640-177: Stars and Galaxies

Solutions to Problem Sheet 1

1. V1 has period \( T_1 = 0.5 \) days, brightness \( b_1 = 2.3 \times 10^{-13} \text{W m}^{-2} \). V2 has period \( T_2 = 10 \) days, brightness \( b_2 = 3.5 \times 10^{-12} \text{W m}^{-2} \).

(a) Based on fig 32.4 of text, or figure of page 6 of notes, \( T_1 = 0.5 \) days implies V1 is an RR Lyrae, while \( T_2 = 10 \) days implies V2 is a Cepheid.

(b) i. \( L_1 \approx 50L_\odot \), but anything between perhaps 25 and 75 would be consistent. Hence we can (very roughly) say \( L_1 \approx 50 \pm 25L_\odot \).

   Similarly, \( L_2 \approx 1500 \pm 400L_\odot \).

   ii. the uncertainties are (again very roughly) \( \Delta L_1 \approx 25L_\odot \) (50%) and \( \Delta L_2 \approx 400L_\odot \) (25%).

   iii. 

   

   \[
   d = \sqrt{\frac{L}{4\pi b}}
   \]

   \[
   d_1 = \sqrt{\frac{50L_\odot}{4\pi \times 2.3 \times 10^{-13}}}
   \]

   \[
   = 8 \times 10^{19} \text{m}
   \]

   using \( L_\odot = 4 \times 10^{26} \text{W}. \) \( 1 \text{pc} = 3 \times 10^{16} \text{m} \) so \( d_1 \approx 2.8 \text{kpc} \pm 1.4 \text{kpc} \).

   Similarly,

   \[
   d_2 = \sqrt{\frac{1500L_\odot}{4\pi \times 3.5 \times 10^{-12}}}
   \]

   \[
   = 1.2 \times 10^{20} \text{m}
   \]

   \[
   = 3.9 \pm 1 \text{kpc}
   \]

   (c) Our galaxy is approx. 20 to 30 kpc across, therefore these stars are within our galaxy.

   (d) These stars are, to within experimental error, at the same distance from us. Since we are told that they appear in the same region of the sky, and they are the same distance away, then we must assume that (to within experimental error) they are indeed close to each other.

2. Melbourne is a terrible site, due to:

   (a) low altitude

   (b) high moisture content in air

   (c) high light pollution

   (d) turbulent air flow

   These factors would reduce the “seeing” to a point that any telescope bigger than about 0.5 m in diameter would be wasted.

   In addition, such large (12 m) mirrors are subject to distortions due to the mirror weight, and difficulties in controlling temperature (hence thermal distortions), independent of the site.

3. Light is collected in proportion to the area of the reflector, and lost in proportion to the area of any blockages, or apertures in the reflector. Hence the wastage is

   \[
   \frac{\pi 0.5^2}{\pi 12^2} = 0.00174
   \]

   or 0.174%.
4. Angular resolution is approximately \( \Delta \theta = \frac{\lambda}{D} \) where \( \lambda \) is the wavelength of the light, and \( D \) is the diameter of the optic (i.e. the lens or mirror). So \( D = \frac{\lambda}{\Delta \theta} \). Visible light has a wavelength of about \( \lambda = 0.5 \mu m = 0.5 \times 10^{-6} \text{ m} \) and \( \Delta \theta = 1 \) arc second = \( (1/3600)^\circ \).

1° is equivalent to \( \pi/180 \) radians so

\[
\Delta \theta = \frac{\pi}{180 \times 3600} \text{ radians}
\]

and therefore

\[
D = \frac{0.5 \times 10^{-6}}{5 \times 10^{-8}} = 0.1 \text{ m}.
\]

That is they must have 10 cm eyes (hence, bug-eyed aliens!).

Our eyes have \( D \approx 1 \text{ cm} \) and

\[
\frac{\lambda}{D} = \frac{\Delta \theta D}{\pi} = \frac{1}{180} \left( \frac{1}{3600} \right) 0.01
\]

\[
= 5 \times 10^{-8} \text{ m}
\]

which is in the ultra-violet or soft X-ray region of the spectrum.

5. Heavy elements are associated with young stars, i.e. those in the galactic disk. Note that you heavy elements are produced by supernovae, and hence would be associated with older stars. However, young stars (as in the disk) were formed relatively recently and therefore from material which includes supernovae remnants. Old stars (as in the bulge and halo) were formed a long time ago, from material that hadn’t yet been through the supernova experience.

6. Hydrogen emits at 21 cm. \( \lambda = 22 \text{ cm} \) is longer than rest hydrogen, i.e. red-shifted, so the gas is moving away from the rocket. That is, the rocket will not reach the gas – no matter how fast the rocket is going, the gas must be moving away faster.

7.

\[
M = \frac{v^2 r}{G}
\]

where

\[
r = 1 \text{ pc} = 3.086 \times 10^{16} \text{ m}
\]

\[
G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2
\]

\[
v = 80 \text{ km/s}
\]

so

\[
M = \frac{(80000)^2 (3.086 \times 10^{16})}{6.67 \times 10^{-11}} = 3 \times 10^{36} \text{ kg}
\]

The sun’s mass is \( M_\odot = 2 \times 10^{30} \) kg so \( M = 1.5 \times 10^6 M_\odot \).

8. If the outer mass rotates more slowly, then using \( v^2 = \frac{GM}{r} \) implies \( v \) is slower as \( r \) gets large. Therefore the mass is “inside” all radii, and the mass must be centrally located, rather than evenly distributed.

9. The temperature is \( T = 77 \) K. Using Wien’s Law,

\[
\lambda_{\text{max}} = \frac{3 \times 10^6}{T} \text{ nm}
\]

\[
= \frac{39 000}{T} \text{ nm}
\]

\[
= 39 \mu m
\]

Objects colder than 77 K will emit at wavelengths longer than this. That is, the detector will be sensitive to wavelengths longer than 39 \( \mu m \).