Pulsars as gravitational wave sources

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Acknowledgements

Known pulsar search results from LIGO and Virgo are presented on behalf of the LIGO Scientific Collaboration and Virgo Collaboration (Abbott *et al*., ApJ, 839, 12, 2007)

Results from the proposed lower ellipticity cut-off for millisecond pulsars are presented on behalf of Graham Woan, Bryn Haskell, Ian Jones, Paul Lasky and myself (Woan *et al*., arXiv:1806.02822)

Results from the ellipticity distribution work are presented on behalf of Chris Messenger, Xilong Fan and myself (Pitkin *et al*., arXiv:1807.06726)

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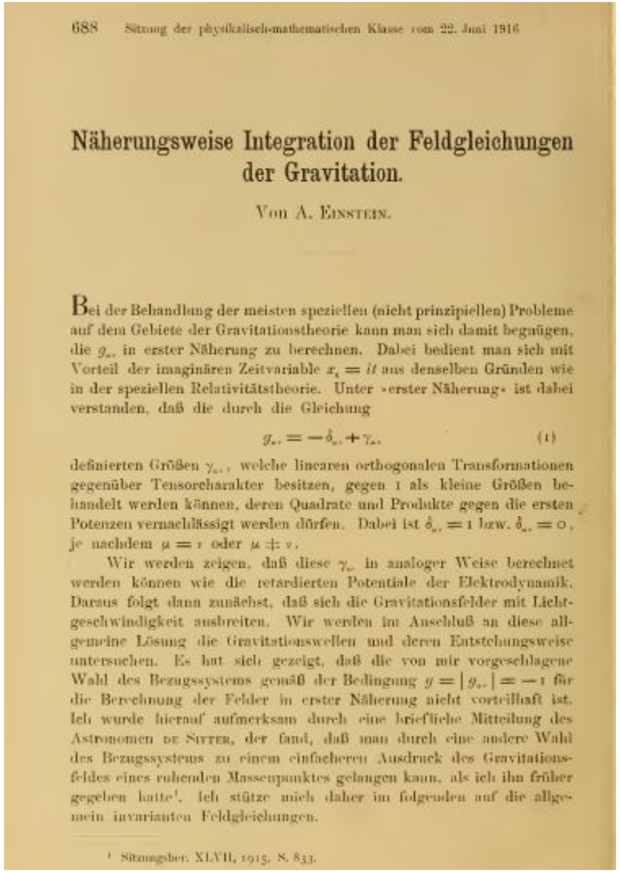
I would like to acknowledge the Wurundjeri people who are the Traditional Custodians of this Land. I would also like to pay respect to the Elders both past and present of the Kulin Nation and extend that respect to other Indigenous Australians present.

Gravitational waves

- Direct prediction of Einstein's General Theory of Relativity
- Solutions to Einstein equation in vacuum are wave equations
- "Ripples in space-time"

$$\underbrace{\left(-\frac{\partial^2}{\partial t^2}+\nabla^2\right)h^{\mu\nu}=\cancel{-16\pi T^{\mu\nu}}=0}$$

solution for tensor h is a wave equation



688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \tag{1}$$

definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformationen gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen behandelt werden können, deren Quadrate und Produkte gegen die ersten Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

Wir werden zeigen, daß diese $\gamma_{\mu\nu}$ in analoger Weise berechnet werden können wie die retardierten Potentiale der Elektrodynamik. Daraus folgt dann zunächst, daß sich die Gravitationsfelder mit Lichtgeschwindigkeit ausbreiten. Wir werden im Anschluß an diese allgemeine Lösung die Gravitationswellen und deren Entstehungsweise untersuchen. Es hat sich gezeigt, daß die von mir vorgeschlagene Wahl des Bezugssystems gemäß der Bedingung $g = |g_{\mu\nu}| = -1$ für die Berechnung der Felder in erster Näherung nicht vorteilhaft ist. Ich wurde hierauf aufmerksam durch eine briefliche Mitteilung des Astronomen de Sitter, der fand, daß man durch eine andere Wahl des Bezugssystems zu einem einfacheren Ausdruck des Gravitationsfeldes eines ruhenden Massenpunktes gelangen kann, als ich ihn früher gegeben hatte[1]. Ich stütze mich daher im folgenden auf die allgemein invarianten Feldgleichungen.

[1] Sitzungsber. XLVII, 1915, S. 833.

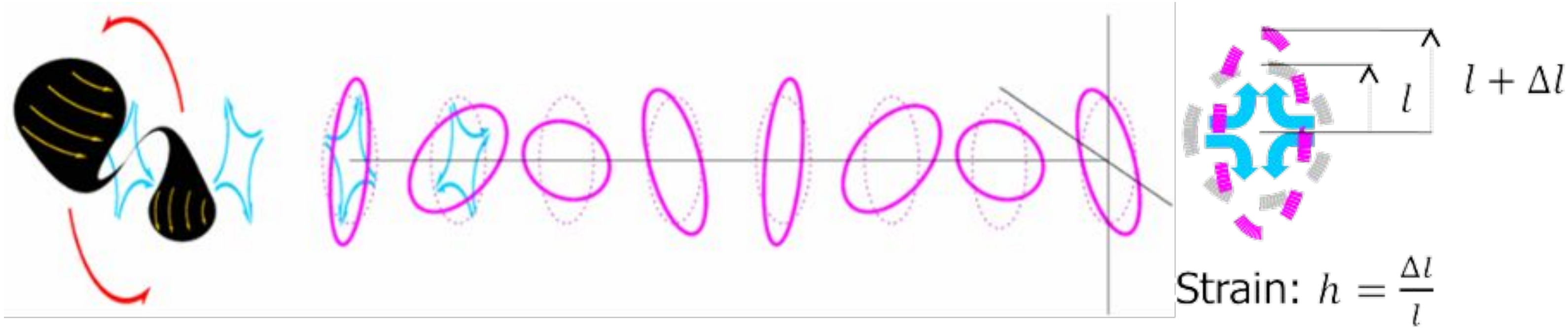
Einstein, "Näherungsweise Integration der Feldgleichungen der Gravitation", Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften, 1916

Gravitational waves (production)

Source: Bulk motions (accelerations) produce changing tidal field

Oscillating field propagates unobstructed to observer

Observer detects distortion strain



Quadrupole formula:

$$h \lesssim \frac{1}{r} \boxed{\frac{G}{c^4}} \boxed{M v_{\rm ns}^2}$$

Source distance

8×10^{-45} $s^2 m^{-1} kg^{-1}$ (small number!)

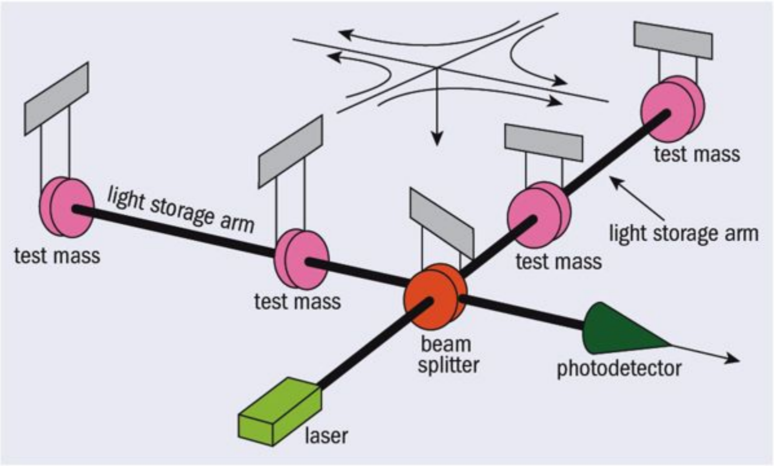
"Non-spherical" kinetic energy (must be large to give detectable strain)

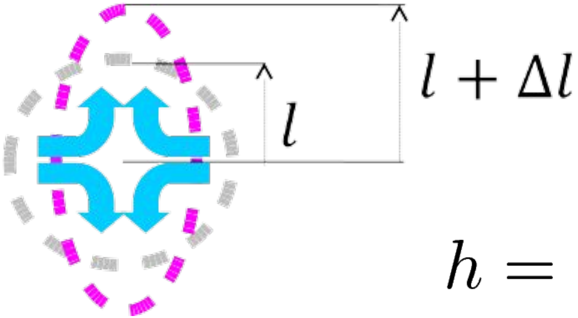
E.g. for orbiting mass at radius A with period P:

$$\boxed{v_{\rm ns} \approx \frac{2\pi A}{P} = A\omega}$$

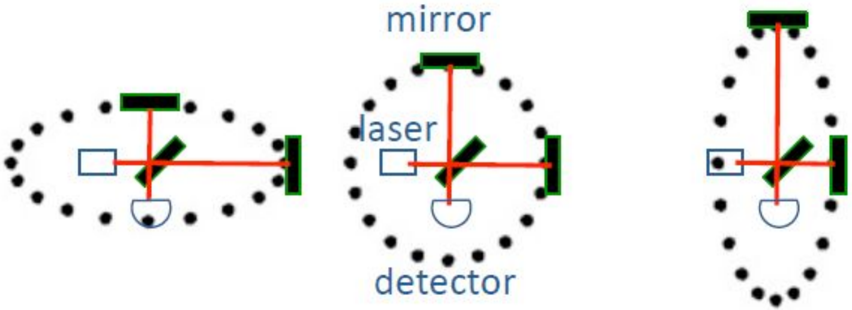
Gravitational waves (detection)

- Measure proper distance between two freely falling test masses (i.e. the suspended mirrors at the end of an interferometer’s arms)





$$h = \frac{\Delta l}{l}$$



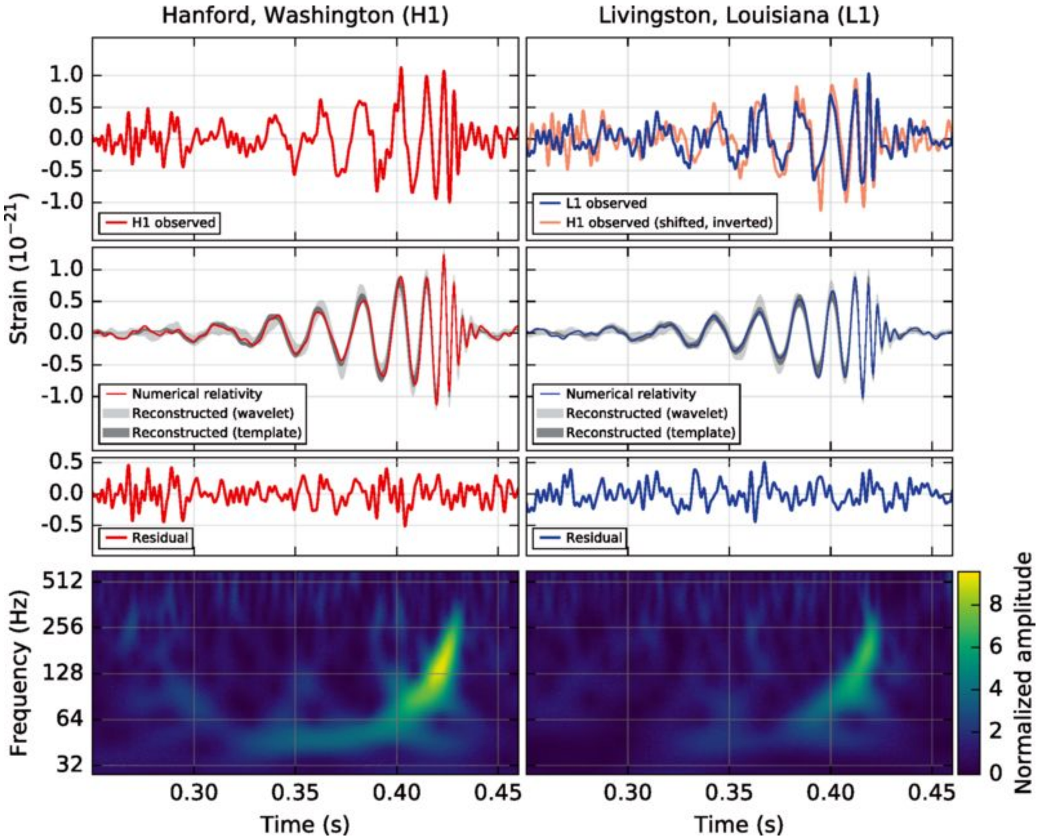
Gravitational waves (detectors)

https://arxiv.org/abs/1206.6163

LIGO Scientific Collaboration (LSC) and Virgo Collaboration

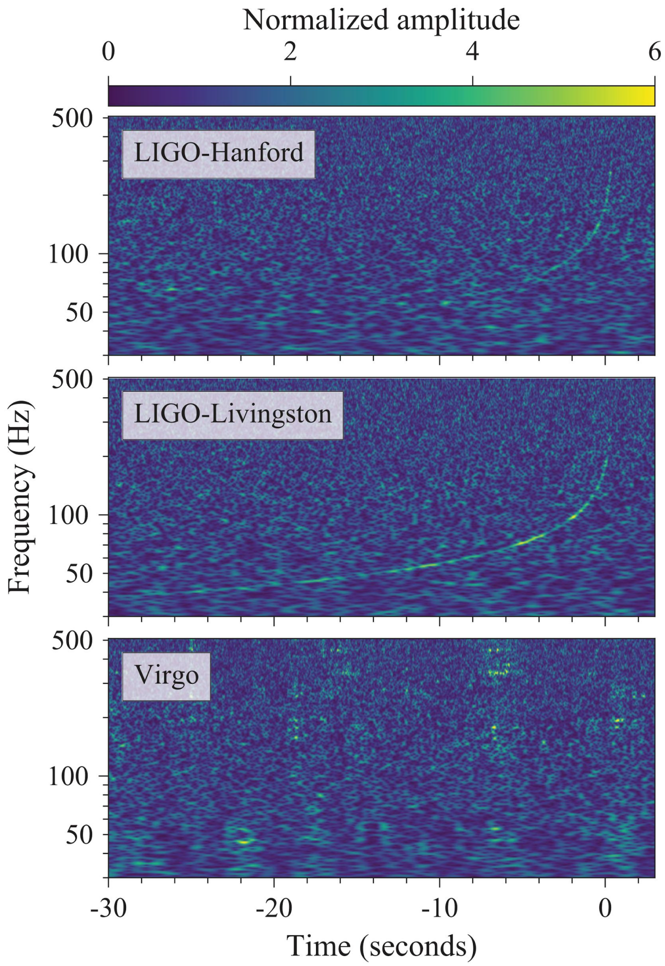
Gravitational waves (detections)

