1. If stars form early we expect an elliptical galaxy because ellipticals contain all old stars. Spirals contain both young and old stars, therefore we expect spirals when stars form both early and late.

2. G1 has low apparent brightness and broad spectral output: 19 cm – 23 cm. G2 is bright with narrow spectral output: 20 cm – 22 cm. Hydrogen emits at 21 cm. If we observe output at 19, 20, ..., 23 cm (and everything in between) then the H must be moving at a broad range of velocities with some high velocities.

G1 has greater spectral output, therefore greater Doppler spread and higher velocities. Assume that rotation about the galactic centre produces the Doppler spread, so higher velocities implies greater mass \( M = \frac{v^2 r}{G} \) so G1 is “larger” (more mass).

Hence, Dr Butterfly, searching for large galaxies, would collect G1.

3. Helium at 164.0 nm in lab; measure \( z = 0.06 \).

\[ z = \frac{\Delta \lambda}{\lambda} \rightarrow \Delta \lambda = z \lambda = 0.06 \times 164 = 9.8 \text{ nm} \]

Redshift implies measured wavelength more “red” i.e. larger so \( \lambda \) measured is 164 + 9.8 = 173.8 nm.

\( z = 5 \) gives \( \Delta \lambda = 5 \times 164 = 820 \text{ nm} \) so \( \lambda \) measured is 164 + 820 = 984 nm.

4. This question is rather tricky to answer correctly, but there are a couple of reasonable answers that are good enough. A fully detailed answer is beyond the scope of a first-year course. Before starting, I’d like to stress that a measurement of the redshift gives the distance to a source now, not the distance when it actually emitted the light.

(a) Let’s work it out based on the current distance as determined from the redshift.

The universe is expanding at 75 km/s per Mpc. Distance 10 billion Ly is \( 10^{10}/(3.26 \times 10^6) = 3067 \text{ Mpc} \), expanding at 3067 \times 75000 = 2.3 \times 10^8 \text{ m/s}.

In 5 billion years, the expansion would be \( (2.3 \times 10^8) \times (5 \times 10^9) \times (3600 \times 24 \times 365) = 3.7 \times 10^{25} \text{ m} \), or \( 3.9 \times 10^9 \text{ Ly} \) (one Ly is \( 9.5 \times 10^{15} \text{ m} \)).

Hence, 5 billion years ago, this galaxy was only 6 billion Ly away.

(b) The rate at which the distance changes with time depends on the distance. Recall \( H \approx 2.4 \times 10^{-18} \text{ m/m/s} \), and the rate of change of distance is \( H d \). If \( d \) is small, then the change in distance per second is small. If \( d \) is large, then so too will the change of distance per second be large.

5 billion years ago, the distance \( d \) was much less than now, and so the rate of change of distance \( H d \) was also less.

Let’s call \( d_{10} \) the distance now, and \( d_5 \) the distance 5 billion years ago. We can write \( d_{10} = d_5 + \Delta d \) where \( \Delta d \) is the change in distance over 5 billion years from then to now.

The change in distance is given by the rate, \( H d_5 \), times the time \( t = 5 \text{ billion years} = (5 \times 10^9) \times (3.2 \times 10^7) \text{ seconds} \).
So:

\[
\begin{align*}
d_{10} &= d_5 + \Delta d \\
 &= d_5 + Hd_{st} \\
 &= d_5(1 + Ht)
\end{align*}
\]

so

\[
d_5 = \frac{d_{10}}{1 + Ht}.
\]

We usually use \( H = 75 \text{ km/s per Mpc} \), but we can also use \( H = 2.4 \times 10^{-18} \text{ m/m/s} \) (metres per metre per second). Then \( Ht \) is given by:

\[
Ht = 2.4 \times 10^{-18} \times (5 \times 10^6) \times (3.2 \times 10^7) = 0.4.
\]

This result is an expansion factor, i.e. over 5 billion years, any distance increases by 0.4 times the original distance. Think of it as 0.4 metres per metre.

