr'} \rightarrow E_{r't'} \rightarrow E_{t'} \rightarrow E_{t'} \\
\end{align*} \]

- We know \( r = -r' \) (phase shift on reflection)
- Phase difference between adjacent rays is \( \delta = k\Lambda \) where \( \Lambda = 2\pi/d\cos\theta \)
- Incident and subsequent reflected rays:
  \[ 
  \begin{align*}
  E_0 &= E_0 e^{im\phi} \\
  E_1 &= E_1 e^{im\phi} \\
  E_{r1} &= E_{r1} e^{im\phi} \\
  E_{t1} &= E_{t1} e^{im\phi} \\
  E_{r2} &= E_{r2} e^{im\phi} \\
  E_{t2} &= E_{t2} e^{im\phi} \\
  
  \end{align*} \]

- Take sum, rearrange → geometric series

\[ I_k = I_k \left( \frac{1}{1 - r^2} \right) \]

- Reflected irradiance (using \( 2\cos^2(x) = e^{ix} + e^{-ix} \)):

\[ I_k = I_k \left( \frac{2r^2(1 - \cos\delta)}{1 + r^2 - 2r^2 \cos\delta} \right) \]

Airy function

- Fringe visibility
  - Maximum reflected when \( \delta = (2m + 1)\pi \) (as for 2-beam)
    \[ I_k = I_k \left( \frac{4r^2}{(1 + r^2)} \right) \]
    \[ I_r = I_r \left( \frac{1 - r^2}{(1 + r^2)} \right) \]

  - Define coefficient of finesse, \( F \):
    \[ F = \frac{2r^2}{(1 - r^2)} \]
    \[ I_k = I_k \left( \frac{F \sin^2(\delta/2)}{1 + F \sin^2(\delta/2)} \right) \]
    \[ I_r = I_r \left( \frac{1}{1 + F \sin^2(\delta/2)} \right) \]

  - The term \( \phi \theta / (1 + F \sin^2(\delta/2)) \) is known as the Airy function which represents the transmitted flux-density distribution
  - Multi-beam interference redistributes the energy from sinusoidal (2-beam) to Airy function
  - Note \( \delta \) is related to \( \theta \) or \( \phi \)
Fabry-Perot interferometer

- Note narrow spikes of Airy function: wavelength selection \( \rightarrow \) spectroscopic tool
- Charles Fabry and Alfred Perot, late 1800’s
  - Used to discover ozone layer
  - Also important in spectroscopy of stars
    - \( \rightarrow \) velocities
    - \( \rightarrow \) galactic structure
- Optical basis of lasers
  - Narrow source
  - Rays at same angle from source
    - \( \rightarrow \) single circular fringe at screen
  - Different angles
    - \( \rightarrow \) concentric circles

\[
\text{Fringe width measured as FWHM (full width at half maximum).}
\]

\[
\text{Airy function drops to half max when } \delta = \delta_{\text{max}} \pm \frac{\delta}{\sqrt{2}}:
\]

\[
\text{Fringe spacing or free spectral range, } \text{FSR} \approx \frac{c}{2n_f d}
\]

\[
\text{Often refer to ratio of FSR to width, finesse } \mathcal{F} = \frac{2\pi}{\gamma}:
\]

\[
\text{Flat mirrors } \rightarrow \mathcal{F} \text{ about 30}
\]

\[
\text{Curved mirrors now } \mathcal{F} > 1 \text{ million}
\]

\[
\text{Homework: what is reflectivity } R = r^2 \text{ in that case?}
\]

\[
\text{Homework: if input power is } P_{\text{in}}, \text{ what is } P_{\text{c}}, \text{ power in cavity?}
\]

Frequency selection

- Laser gain bandwidth typically broad compared to cavity lineshape
- Resonator cavity selects laser frequency to one or more longitudinal modes
- Thus Fabry-Perot cavity is the basis of laser monochromaticity

\[
\text{Example #1}
\]