Gravitational waves detected from *binary black hole* coalescence on 14th Sep 2015 using the LIGO detectors



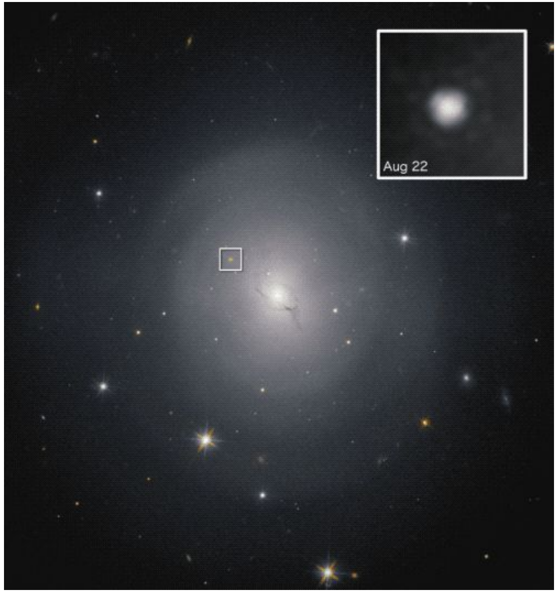
Abbott et al., *Phys. Rev. Lett.* 116, 061102 (2016)

Gravitational waves (detections)



Abbott et al., *Phys. Rev. Lett.* 119, 161101 (2017)

Gravitational waves detected from *binary neutron star* coalescence on 17th Aug 2017 using the LIGO & Virgo detectors



Credit: NASA and ESA

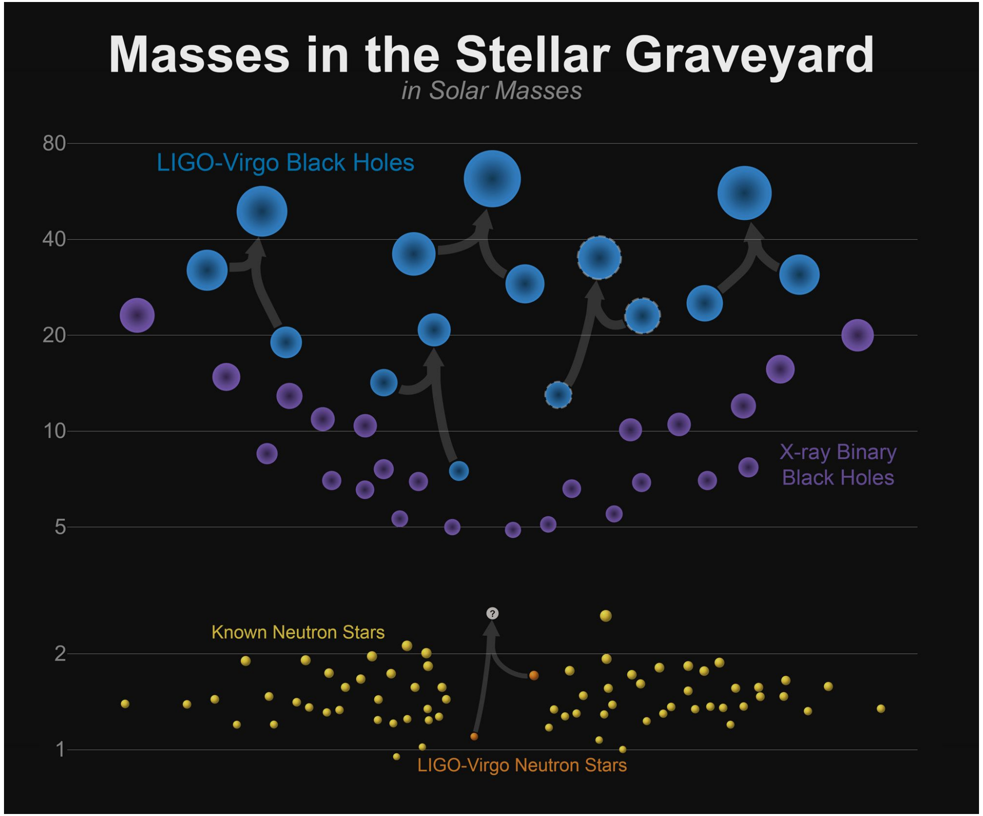
Gravitational waves (detections)

List of gravitational wave events [edit]

List of binary merger events

GW event	Detection time (UTC)	Date published	Location area[n 1] (deg^2)	Luminosity distance (Mpc)[n 2]	Energy radiated ($c^2 M_\odot$)[n 3]	Chirp mass ($M_\odot$)[n 4]	Primary		Secondary		Remnant			Notes	Ref
							Type	Mass ($M_\odot$)	Type	Mass ($M_\odot$)	Type	Mass ($M_\odot$)	Spin[n 5]		
GW150914	2015-09-14 09:50:45	2016-02-11	600; mostly to the south	440^{+160}_{-180}	$3.0^{+0.5}_{-0.5}$	$28.2^{+1.8}_{-1.7}$	BH [n 6]	$35.4^{+5.0}_{-3.4}$	BH [n 7]	$29.8^{+3.3}_{-4.3}$	BH	$62.2^{+3.7}_{-3.4}$	$0.68^{+0.05}_{-0.06}$	First GW detection; first BH merger observed; largest progenitor masses to date	[3][4][2]
LVT151012 (fr)	2015-10-12 09:54:43	2016-06-15	1600	1000^{+500}_{-500}	$1.5^{+0.3}_{-0.4}$	$15.1^{+1.4}_{-1.1}$	BH	23^{+18}_{-6}	BH	13^{+4}_{-5}	BH	35^{+14}_{-4}	$0.66^{+0.09}_{-0.10}$	Not significant enough to confirm (~13% chance of being noise)	[5]
GW151226	2015-12-26 03:38:53	2016-06-15	850	440^{+180}_{-190}	$1.0^{+0.1}_{-0.2}$	$8.9^{+0.3}_{-0.3}$	BH	$14.2^{+8.3}_{-3.7}$	BH	$7.5^{+2.3}_{-2.3}$	BH	$20.8^{+6.1}_{-1.7}$	$0.74^{+0.06}_{-0.06}$		[6][7]
GW170104	2017-01-04 10:11:58	2017-06-01	1200	880^{+450}_{-390}	$2.0^{+0.6}_{-0.7}$	$21.1^{+2.4}_{-2.7}$	BH	$31.2^{+8.4}_{-6.0}$	BH	$19.4^{+5.3}_{-5.9}$	BH	$48.7^{+5.7}_{-4.6}$	$0.64^{+0.09}_{-0.20}$	Farthest confirmed event to date	[8][9]
GW170608	2017-06-08 02:01:16	2017-11-16	520; to the north	340^{+140}_{-140}	$0.85^{+0.07}_{-0.17}$	$7.9^{+0.2}_{-0.2}$	BH	12^{+7}_{-2}	BH	7^{+2}_{-2}	BH	$18.0^{+4.8}_{-0.9}$	$0.69^{+0.04}_{-0.05}$	Smallest BH progenitor masses to date	[10]
GW170814	2017-08-14 10:30:43	2017-09-27	60; towards Eridanus	540^{+130}_{-210}	$2.7^{+0.4}_{-0.3}$	$24.1^{+1.4}_{-1.1}$	BH	$30.5^{+5.7}_{-3.0}$	BH	$25.3^{+2.8}_{-4.2}$	BH	$53.2^{+3.2}_{-2.5}$	$0.70^{+0.07}_{-0.05}$	First detection by three observatories; first measurement of polarization	[11][12]
GW170817	2017-08-17 12:41:04	2017-10-16	28; NGC 4993	40^{+8}_{-14}	> 0.025	$1.188^{+0.004}_{-0.002}$	NS	1.36 - 1.60[n 8]	NS	1.17 - 1.36[n 9]	BH [n 10]	< $2.74^{+0.04}_{-0.01}$ [n 11]		First NS merger observed in GW; first detection of EM counterpart (GRB 170817A; AT 2017gfo); nearest event to date	[1][15][16]

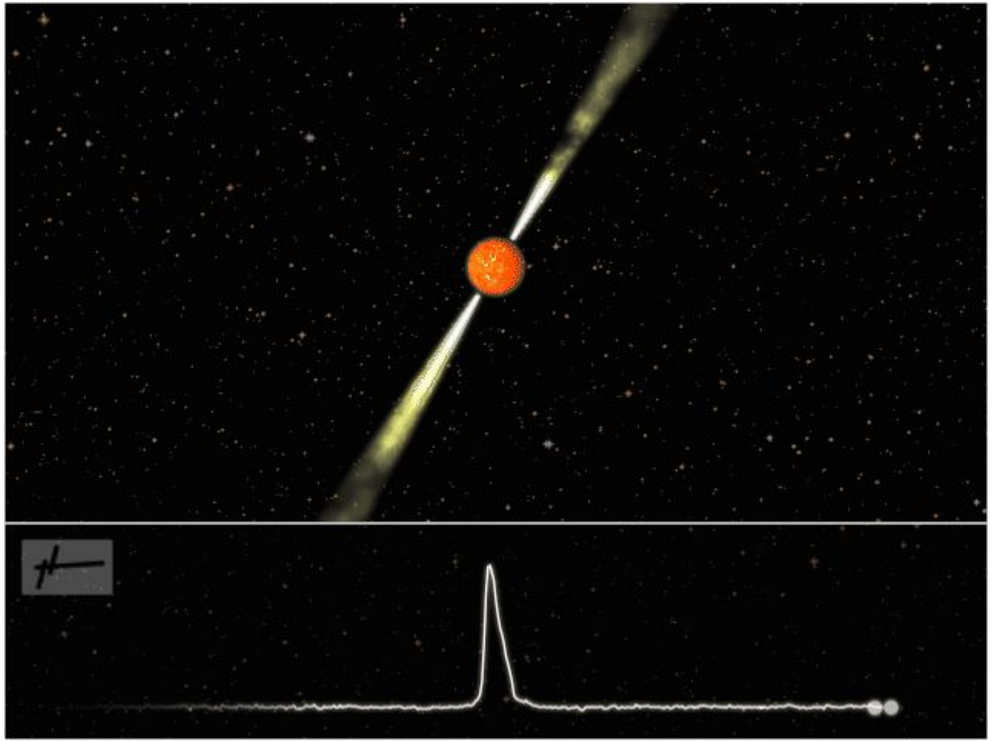
Gravitational waves (detections)



Credit: LIGO-Virgo/Frank Elavsky/Northwestern University

Pulsars

- Rapidly rotating neutron stars observed through lighthouse-like pulses of beamed emission from magnetic poles
- Over 2500 pulsars observed (~200000 active pulsars in the Milky Way, and $\sim 10^8$ neutron stars)

