Problems Lecture 13

1. (a) Calculate the Q value for the reaction $^4\text{He}(p,d)^3\text{He}$
   (b) What is the threshold energy (lab energy) for the protons?
   (c) If the experiment was done by firing $\alpha$’s on to a target of protons, what would be the threshold energy (i.e. lab. Energy) of the $\alpha$’s?

2. Consider the creation of $^{41}\text{Ca}$ by firing a 6 MeV (Lab.) deuteron on to a target of $^{39}\text{K}$.
   (a) Show that the energy of the deuteron in the CM system is 5.71 MeV
   (b) Explain why, on energy-conservation arguments, all the reactions shown above except $^{39}\text{K} + d \rightarrow ^{38}\text{K} + t$ ($t = ^3\text{H}$) are allowed
   (c) Show, on the assumption that the deuteron approaches the target with an impact parameter equal to the target-nucleus radius, that the maximum AM brought in by the deuteron is $l=3$.
   (d) Show that the compound nucleus state can have a total AM with values up to $I = 11/2$
   (e) Show that for decay by proton emission to $^{39}\text{K}$, states with spins and parities of 0 to 8 and 0$^+$ to 9$^+$ can be formed.
Review Lecture 13

**Gamma-Ray decays**

2 Occur when a nucleus loses energy from a higher Q state to a lower one in order to gain stability.

3 \(\gamma\)-rays carry AM (QN \(\ell\))

4 The probability of a transition between states (emission of a \(\gamma\)-ray) is strongly dependent on (a) the energy (b) the AM Q No. \(\ell\) of the photon required to conserve AM in the nucleus. (Krane 10.3) Note that transition 0 \(\rightarrow\) 0 cannot occur, since the photon must carry away AM.

5 Specification of \(\gamma\)-rays by type

<table>
<thead>
<tr>
<th>Type</th>
<th>symbol</th>
<th>AM change</th>
<th>Parity change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric dipole</td>
<td>E1</td>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>Magnetic dipole</td>
<td>M1</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>Electric quadrupole</td>
<td>E2</td>
<td>2</td>
<td>No</td>
</tr>
<tr>
<td>Magnetic quadrupole</td>
<td>M2</td>
<td>2</td>
<td>Yes</td>
</tr>
</tbody>
</table>

7 **Internal Conversion (Krane section 10.6)**

When the \(\ell\) of the photon required to allow a transition is high, the transition rate may be very small. (for 0 \(\rightarrow\) 0 transition zero). For heavy nuclei (large Z) the energy quantum can be given to an orbital (most likely a k-shell) electron, and this electron is emitted with a discrete kinetic energy of E -BE.

This is more likely if the orbital electrons are close to the nucleus as they are for K-shell electrons in heavy atoms (high Z and \(r_{atom}\) is prop 1/Z).
Nuclear Reactions

In mediaeval times, the alchemists had hoped to turn lead into gold. The chemists have never succeeded. However we nuclear physicists can do that. 23% of lead is $^{205}\text{Pb}_{82}$ and gold is $^{197}\text{Au}_{79}$.

$$\begin{array}{ccc}
\text{Z} & \text{N} \\
\text{Pb} & 82 & 123 \\
\text{Au} & 79 & 118
\end{array}$$

All we need do is remove 3 protons and 5 neutrons. So a reaction like

$$^{205}\text{Pb}_{82} + \text{p}_1 \rightarrow ^{197}\text{Au}_{79} + ^4\alpha_2 + ^4\alpha_2 + ^1n$$

would do it (not very well).

However in nuclear reaction transformations of the innate nature of the nucleus occur.

Nuclear reactions are an essential part of the study of the nucleus, and are designed to reveal the spins, parities, energies of the nuclear states. Knowing these one can try to model the nucleus and hence refine our understanding of the basic N-N force between the nucleons.

The general form of a nuclear reaction is

$$\text{X} + \text{a} \rightarrow \text{Y} + \text{b}$$

or

$$\text{X(a,b)Y}$$

The projectile may be a particle (p, n, $\alpha$), a photon (photonuclear reaction) or even another nucleus (heavy ion reaction).

The ejectile may be a particle, a photon, or again a nucleus.

General rules for nuclear Reactions

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- Mass/energy is conserved

$$m_Xc^2 + T_X + m_ac^2 + T_a = m_Yc^2 + T_Y + m_bc^2 + T_b$$

That is

$$Q = [(m_X + m_a) - (m_Y + m_b)]c^2$$

Which is the same as the excess energy of the FINAL products
\[ Q = T_{\text{final}} - T_{\text{initial}} \]

if \( Q \) is positive it is exoergic.
Binding energy released as KE of products

if \( Q \) is negative the reaction is endoergic,
Initial KE becomes mass or BE

- Charge is conserved
- Baryon number is conserved.
  The number of p+n is conserved
- Linear momentum is conserved
- AM is conserved
- Parity is conserved

What is the object of performing nuclear reactions?

Ultimately it is to observe the quantum states of a nucleus, that is to define the AM, energy and parity of the quantum states that exist in a given nucleus. From this information one can use models to work backwards to better define the N-N potential, and particularly how these nucleons might be effected by being in close proximity with other nucleons.