We know \( d_{10} \) is 10 billion Ly or 3067 Mpc, so

\[
d_5 = \frac{d_{10}}{1 + Ht} = \frac{3067}{1.4} = 2191 \text{ Mpc}.
\]

The redshift \( z \) is given by \( v/c \) and we have calculated above \( v = 2.3 \times 10^8 \text{ m/s} = 0.77c \), so \( z = 0.77 \).

We also know \( z = \frac{\Delta \lambda}{\lambda} \) so \( \Delta \lambda = 0.77 \times 164 = 126 \text{ nm} \). The measured wavelength is then \( \lambda_{\text{measured}} = \lambda_{\text{lab}} + \Delta \lambda = 164 + 126 = 290 \text{ nm} \).

Note that this calculation of the measured wavelength is non-relativistic, which is not reasonable given the high redshift. We should use:

\[
\frac{\Delta \lambda}{\lambda} = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1 = 1.75
\]

so

\[
\Delta \lambda = 1.75 \times 164 = 287 \text{ nm}
\]

and the measured wavelength will then be \( 164 + 287 = 451 \text{ nm} \). Relativity really does matter! Relativistic corrections are important for any redshift bigger than about \( z = 0.1 \).

5. The intensity at Earth is \( I = 1.4 \text{ kW/m}^2 \) for \( 1L_{\odot} \times 4 \times 10^{26} \text{ W} \). The irradiated area is half the area of our planet, approximately a sphere with radius 6400 km \((6.4 \times 10^6 \text{ m})\). \( A = \frac{1}{2}4\pi r^2 \).

However, the illumination is not even – the sun’s rays are perpendicular to the surface at the equator, but nearly parallel to the surface at the poles (ignoring the tilt of the Earth’s axis). The sun’s rays are therefore spread out over a large area at the poles. To calculate the total power absorbed, we should use the area of a disk, with radius equal to the Earth’s radius, i.e. \( A = \pi r^2 = \pi(6.4 \times 10^6)^2 = 1.3 \times 10^{14} \text{ m}^2 \). The net power absorbed is then \( IA = 1.8 \times 10^{17} \text{ W} \).

This means Earth absorbs roughly \( 1.8 \times 10^{17} \text{ W} \), or \( 1.8 \times 10^{17} \text{ J} \) every second. This is, of course, re-radiated. If it was not, then the Earth would vaporise after absorbing \( 5 \times 10^{31} \text{ J} \) in a time of

\[
\frac{5 \times 10^{31}}{1.8 \times 10^{17}} = 2.8 \times 10^{14} \text{ s}
\]

or about 10 million years. Quasars are up to \( 10^{15} \) times as luminous, therefore Earth would vaporise after about 0.3 seconds!
6. Syd: giant single dish. Poor resolution, but high collection efficiency. Good for dim (distant) objects, where separation from other nearby (in angle) sources is not crucial – e.g. quasars, distant galaxies, pulsars, etc.

Mel: multiple dishes with VLBI. High resolution, but poor collection. Therefore good for bright sources, particularly where it is important to resolve close (in angle) sources – e.g. binary stars, galactic centres, masers, etc.

7. Our galaxy is approx 30 kpc across, $30000 \times 3.26 \text{Ly} = 100000 \text{Ly}$. Therefore the most rapid change will take 100 000 years.

8. 10 km per $10^8$ J is $10^{-4}$ m/J (i.e. 0.1 mm per Joule!).

Typical fusion efficiency is 0.01% (1 part in $10^4$).

$E = mc^2$ for 100% efficiency, $E = \eta mc^2$ where $\eta$ is the efficiency, $10^{-4}$. So the energy per kg (i.e. $m = 1 \text{kg}$) is $10^{-4} (3 \times 10^8)^2 = 9 \times 10^{12}$ Joules.

Assume 1 litre has mass of approx 1 kg, so $E \approx 9 \times 10^{12}$ J/kg. This is approximately $9 \times 10^4$ times the chemical output, i.e. roughly a factor of $10^5$. Therefore the car will travel approximately 10 km times $10^5 = 10^6$ km per litre.