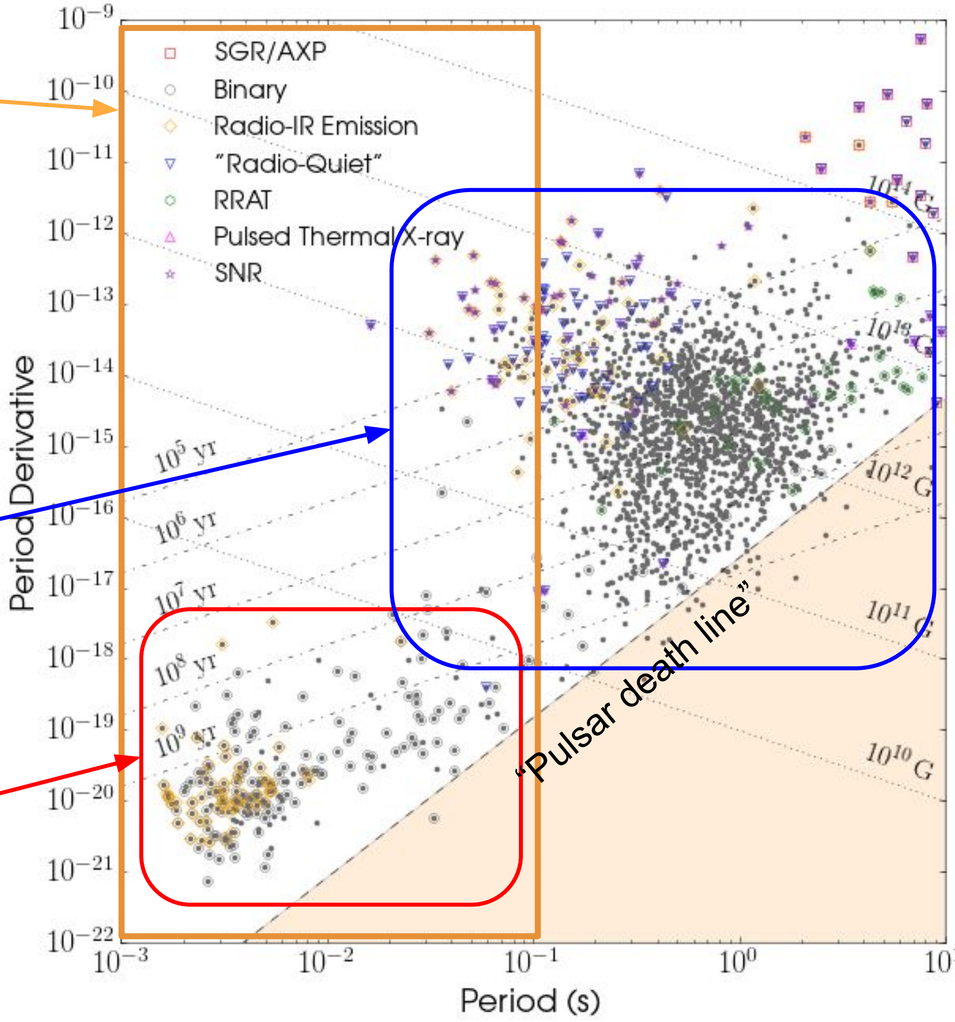


Credit: Joeri van Leeuwen

Pulsars

Pulsars with periods accessible to the LIGO/Virgo gravitational wave detectors

Population of pulsars if often shown in a P-Pdot (period vs. period derivative) diagram



“Young” pulsars: slow, large spin-down, large dipole fields

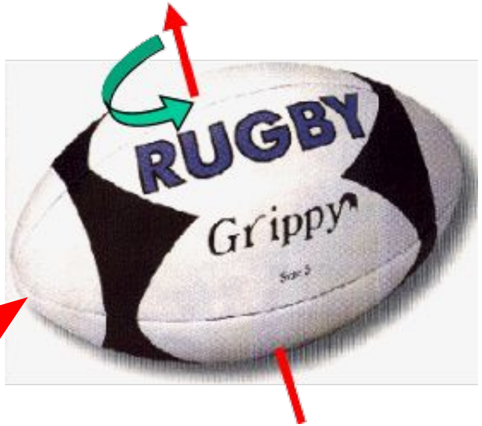
Millisecond/recycled pulsars: fast, small spin-down, “small” dipole fields

Image produced with psrqpy, Pitkin, JOSS (2018)

Gravitational waves from pulsars

Pulsars will emit gravitational waves if they have some non-axisymmetry to produce a time varying mass (or mass current) quadrupole, e.g., they:

- **have a triaxial moment of inertia (a “mountain”!); emission at twice the rotation frequency**
- are undergoing free precession; emission at approximately the rotation frequency
- have r-modes (Rossby waves); emission at approximately 4/3 rotation frequency
- have an excited, and quickly damped, resonant mode; emission in the kHz.



Part I: searches for gravitational waves from known pulsars

Credit: X-ray: NASA/CXC/SAO; Optical: NASA/STScI; Infrared: NASA-JPL-Caltech

Searches for gravitational waves from pulsars

Known pulsars are great GW targets; precise phase evolution from EM observations mean that long duration (~year) coherent searches are possible.

Known pulsar searches carried out by the LIGO Scientific Collaboration & Virgo Collaboration (LVC) have made the following assumption:

- signals are emitted from a triaxial star ($l=m=2$ mass quadrupole mode) rotating about its principal moment of inertia I_{zz} (no precession)
- GW signals are phase locked with the electromagnetic emission (which is itself locked to the star's rotation)† giving emission at twice the rotation rate f_{rot}

†Some targeted searches have been performed relaxing the very strong assumption about GW emission being phase locked to the rotation, e.g., Abbott *et al*, *ApJL*, 683 (2008) & Abbott *et al*, *PRD* 96, 122006 (2017)

Searches

For each pulsar, searches attempt to evaluate the probability distribution of the unknown GW parameters:

- h_0: the gravitational wave strain detected at the Earth
- $\cos\iota$: the cosine of the inclination of the rotation axis to the line-of-site
- ϕ_0: the phase of the signal at some epoch
- ψ: the polarisation angle

When no signal is found an upper limit on h_0 can be set (often at 95% credible level). This can be compared to the *spin-down limit* set by assuming all rotational kinetic energy is dissipated through l=m=2 mass quadrupole GW emission:

$$h_0^{\rm sd} = 2.55\times10^{-25}\left(\frac{I_{zz}}{10^{38}\,{\rm kg\,m^2}}\right)^{1/2}\left(\frac{1\,{\rm kpc}}{d}\right)\left(\frac{100\,{\rm Hz}}{f_{\rm rot}}\right)^{1/2}\left(\frac{|\dot{f}_{\rm rot}|}{10^{-11}\,{\rm Hz}\,s^{-1}}\right)^{1/2}$$

Searches

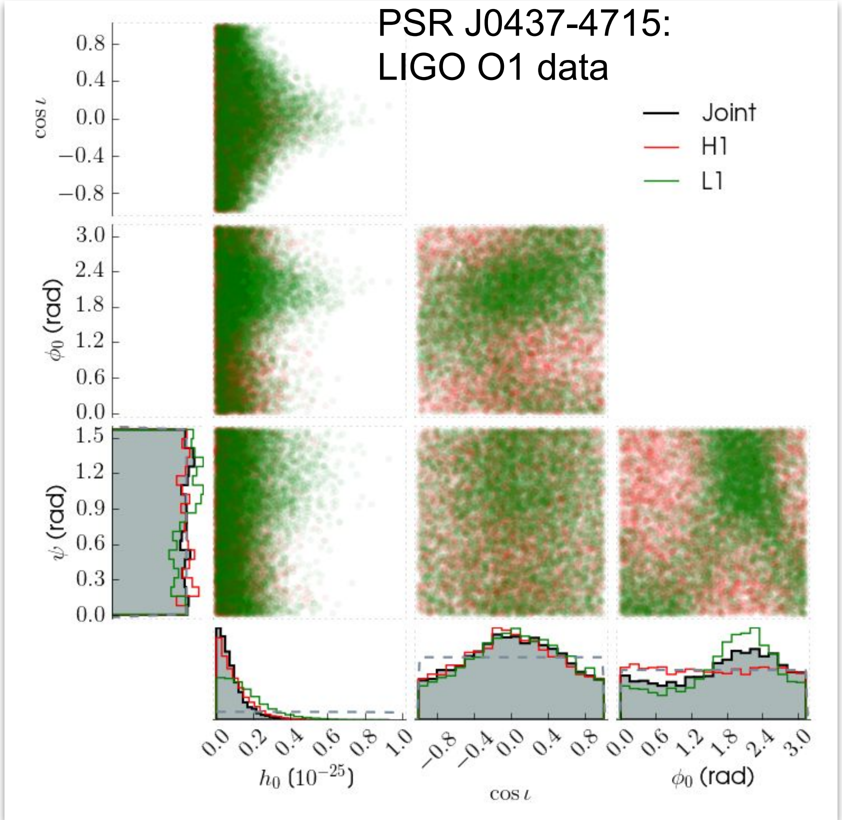
Example posteriors

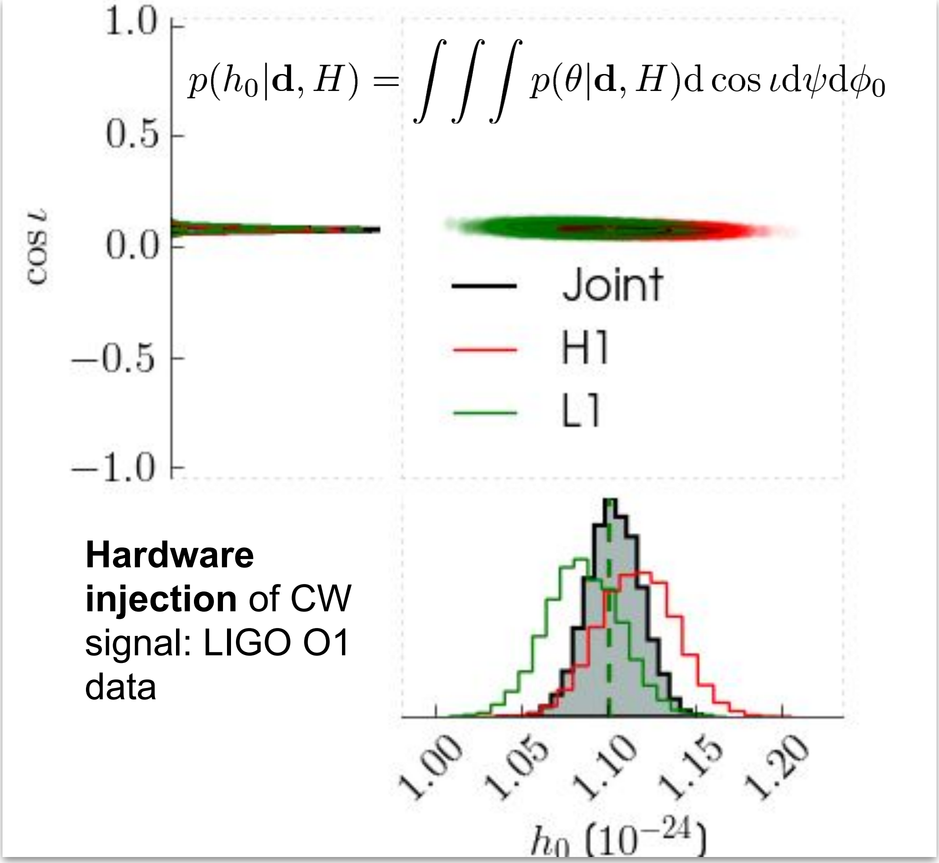
likelihood prior

$$p(\theta|\mathbf{d}, H) = \frac{p(\mathbf{d}|\theta, H)p(\theta|H)}{p(\mathbf{d}|H)}$$