Tools: Accelerators, detectors, Ref. Part3 Nuclear Lab.
The measurements will generally be made by bombarding the target nucleus with a projectile (say a proton, neutron, \( \alpha \) or \( \gamma \)-ray) of known energy, and observing the various parameters of the ejectile (say its energy, and angular distribution).

Some of the possible reactions and the results one might expect to obtain are:

Measuring the reaction \( Q \). That is determining the masses of either the target or daughter nucleus.

\( \text{e.g. } ^{12}\text{C}(\gamma, n) \). In an experiment one sets up to detect neutrons. The \( \gamma \)-ray energy is increased and the yield of neutrons is plotted as a function of \( E_\gamma \). The threshold can be determined quite accurately.
Study the yield of a given reaction e.g. study the yield of Y or y above, as a function of the energy of the bombarding particle x. This reveals features in the quantum system x + X, as revealed in the reaction cross section.

e.g. $^{11}$B(p,n) $^{11}$B + p $\rightarrow$ $^{11}$C + n
or $^{11}$B(p,$\gamma$) $^{11}$B + p $\rightarrow$ $^{12}$C + $\gamma$

or $^{11}$B(p,$\alpha$) $^{11}$B + p $\rightarrow$ $^{12}$C + $\alpha$

comment on the same features being revealed $\rightarrow$ compound nucleus picture.

Study the energy spectrum of the ejectile This reveals features such as energy levels in the residual nucleus.

Mark Boland wanted to find evidence of a new state in the nucleus $^6$He. The plan was to study the reaction $^7$Li($\gamma$,p)$^6$He.

We fired 60 Mev $\gamma$'s onto a lithium target and measured the energy of the protons emitted. The energy of the protons was determined by the energy of the state in 6He that was populated.

We succeeded in confirming the new state
Study the angular relation of the ejectile relative to the incident beam

In simple terms this is related to the AM of the incident particle rel to the nucleus, and the AM of the nucleon with which the interaction occurs. In practice the AD is very sensitive to phase changes produced by components in the nuclear potential due to all the nucleons in close proximity. In other words the properties of a nucleon depend on its environment.

The angular relation of one product relative to another (angular correlation)

In our work in the photodisintegration of light nuclei we study e.g. $^{12}$C + $\gamma$. We observe that sometimes a proton is ejected. Sometimes a neutron. But about 50% of the time both a p and n come out. When we look at the angular correlation of the p rel to the n, we observe
that it is peaked at about 150 degrees.

Why should this be? If we had done the experiment $d(\gamma,p)n$ and looked at the correlation between the break up particles we would have seen that the angular correlation looked like

The answer is that it seems that when the photon interacts with the charges in the nucleus, these charges (protons) are generally associated with a neutron. That is, inside the oxygen nucleus we have not 8 p and 8 n individually, but they are associated as 8 quasi-deuterons. The photon then disintegrates the deuterons and the n and p come out with the AD characteristic of $d(\gamma,p)n$. The slight difference is due to the fact that the quasi-deuterons are moving around, and the momentum effect is evident in the slight closing of the angle between the p and n.

Before moving on to discuss the mechanisms involved, I want to indicate the types of nuclear reactions that can be studied.

**Types of nuclear reactions**

1. **Elastic scattering**

   $$X + x \rightarrow X + x$$

   no change in reacting particles
   no energy change

   This is the elastic scattering process that we discussed in early lectures when studying the size of nuclei. The projectile $x$ then was an electron. In Rutherford scattering $x = \alpha$.

   Elastic scattering is always present when there is a nuclear reaction. It is the natural consequence of firing a wave of particles at an object of the size of the wavelength.

2. **Inelastic scattering**

   $$X + x \rightarrow X^* + x + Q \quad (Q \text{ is negative})$$

   $X(x,x')X^*$

   $X^*$ means $X$ is left in an excited state

   (ask Alex for picture of $^{12}\text{C}(e,e')$)

   In an inelastic scattering the incoming particle transfers energy to the target corresponding to the energy of one of its excited states. The projectile then leaves with less energy. By measuring the energy of the scattered particle, and
knowing its initial energy we can deduce the energies of the excited states in the target nucleus. By studying the AD of the inelastically scattered particle we may also deduce something of the spin and parity of the excited state.

3. Radiative capture

$$X + x \rightarrow Y + \gamma + Q$$

Q is positive and is the BE of x in Y

Again I illustrate this with the part 3 nuclear experiment $^{12}$C(p,γ).
4. General Reaction

\[ X + x \rightarrow Y + y + Q \]
\[ X(x,y)Y \]
\[ Q \text{ is negative being the threshold energy} \]

In this case the reaction cross section reveals structure in the compound nucleus \((X + x)\). E.g. \(^{12}\text{C}(p,\alpha)^{8}\text{Be}, \(^{12}\text{C}(p,d)^{11}\text{C}\)

5. Photonuclear Reaction

\[ X + \gamma \rightarrow Y + y + Q \]
\[ X(\gamma,y)Y \]
\[ Q \text{ is -ve and is the reaction threshold} \]
These reactions study the structure of excited states in X, e.g. the GDR. By studying the energies of γ with known E_γ we learn much about the reaction mechanism involved, and the internal structure of the nucleus X.