My laser diodes have gain bandwidth of approx 15 to 20nm yet laser linewidth is smaller than 300kHz. Cavity length \( \approx 30 \) mm so cavity spacing is 5GHz. Diffraction grating selects only one longitudinal mode

\[
\text{Example #2}
\]

Small HeNe laser in my lab (wattmeter), length 125mm, cavity spacing 1.2GHz, gain bandwidth \( =1.5 \) GHz \( \Rightarrow \) only one cavity mode sees gain. High reflectivity mirrors give \( \Delta f = 100 \) kHz

Coherence

- Coherence = correlation in phase between field at two points in time and space
- Temporal coherence measures monochromaticity of light
- Spatial coherence measures uniformity of phase across an optical wavefront

- Conventional source (lightbulb) is broad spatially and emits light at many wavelengths
  - Obtain temporal coherence with a filter
  - Spatial coherence with a pinhole aperture

- Laser light is highly coherent in both senses
- Define coherence time and coherence length
  - Coherence time \( \tau_c \) is time interval over which phase can be predicted
  - Coherence length \( L_c = \frac{c\tau_c}{\Gamma} \)
  - For Lorentzian linewidth \( \Gamma \):
    - \( \tau_c = \frac{1}{\pi \Gamma} \)

\[
\text{Example: HeNe laser } \Gamma = 100 \text{ kHz, coherence length } = 1 \text{ km. Can have interferometer with path-length difference up to 1km and still see fringes!}
\]
Directionality

- Pencil-like laser beams remarkably ray-like
- Possible because of coherence + cavity design
- Curved mirrors produce focused beam, with waist at focal centre of cavity

\[
I = I_0 e^{-2r^2/w^2}
\]
where \( r \) is radius, \( w \) is beam half-width

- Beam width is radius such that beam amplitude falls to \( 1/e \) and intensity to \( 1/e^2 \)
- Minimum beam waist \( w_0 \) is at centre for confocal cavity
  - Confocal means \( R_1 = R_2 = L \) (two foci overlap)
  - In-cavity profile is self-consistent and external beam maintains same profile, with beam width \( w(z) = w_0 \left[ 1 + \left( \frac{zL}{w_0^2} \right) \right]^{1/2} \)

Transverse modes

- Intracavity mode is not uniform, and not (generally) cylindrically symmetric
- Known as TEM\(_{m,n}\) modes where \( m, n \) refer to number of transverse nodal lines along \( x, y \) directions
- Prefer TEM\(_{00}\) Gaussian profile beam:
  - Gaussian cross section \( e^{-2r^2/w^2} \)
  - No phase shifts in electric field across beam (spatially coherent)
  - Smallest angular divergence
  - Can be focused to smallest spot size

\[
\text{Spectral brightness is even more dramatic!}
\]

Source brightness

- 1mW HeNe laser is brighter than the Sun
- “Brightness”:
  - Irradiance = energy flux per unit area
  - Spectral irradiance = irradiance per unit frequency
  - Radiance = energy flux per solid angle
  - Spectral radiance = radiance per unit frequency

High-pressure mercury lamp \( L = 2.5 \times 10^7 \text{ W m}^{-2} \text{ sr}^{-1} \)

HeNe laser

\[
P = 1\text{mW}, \quad w_0 = 0.5\text{ mm}, \quad \text{divergence } \Theta = 1.6\text{ mrad}, \quad \lambda = 633\text{ nm}
\]
Solid angle is area/radius\(^2\)

- \( \Delta \Omega = \frac{1}{4} \pi \Theta^2 \)
- \( \Delta \Omega = \frac{4L}{\pi} \pi^2 \Theta^2 \)
- \( \Delta \Omega = \frac{4L}{\pi} \left( \frac{2.5 \times 10^{-15}}{6.5 \times 10^{-10}} \right) = 2.5 \times 10^4 \text{ W m}^{-2} \text{ sr}^{-1} \)