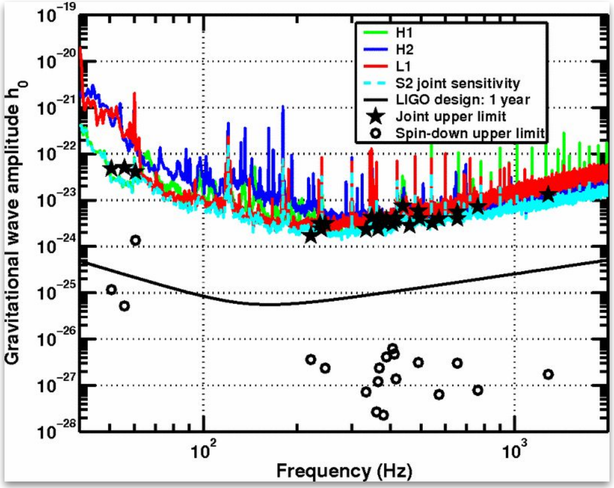
posterior

$$\theta = \{h_0, \cos\iota, \phi_0, \psi\}$$

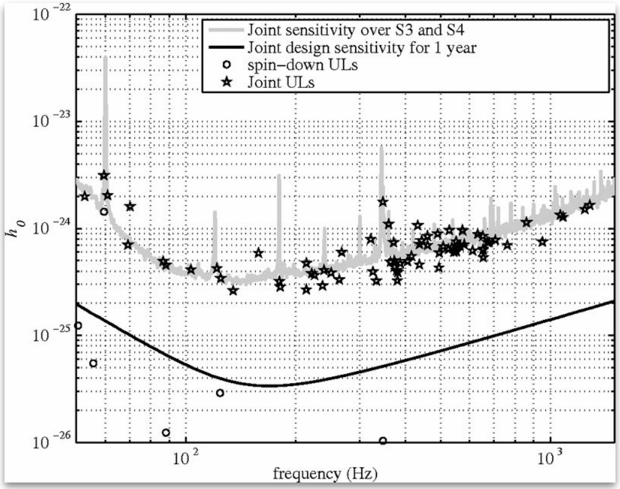




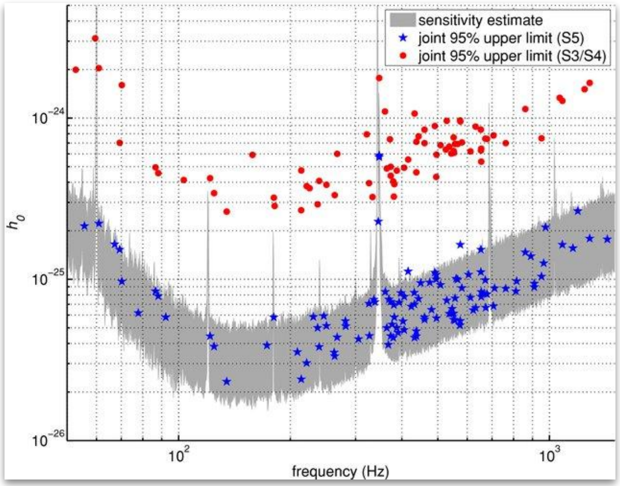
Searches



LIGO S2: 28 pulsars. Abbott *et al*, *PRL* 94, 181103 (2005)



LIGO S3+S4: 78 pulsars. Abbott *et al*, *PRD* 76, 042001 (2007)



LIGO S5: 116 pulsars. Abbott *et al*, *ApJ*, 713 (2010)

Rely on up-to-date ephemerides from EM pulsar observations (radio, X-ray, γ-ray) preferably overlapping GW observing runs.

[†]see, e.g., Johnson-McDaniel & Owen, *PRD* 88, 044004 (2013)

Searches

The probability distribution of h_0 can be converted into a distribution on the mass quadrupole moment Q_{22}, or fiducial ellipticity[†] ε assuming a known distance (often known to ~20%):

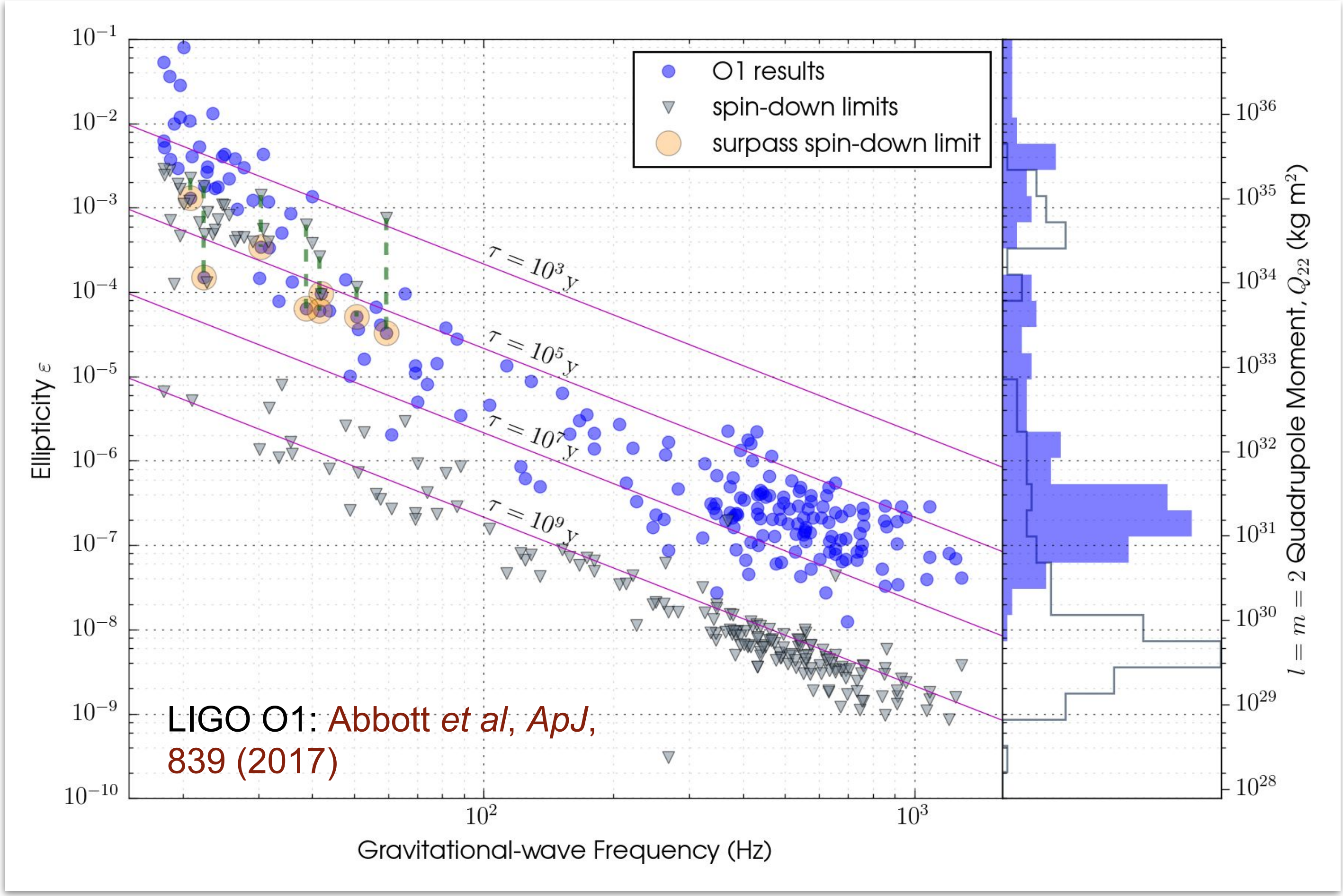
$$\varepsilon = 10^{-6} \left(\frac{h_0}{4.2 \times 10^{-26}} \right) \left(\frac{100\,\mathrm{Hz}}{f_{\mathrm{rot}}} \right)^2 \left(\frac{10^{38}\,\mathrm{kg\,m^2}}{I_{zz}} \right) \left(\frac{d}{1\,\mathrm{kpc}} \right)$$

and (following Ushomirsky, Cutler & Bildsten, *MNRAS* 319 (2000))

$$Q_{22} = \sqrt{\frac{15}{8\pi}} I_{zz} \varepsilon$$

This can in-turn be converted to a limit on the model-dependent internal B-field strength (e.g. Cutler, *PRD* 66, 084205 (2002) for toroidal field with $B < 10^{15}$ G):

$$\varepsilon_B \approx -1.6 \times 10^{-6} \left(\frac{\langle B_t \rangle}{10^{15}\,\mathrm{G}} \right)$$

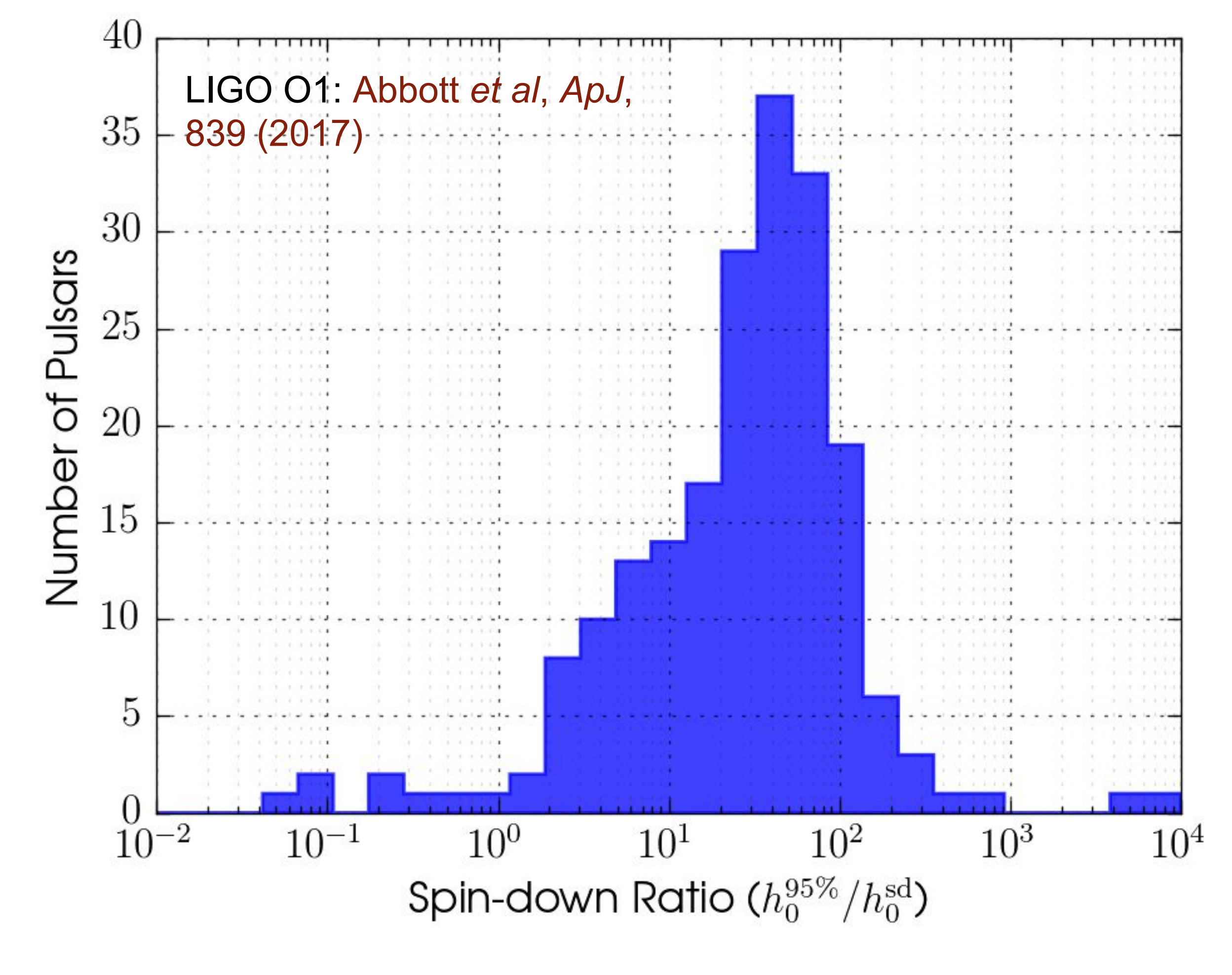


LIGO O1: Abbott *et al*, *ApJ*, 839 (2017)

We can convert to surface deformation, maximised over EoS, using†:

$$R\varepsilon_{\mathrm{surf},22} \approx 25 \left(\frac{\varepsilon}{10^{-4}}\right) \mathrm{cm}$$

†Johnson-McDaniel, *PRD* 88, 044016 (2013)



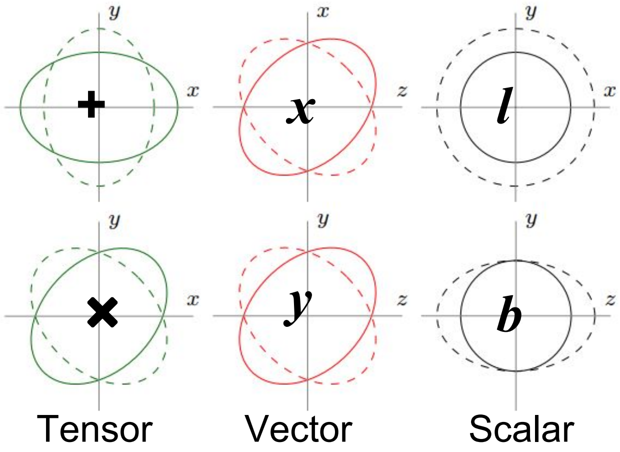
Smallest spin-down ratio: Crab pulsar, $\varepsilon < 3.6 \times 10^{-5}$, which is 20 times below the spin-down limit (less than ~0.003 of the spin-down luminosity is emitted via GWs)

