Nonequilibrium Statistical Mechanics (Honours)

Syllabus

The course is divided into two parts. The first part introduces you to three *practical* techniques in nonequilibrium statistical mechanics: fluctuations, birth-death processes, and kinetic theory. You are likely to encounter these again and again in the course of your research. The second part introduces you to the (very beautiful and very new) theory of pattern formation in complex systems. This involves a practical element, namely methods for calculating the stability of nonlinear dynamical systems, as well as a taste of how statistical physics today is being applied to solve profound biological problems.

1. Introduction
   (a) Qualitative comparison of equilibrium and nonequilibirum systems
   (b) Review of thermodynamic ensembles, phase space density, Liouville equation
2. Fluctuations
   (a) Brownian motion, Langevin equation, fluctuation-dissipation theorem, fluctuation spectrum
   (b) Markov processes, Chapman-Kolmogorov equation, Fokker-Planck equation, detailed balance, Einstein-Smoluchowski equation
   (c) Examples of stochastic processes and boundary conditions, Kramers escape problem
3. Birth-death processes
   (a) Master equations, van Kampen's system size expansion
   (b) Applications: predator-prey ecologies, autocatalytic chemical reactions, bistability, nuclear reactions
4. Kinetic theory
   (a) BBGKY hierarchy, Boltzmann collision integral, Boltzmann H theorem
   (b) Transport processes, relaxation time approximation, electrical conductivity
   (c) Vlasov equation, Landau damping in a collisionless plasma
5. Stability of far-from-equilibrium systems
   (a) Linear stability of systems of nonlinear ordinary differential equations
   (b) Limit cycles, bifurcations, long-term behaviour of multi-species predator-prey ecologies
   (c) Introduction to Landau-Ginzburg theory, noise-induced phase transitions
6. Pattern formation
   (a) Reaction-diffusion (Turing) mechanism, 1-dim and 2-dim spatial patterns, travelling wave fronts

(b) Applications: BZ chemical clock, chemotaxis in slime moulds, mammalian coat patterns, pursuit and evasion in predator-prey ecologies

(c) Symmetries, normal modes, centre manifold reduction

Assessment

- Four assignments, one question each, $4 \times 10\% = 40\%$
- Exam, open book, 6 hr, 60%

Reference books

There is no single textbook that is suitable for this course, largely because the subject of nonequilibrium statistical mechanics is still in an early stage of development. I have collected a set of general references in the bibliography below.

1. K. Huang, 1987, *Statistical Mecahnics*, Wiley. Elegantly written, encyclopaedic reference book which covers thermodynamics, equilibrium statistical mechanics (classical and quantum), and phase transitions (including renormalisation group). Apart from three good chapters on kinetic theory, it does not discuss nonequilibrium systems.

2. N. G. van Kampen, 1992, *Stochastic Processes in Physics and Chemistry*, North-Holland. Thorough discussion of fluctuations (Brownian motion) and birth-death processes (master equations), albeit in a slightly baroque notation.

3. R. Kubo, N. Toda, & N. Hashitsume, 1991, *Statistical Physics II: Nonequilibrium Statistical Mecahnics*, Springer. Covers similar ground to van Kampen, as well as the theory of linear nonequilibrium thermodynamics (e.g. thermoelectric effects and the Onsager relations) which we do not have time to cover in our course.

4. Yu. L. Klimontovich (translated by G. Pontecorvo), 1986, *Statistical Physics*, Harwood. A classic by a founder of the subject.

5. G. Nicolis & I. Prigogine, 1977, *Self-organisation in nonequilibrium systems*, Wiley. Another classic by two inventors of the theory of pattern formation and the spontaneous development of 'dissipative structures' in complex systems.

6. H. S. Wio, 1994, *An Introduction to Stochastic Processes and Nonequilibrium Statistical Physics*, World Scientific. I follow this book quite closely in parts of the course, especially the sections on stability and pattern formation. Well written, but the 'typewriter font' is hard to read!

Mathematical preliminaries

You should take the opportunity during the early weeks of the course to look back over your lecture notes from previous mathematics courses and refresh your memory on the following (very useful!) mathematical techniques. Selected references are given for each topic, together with a practice problem or two. The books referred to are J. Mathews & R. L. Walker, 1970, *Mathematical Methods of Physics*, Addison-Wesley (MW70) and D. Zwillinger, 1997, *Handbook of Differential Equations*, Academic (Z97).

1. Elementary statistics, including the definitions of moments (mean, standard deviation), autocorrelation functions, probability distributions (e.g. Gaussian), and characteristic functions. (MW70; §14.4–§14.6; Q14.5–Q14.7.)

2. Fourier series and transforms, including the convolution theorem and Parseval's theorem. (MW70; §4.1, §4.2; Q4.3, Q4.4, Q4.8, Q4.9.)

3. 1st order partial differential equations, especially the method of characteristics. (Z97; §98, §99.) This is useful when solving master equations.

4. 2nd order partial differential equations, especially (i) the diffusion equation and its solution by the spectral method (Fourier transforms), and (ii) the Laplace equation and its solution by orthogonal polynomials. (Z97; §78; MW70; §8.2–§8.4; Q8.1–Q8.4, Q8.6–Q8.10, Q8.18–Q8.20, Q8.22–Q8.24.) This is useful when solving problems in (i) Brownian motion and (ii) pattern formation.

5. Solution methods for ordinary differential equations, including by integrating factors, variation of parameters, and eigenfunction expansions. (Z97; §58, §79, §95.)

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