6. (a) \( c = f \lambda \) so \( \lambda = c / f \) where \( f = 22.23508 \times 10^9 \) Hz. Hence \( \lambda = 0.0135 \text{ m} = 1.35 \text{ cm} \).

(b)

\[
\Delta \theta = 4 \text{ mas} \\
= 4 \times 10^{-3} \text{arc seconds} \\
= 4 \times 10^{-3} \times \frac{\text{degrees}}{3600} \\
= 4 \times 10^{-3} \times \frac{\pi}{180} \text{ radians} \\
= 1.9 \times 10^{-8} \text{ radians}
\]

Using \( \Delta \theta \approx \lambda / D \),

\[
D = \frac{\lambda}{\Delta \theta} = \frac{1.35 \times 10^{-2}}{1.9 \times 10^{-8}} = 7 \times 10^5 \text{ m} = 700 \text{ km}
\]

(c) Relative velocity between source and observer causes an apparent frequency shift in the observed wave. If the source-observer distance is *decreasing* (object moving towards observer, or observer moving towards source) then the observed frequency is increased, and vice versa. The observer “sees” more oscillations per unit time and therefore “sees” a higher frequency.

The number of extra waves “seen” by the observer is \( v / \lambda \) per unit time \( (v = \text{distance/time}) \). That is,

\[
\Delta f = \frac{v}{\lambda}
\]

or, equivalently,

\[
\frac{\Delta \lambda}{\lambda} = \frac{v}{c}.
\]

Thus there is a link between frequency and velocity, and Doppler gives us a “handle” on the velocity of distant sources.

(d) \( \Delta f = \frac{v}{\lambda} \) so \( v = \lambda \Delta f = 0.0135 \times 80 \times 10^6 = 1.08 \times 10^6 \text{ m/s} \).

\[
M = \frac{v^2 r}{G} = \frac{(1.08 \times 10^6)^2 \times (3 \times 10^{16} \times 0.13)}{6.67 \times 10^{-11}} = 6.82 \times 10^{37} \text{ kg}. \text{ One solar mass is } 2 \times 10^{30} \text{ kg}
\]

so \( M = \frac{6.82 \times 10^{37}}{2 \times 10^{30}} = 3.4 \times 10^7 M_\odot \).

(e) \( i. \ \frac{\Delta \lambda}{\lambda} = \frac{v}{c} \quad \therefore \quad \frac{v}{c} = \frac{476 \times 10^3}{3 \times 10^8} = 1.6 \times 10^{-3} \)
ii.  \[ z = \frac{\Delta \lambda}{\lambda} \]  
\[ \therefore \Delta \lambda = \lambda z = 0.0135 \times 1.6 \times 10^{-3} = 2.14 \times 10^{-5} \text{m}. \]

Hence \( \lambda' = \lambda + \Delta \lambda = 0.0135 + 2.14 \times 10^{-5} \approx 0.01352 \text{m}. \)

iii. \( v = H d \)  
\[ d = \frac{v}{H} = \frac{476}{75} \text{Mpc} = 6.3 \text{Mpc}. \]

(f) \( s = r \theta \) where \( s = 0.13 \text{pc}, \theta = 1.9 \times 10^{-8} \) and \( r \) is the distance to NGC4258.

Hence \( r = \frac{0.13}{1.9 \times 10^{-8}} = 6.54 \text{Mpc}. \)

(g) \( L = 10^{11} L_{\odot} = 10^{11} \times 4 \times 10^{26} \text{W}. \)

\[ d = \sqrt{\frac{L}{4\pi b}} \]  
\[ \therefore b = \frac{L}{4\pi d^2} = \frac{10^{11} \times 4 \times 10^{26}}{4\pi(6.5 \times 10^{6} \times 3 \times 10^{16})^2} = 8.4 \times 10^{-11} \text{W/m}^2. \]

7. (a) \( r = 11 \text{kpc} \)  
\( v = 635 \times 10^3 \text{m/s} \)

\[ M = \frac{v^2 r}{G} = \frac{(635 \times 10^3)^2 \times (11000 \times 3 \times 10^{16})}{6.67 \times 10^{-11}} = 2 \times 10^{42} \text{kg} = 10^{12} M_{\odot}. \]

Hence, about 90%.

(b) \( M = \frac{v^2 r}{G} \) so \( v = \sqrt{\frac{GM}{r}}. \) That is, \( v \propto \frac{1}{\sqrt{r}}. \)

(c) \( M \propto \text{area}, \) so \( M \propto r^2. \)

\( v = \sqrt{\frac{GM}{r}} \propto \sqrt{r^2} \text{ i.e. } v \propto \sqrt{r}. \)

(d) See figure 32.14 of seventh edition of text book.

\( v \) does not fall off for large \( r \) as expected, so the mass is not “inside” some edge radius. That is, there must be invisible mass at large radius (beyond the visible edge).

(e) Strong evidence for microlensing by invisible objects in the halo. Looking at a distant point
source (star, e.g. in Magellanic Clouds), if a dark object passes through our line of sight, gravitational lensing will bend light towards us (light that would otherwise not reach us).

\( \lambda \leq 100\mu m = 100 \times 10^{-6} = 10^{-4} \text{ m or } 10^{5} \text{ nm.} \)

\[ \lambda_{\text{max}} \approx \frac{10^{6}}{T} \text{ nm so } T = \frac{3 \times 10^{6}}{\lambda} = 30 \text{ K.} \]

Hence MACHOs must have a temperature below 30 K (roughly).

8. IRAS ↔ centre of Milky Way (sees through dust)
   VLT ↔ large faint galaxy (light gathering power)
   HST ↔ distant Cepheid variables (resolution)
   X-ray ↔ accretion desk (high energy)

9. (a) Quasar evolution: very few young quasars, so the Universe is not steady-state.
    
    (b) Cosmic background: again, proves the Universe has changed.