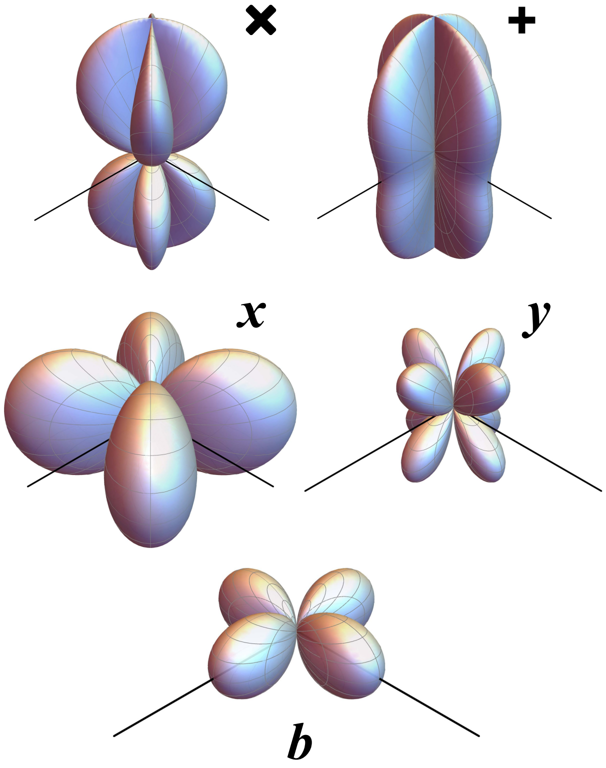
MSP closest to spin-down limit: J0437-4715 (GW frequency 347 Hz, at 0.16 kpc) $\varepsilon < 2.8 \times 10^{-8}$, which is only 1.4 times spin-down limit. Limits internal toroidal B field to $\lesssim 10^{13}$ G

Isi, Pitkin & Weinstein, *PRD* 96, 042001 (2017)

Non-GR signals

Generic metric theories of gravity allow six different GW polarisation modes: tensor ('+' and '×'), vector ('*x*' and '*y*'), and scalar (longitudinal and breathing - degenerate for current interferometers)



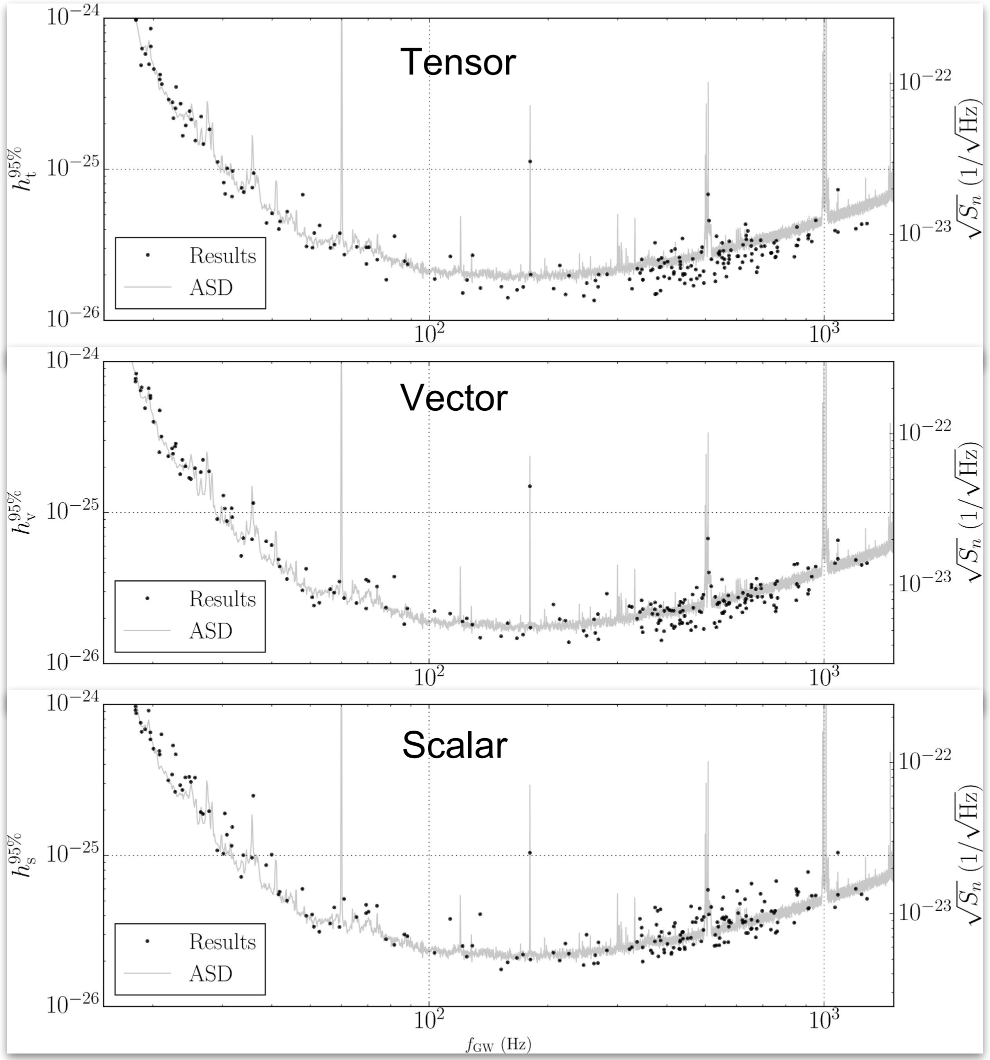


Non-GR signals

Searched the same pulsars as standard O1 analysis (and assuming emission at twice the rotation frequency!)

No signal from non-GR polarisation seen, but limits set on GW amplitude for tensor, vector and scalar modes (no simple spin-down-like limit for comparison!)

LIGO O1: Abbott *et al*, *PRL*, 120 (2018)



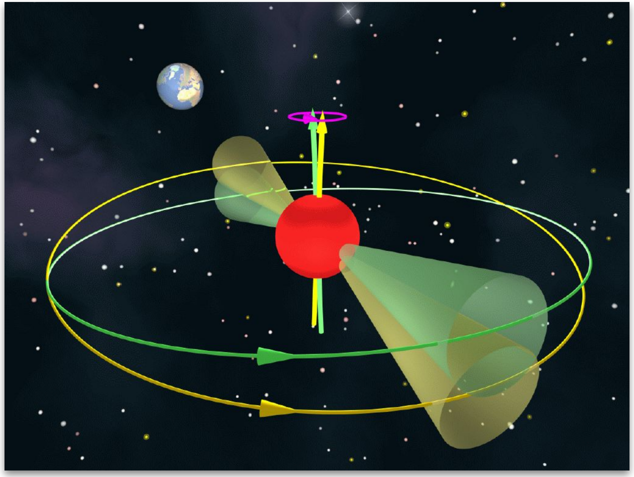
Emission from other modes (the rotation frequency)

Pulsars will emit GWs at (close to) their rotation frequency if undergoing free precession (e.g, Zimmermann & Szedenits, *PRD*, 20, 351 (1979))

- No *strong* evidence for free precession of any pulsar

Potential mechanism to get emission at rotation frequency without precession proposed by Jones, *MNRAS*, 402 (2010).

- Superfluid pinning of the core, but with pinning axes not aligned with a principal moment of inertia
- Adds two additional (non-degenerate) parameters to the waveform model



Credit: M. Kramer

95% credible upper limits set on amplitude at both once and twice rotation frequency for isolated pulsars using LIGO S5 data (Pitkin *et al.*, *MNRAS*, 453, 2015) - no signal seen

Narrow-band searches

The assumption of GW and EM signals begin phase locked may not be correct, e.g., if there is precession or if the EM & GW producing components of the star are not tightly coupled (see discussion in, e.g., Abbott *et al*, *ApJL*, 683 (2008))

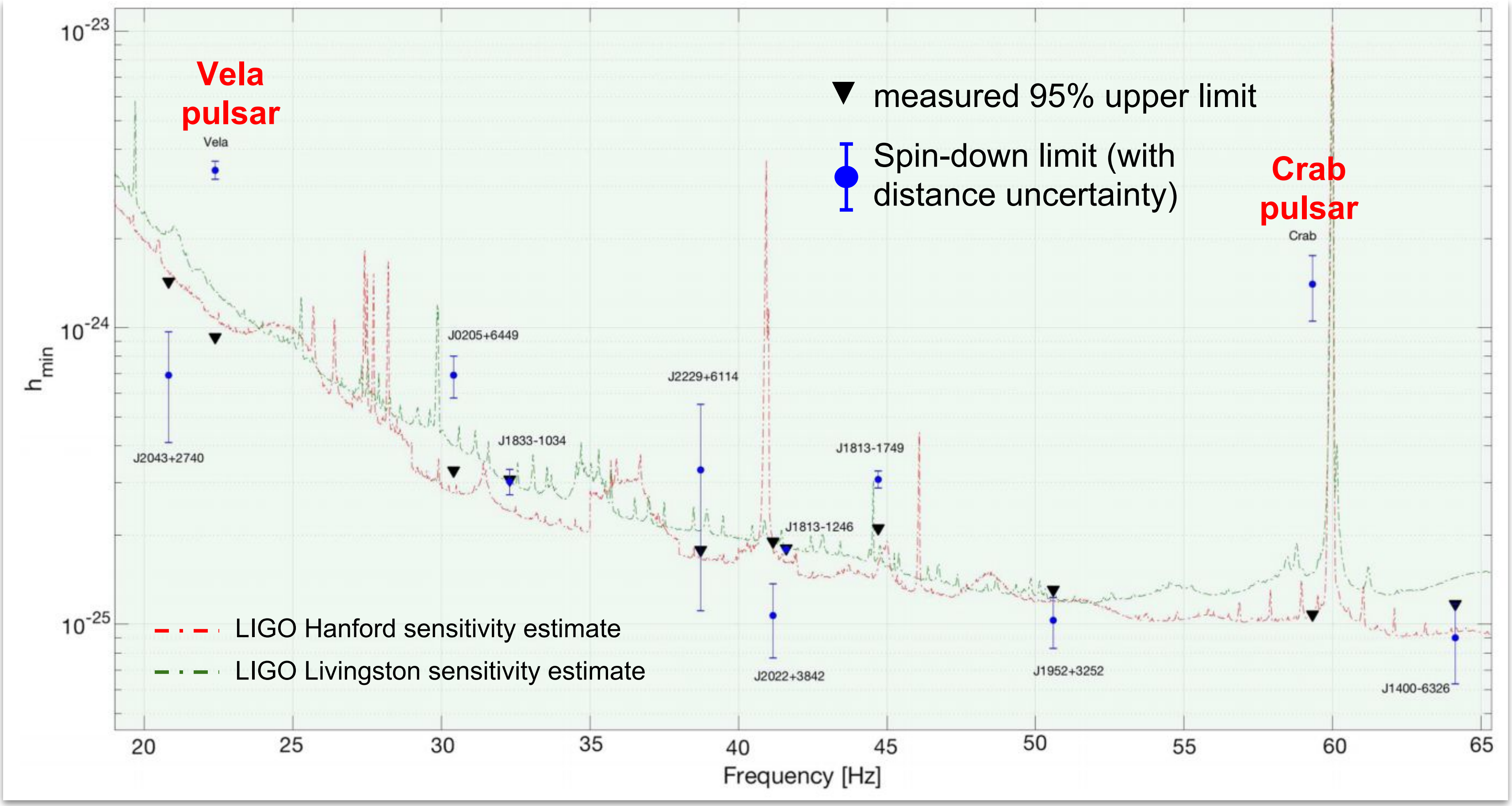
There are also pulsars that are not well timed, so have poor ephemerides with large(ish) uncertainties on frequency and spin-down.

Eleven pulsars have been searched for - frequency band of few 0.01 Hz, and spin-down range of ~few % of spin-down value.

No strong evidence for signals, but upper limits surpass spin-down limits for 5 of the targets.