Sun

\[
P = 4 \times 10^{26} \text{ W}, \quad R = 7 \times 10^8 \text{ m}, \quad \text{radiant exitance } \frac{P}{4\pi R^2} = 6.5 \times 10^7 \text{ W m}^{-2} \text{ sr}^{-1}
\]

But light from sun into human eye (pupil \( D = 5 \text{ mm} \)): solar irradiance at Earth \( \sim 1000 \text{ W m}^{-2} \) so \( P \sim 20\text{ mW} \)
Focusability

- Minimum “diffraction limited” spot size is (Airy disc) $\sim \lambda$.
- Classical sources are incoherent: extended (not point) and broad spectrum hence do not achieve minimum.
- Laser is point source and monochromatic.
- Focus laser with lens (diameter several times bigger than laser beam diameter).
- If focal length is $f$ and laser divergence is $\Theta$ then min spot diam is $\sim f\Theta$.

Recall laser divergence is $\Theta = 2.44\frac{\lambda}{D}$ where $D$ is beam diameter.
- Best spot size then $w = 2.44\frac{f\lambda}{D}$.
- Lens defined by NA (numerical aperture) $\text{NA} = D/2f$ so $w = \frac{1.22\lambda}{\text{NA}}$.
- Best NA $\sim 1$ so spot size $\sim \lambda$.
- Note that area is therefore very small, leading to extremely high irradiance.

The HeNe laser

- HeNe laser the first CW (continuous wave) laser, $\lambda = 1.1523\mu m$.
- Requires two gases: helium is excited by discharge, neon excited by collision with helium.
- Efficiency poor ($\sim 0.01\%$).
  - Excitation of He ($20\text{eV}$).
  - Collision with Ne rather than tube.
  - Many collision outcomes; need excitation of Ne by $20\text{eV}$.
  - All to achieve $\sim 2\text{eV}$ photon.
- Many other laser types, mostly inefficient, until advent of semiconductor lasers.

Semiconductor lasers

- Also known as diode lasers, invented 1962 soon after light emitting diode (LED).
- Crucial properties:
  - Cheap.
  - Efficient (near 100\% differential efficiency).
  - Small.
  - Fast (can be modulated at GHz).
  - Long lifetime.
  - High power (KW).
- Broad wavelength range (370nm to $\sim 30\mu m$ or more).

Semiconductor lasers

- Bias voltage $\rightarrow$ electrons ($n$ layer) & holes ($p$ layer) injected into active layer.
- Direct population inversion:
  - Electrons in “excited state” i.e. conduction band.
  - Holes in “ground state” i.e. valence band.
- Electron/hole recombination in active layer produces photons.

- Gain is enormous: note that the front facet is anti-reflection coated.
- Due to perfect population inversion and very fast “pumping” rate.
Semiconductor lasers

- Not all good…
  - Very short cavity ⇒ large divergence (up to 20°)
  - Asymmetric cavity ⇒ astigmatism (e.g. 8° × 16°)
  - Band gap energy and cavity length are temperature and current sensitive
    - ~30MHz/mK and ~3MHz/µA
  - Very wide bandwidth (broad conduction band due to phonon interactions with solid)
    - Good: 784nm diode lasers just fine at 780.1nm
    - Bad: not inherently 780.1nm

Diode pumped solid-state lasers

- Diode lasers cover many wavelength regions, from blue to far infra red, but not continuous
- High power diode lasers have poor beam quality: not single-mode, poor coherence
- Can access new wavelengths and obtain high quality beams by using diode lasers to pump other materials such as:
  - Ti:sapphire
  - Nd:YAG, Nd:YLF, Nd:YVO₄, Nd:glass… up to petawatt (10¹⁵W)
  - Fibre lasers – now up to 2kW from a single-mode fibre
- Frequency double, triple → even more wavelengths
- Femtosecond pulses + holey fibres → continuum radiation