Narrow-band searches

LIGO O1 data: 11 pulsars. Abbott *et al*, *PRD*, 96 122006 (2017)



Part I: Conclusions

- Searches for gravitational waves from pulsar are mature and produce the tightest limits of gravitational wave amplitude for any source (**but, no signals seen *yet*!**)
    - Search for non-GR modes (without the need to many detectors to break degeneracies)
    - Search for more complex emission mechanisms
- We rely on ephemerides from EM pulsar astronomers to allow these searches to happen
    - We want to search for as many pulsars as possible!
- Search of O2 data (coherently combined with O1) is happening now - O2 sensitivity is comparable to O1, but we also including a search at the rotation frequency

Part II: evidence for a minimum ellipticity

(with G. Woan, B. Haskell, P. Lasky & D. I. Jones,
arXiv:1806.02822)

The P-Pdot diagram

We can plot the period of known pulsars against their period derivative - Pdot (*observed Pdot and true Pdot are not necessarily the same!*)

- lines for different external dipole magnetic field strengths (assuming pure magnetic dipole braking)
- lines for different *characteristic* ages

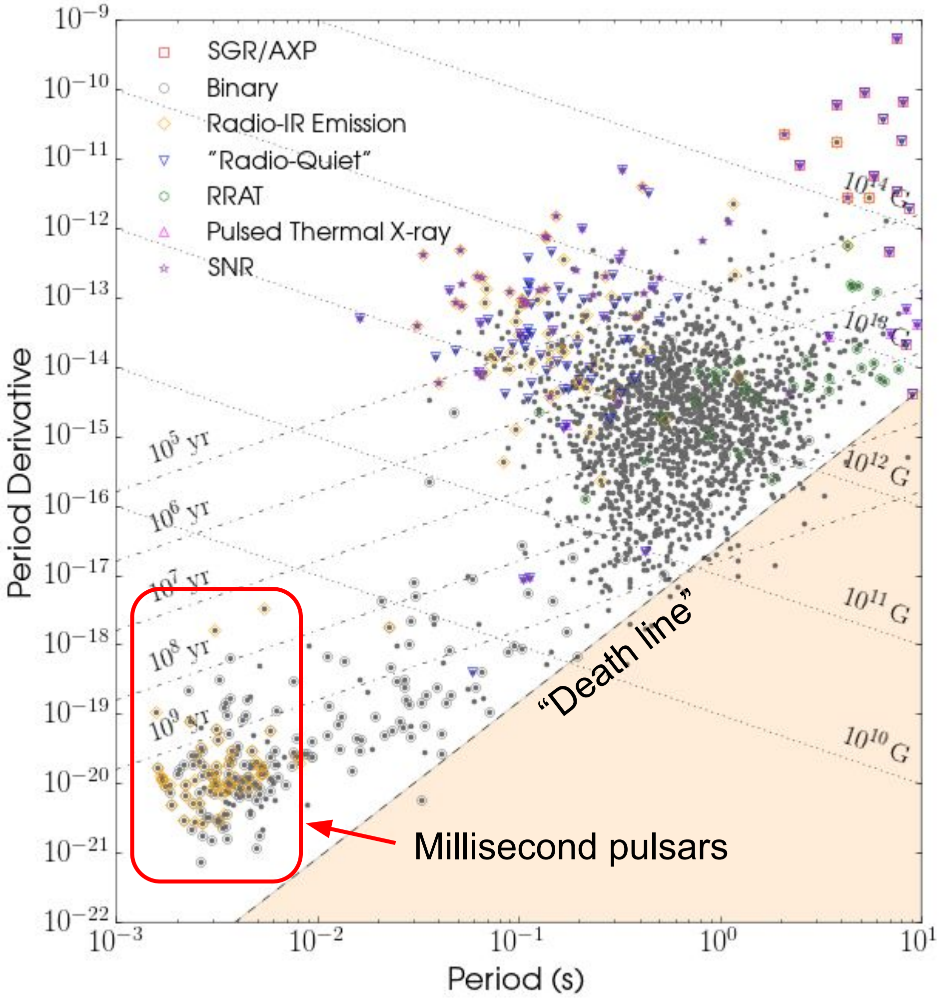
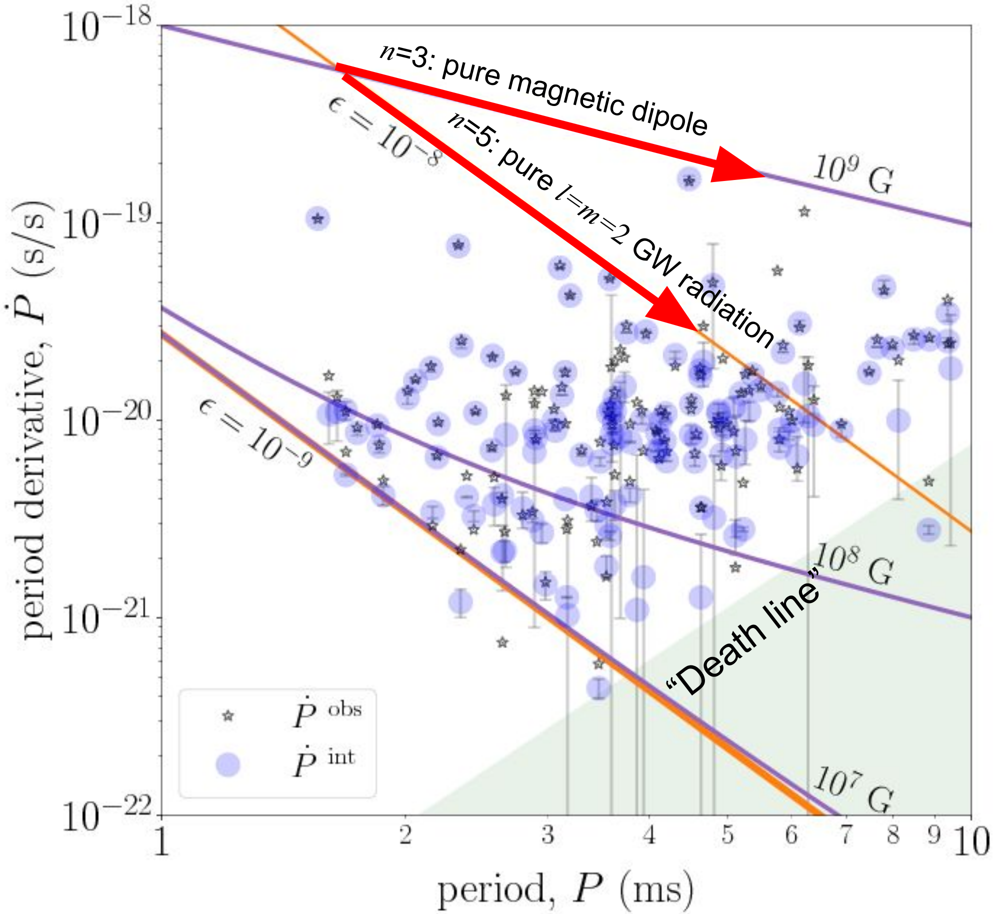


Image produced with psrqpy, Pitkin, JOSS (2018)

P-Pdot diagram

Zoom in on MSPs (showing intrinsic Pdot and uncertainties)

- Lines showing evolution contours for stars spinning-down via
  - pure magnetic dipole radiation
  - pure $l=m=2$ GW emission
  - a combination of both
- Lack of pulsars below contour for GW emission assuming pulsars with ellipticities of 10^{-9}!?

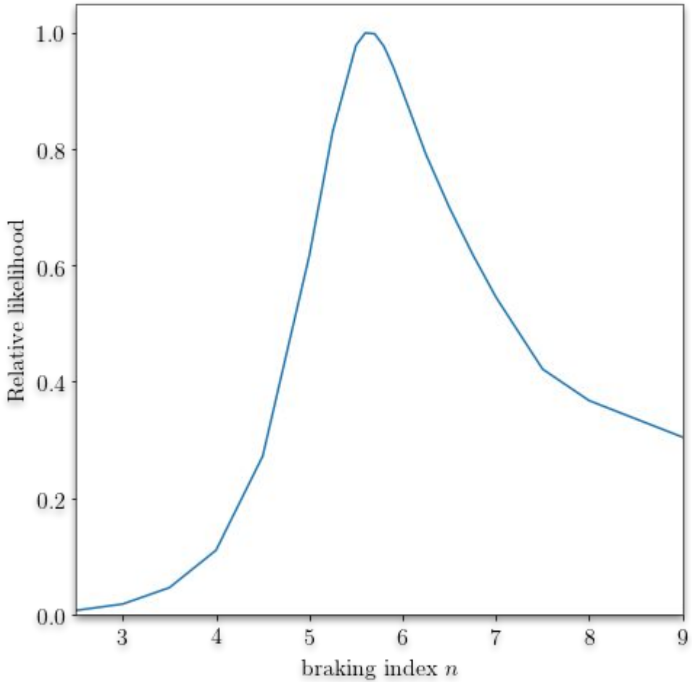


G. Woan *et al.*, arXiv:1806.02822

Ellipticity cut-off

Is the ellipticity cut-off real?

- Observational selection effects?
  - No obvious selection effects that we know of
- Statistical sanity check
  - Prior that MSPs are log-uniformly distributed in Pdot, with a lower Pdot cut-off combining a “death line” (for the r.h.s. of the diagram) and a power law braking process cut-off with unknown braking index (slope) and scale; how do fits to the data including different braking index cut-offs compare to no cut-off?
  - Incorporate uncertainties on Pdot values and in pulsar moment of inertia



Cut-off with n=5 (i.e. pure GW emission) favoured over no cut-off by ~6400

Cut-off with n=5 favoured over n=3 (i.e. pure magnetic dipole emission) by ~35

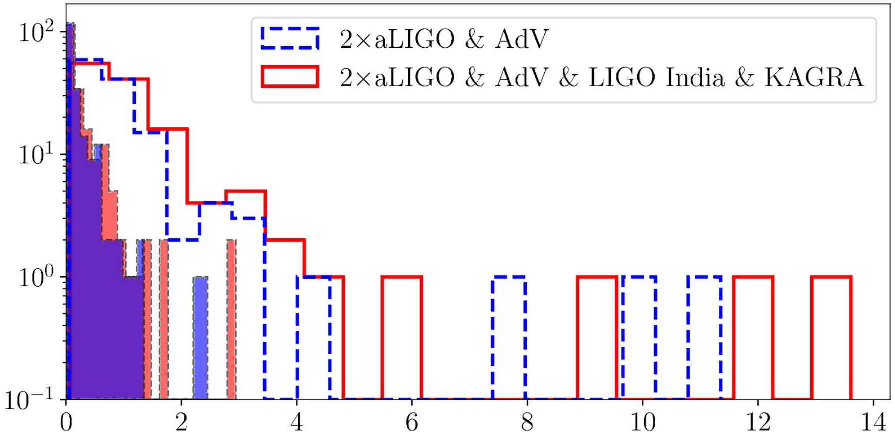
Best fit ellipticity for n=5 is ~10^{-9} (for moment of inertia of 10^{38} kg m^2)

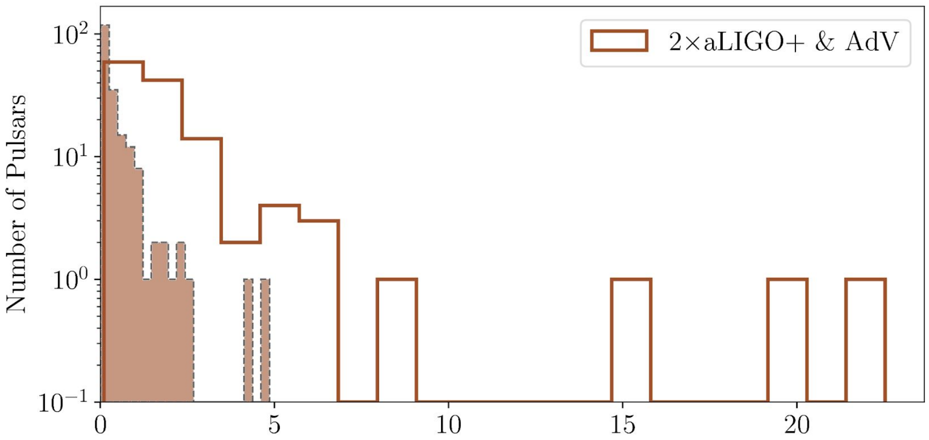
Ellipticity cut-off

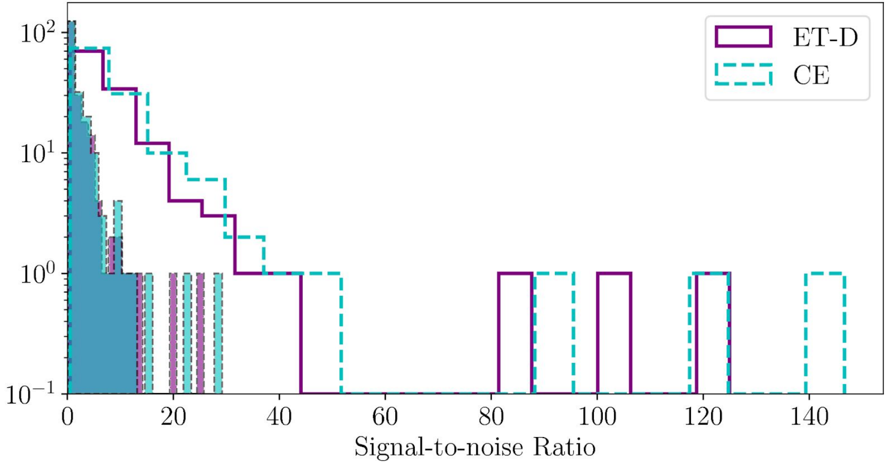
What could cause a minimum ellipticity in MSPs?

- MSPs are recycled; they underwent an accretion phase in a binary system to spin them up
    - Small external magnetic field for MSPs ($\sim 10^8$ Gauss) compared to "young" pulsars ($\gtrsim 10^{11}$ Gauss) suggests field may have been "buried" during accretion (e.g., Vigelius & Melatos, *MNRAS*, 395, 2009)
        - old and cold MSPs may have cores that are type II superconductors, so ellipticity is linear in internal field strength (e.g., Lander, *MNRAS*, 437, 2014) with $\epsilon \sim 10^{-8}(B_i/10^{12}$ Gauss) - so $\epsilon \gtrsim 10^{-9}$
    - Asymmetric crustal fracturing during spin-up (Fattoyev *et al*., arXiv:1804.04952), or spin-down (e.g., Baym & Pines, AnPhys, 66, 1971), could imprint a similar ellipticity in all MSPs

Implications for GW detections







Expected SNR for one year coherent observations of pulsars with various detector networks:

- Filled histograms - all pulsars with ellipticities of 10^{-9}
- Unfilled histograms - all pulsars emitting at their spin-down limits

Part III: the ellipticity distribution

(with C. Messenger & X. Fan, arXiv:1807.06726)

Ellipticity distribution

Known pulsar searches have just treated sources separately. However, we could instead assume that there is an underlying distribution from which their ellipticities are drawn, e.g., an **exponential** distribution

$$p(\varepsilon|\mu_\varepsilon, I) = \frac{1}{\mu_\varepsilon} e^{\varepsilon/\mu_\varepsilon}$$

defined by the *hyperparameter* μ_ε, the mean of the distribution.

We can combined data from all pulsars to estimate the probability distribution of μ_ε: ***hierarchical Bayesian inference*** (already used in GW field for black holes mass and spin distributions, e.g., Abbott *et al.*, *PRX* 6, 041015 (2016) & Stevenson, Berry & Mandel, *MNRAS*, 471 (2017)); estimate **parameters** of distribution *and* use the distribution's evidence as an **ensemble detection statistic**.

Ellipticity distribution

The same kind of analysis could be used on results from a blind all-sky search by changing the prior on the distance

We form a **joint likelihood** of the data from all pulsars marginalised over independent $\cos\iota$, ψ, and ϕ_0 values for each pulsar, and also marginalising over the uncertainty on distance (assuming a 20% Gaussian error):

$$p(\mu_\varepsilon|\{\mathbf{D}\}, I) \propto \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \left(\prod_{i=1}^{N_{\text{pulsars}}} \int^{\theta_i} p(\mathbf{D}_i|\varepsilon, \theta_i, d_i, I) p(\theta_i|I) \boxed{p(d_i|I)} \mathrm{d}\theta_i \mathrm{d}d_i \right) \boxed{p(\varepsilon|\mu_\varepsilon, I) p(\mu_\varepsilon|I)} \mathrm{d}\varepsilon$$

The probability distribution for the ellipticity distribution (and prior on hyperparameters) can be changed to your favourite function

where $\mathbf{D}_i$ is the data for each pulsar, $\theta_i = \{\phi_{0i}, \psi_i, \cos\iota_i\}$, and I assumes knowledge such as pulsar frequencies.

Calculating this for all pulsars at once would be a difficult problem (100s of parameters), but instead we can make use of posteriors distributions on ε (or equivalently Q_{22}) individually marginalised over other parameters.

Simulations

We generated independent realisations of signals from a population of 200 known pulsars (those searched for in O1 data) with ε drawn from exponential distributions with a range of μ_ε values, injected into Gaussian noise based on the aLIGO design sensitivity.

We also generated a set of “background” realisations, in which all pulsars had zero ellipticity (i.e. detector noise only).

For each population realisation we calculate the “evidence” that the data contains pulsars with ε drawn from an exponential distribution, and also that the population is consistent with noise.

Exponential mean, μ_ε, prior (Jeffreys)

$$p(\mu_\varepsilon|I) = \frac{(\ln \mu_{\varepsilon\mathrm{max}} - \ln \mu_{\varepsilon\mathrm{min}})^{-1}}{\mu_\varepsilon} \text{ if } \mu_{\varepsilon\mathrm{min}} < \mu_\varepsilon < \mu_{\varepsilon\mathrm{max}}$$

Bayesian evidence for population with ellipticities drawn from an exponential distribution

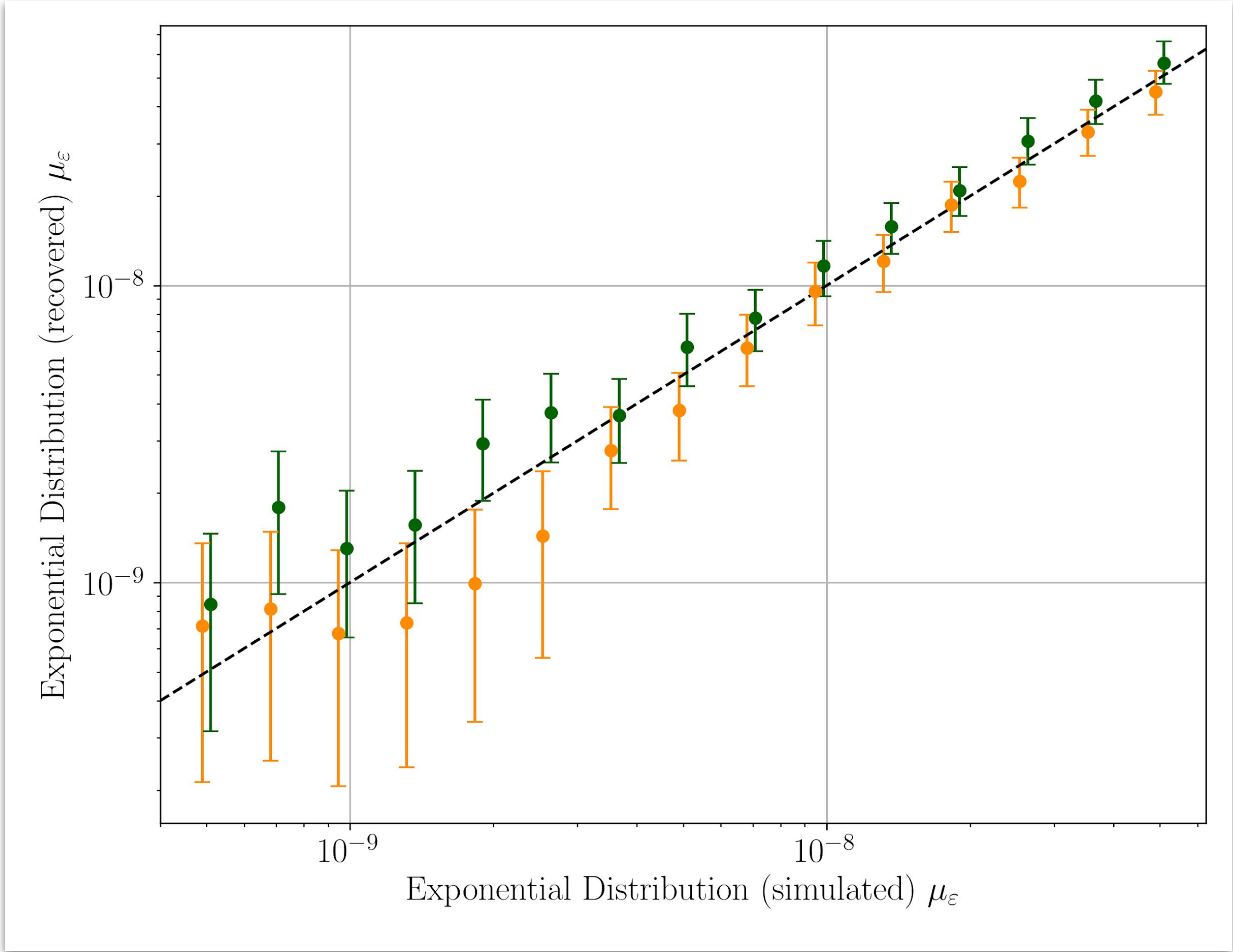
$$p(\{\mathbf{D}\}|H_{\mathrm{exp}}, I) = \int^{\mu_\varepsilon} p(\{\mathbf{D}\}|\mu_\varepsilon, H_{\mathrm{exp}}, I) p(\mu_\varepsilon|H_{\mathrm{exp}}, I) \mathrm{d}\mu_\varepsilon$$

Bayesian odds comparing evidence for exponential distribution to data consistent with noise

$$\mathcal{O} = \frac{p(\{\mathbf{D}\}|H_{\mathrm{exp}}, I)}{p(\{\mathbf{D}\}|H_{\mathrm{noise}}, I)}$$

Results

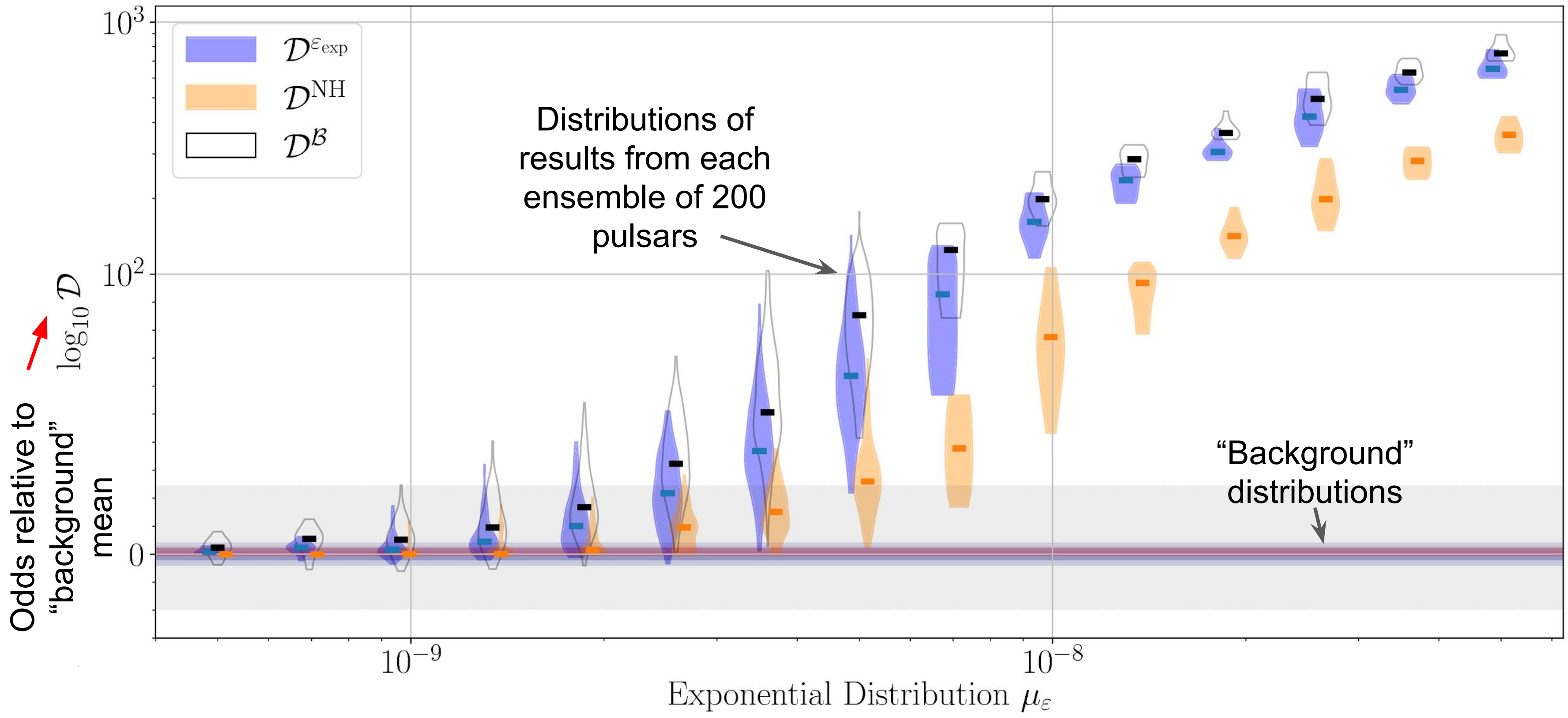
90% credible intervals from the posterior distributions on μ_ε for the simulated population distributions (orange/green intervals are for the ensemble with the largest/smallest odds)



Results

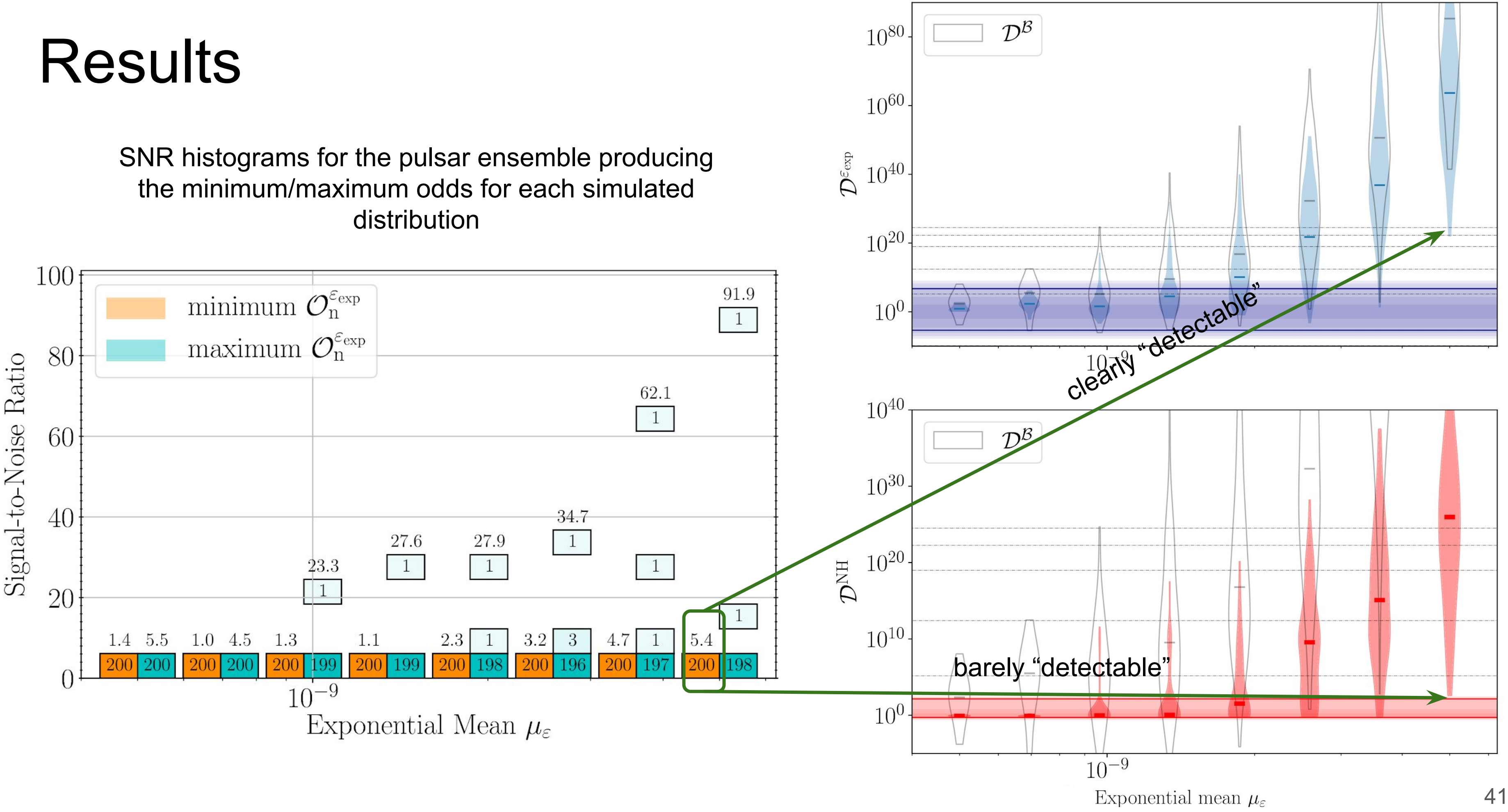
Odds comparing a hypothesis that the data is consistent with signals having ellipticities drawn from an exponential distribution against all data being consistent with noise

Odds comparing a hypothesis that the data is consistent with *any* combination of pulsars containing a signal (signal amplitudes are non-hierarchically marginalised over) against all data being consistent with noise



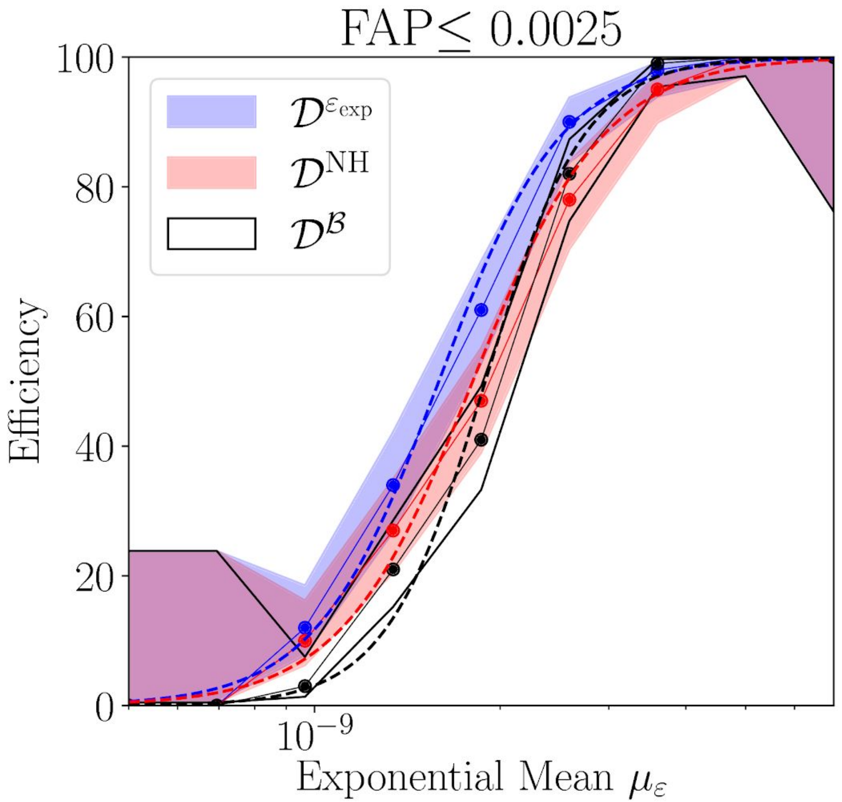
Results

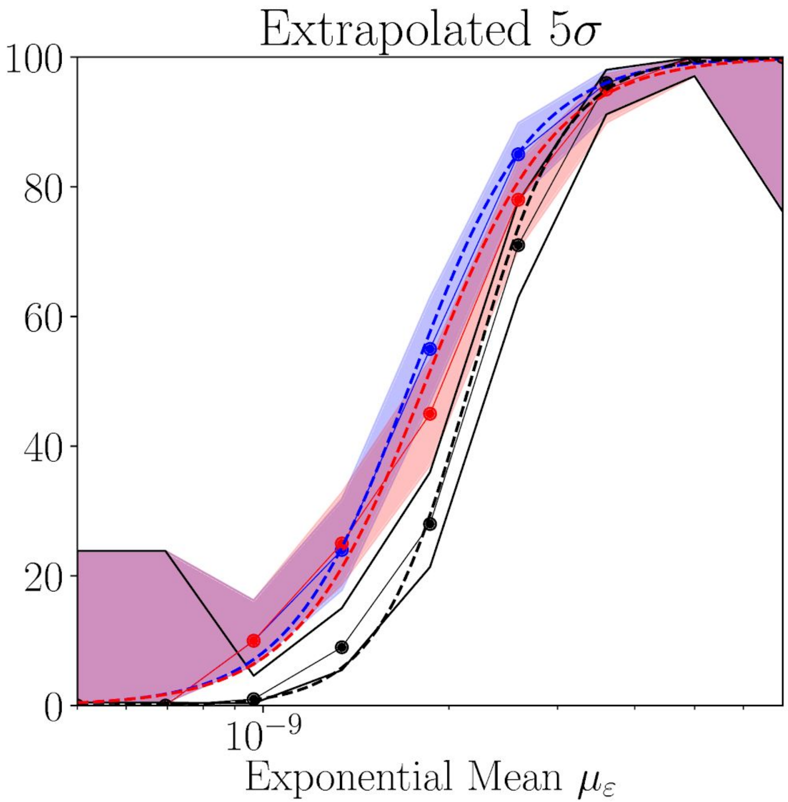
SNR histograms for the pulsar ensemble producing the minimum/maximum odds for each simulated distribution



Results

Efficiency curves for detection of ensemble of pulsars.
Left: using a “false alarm probability” set by the number of background realisations.
Right: extrapolating the background to an equivalent “5σ” level (using a KDE of the background).

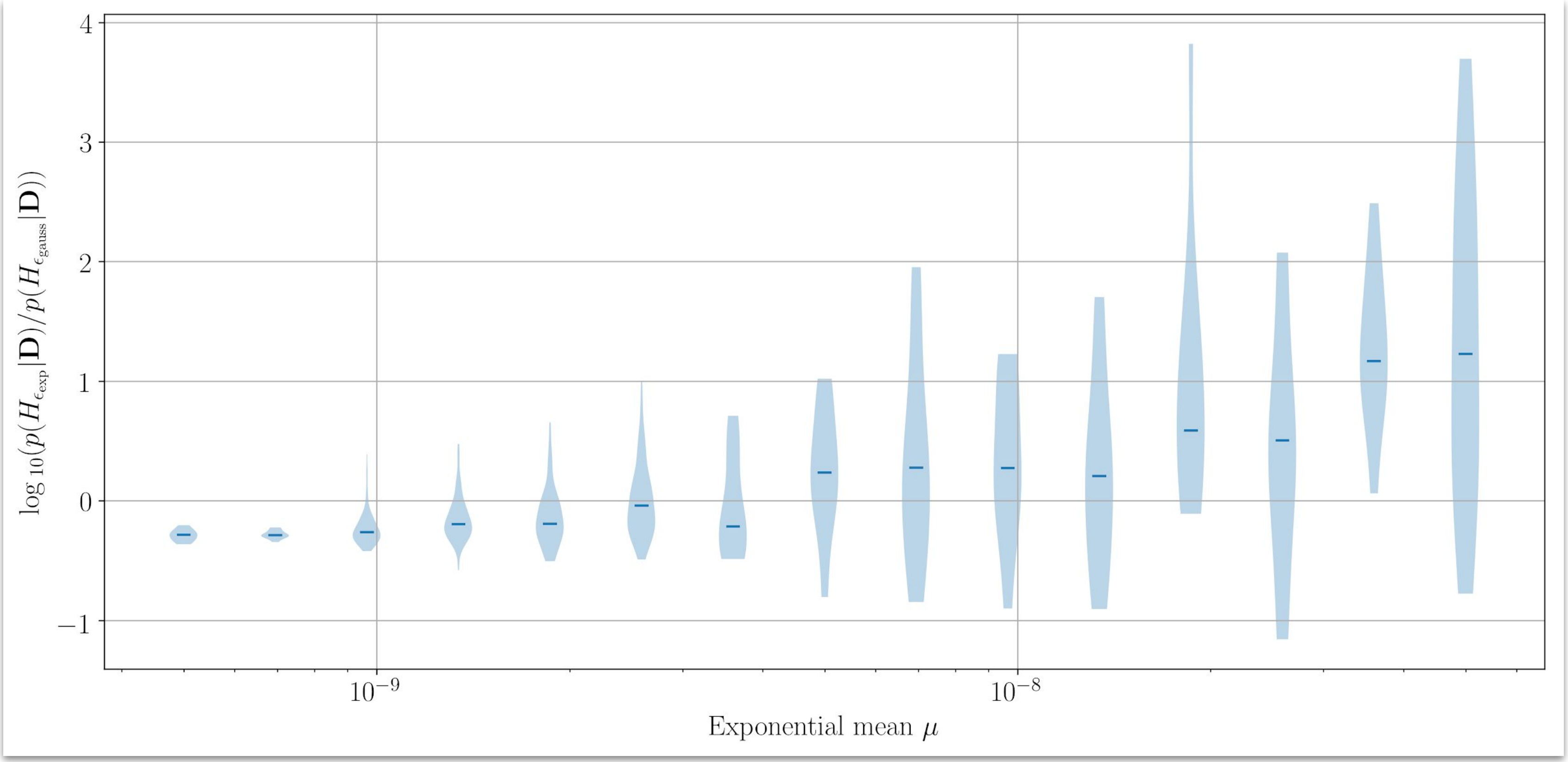




Results

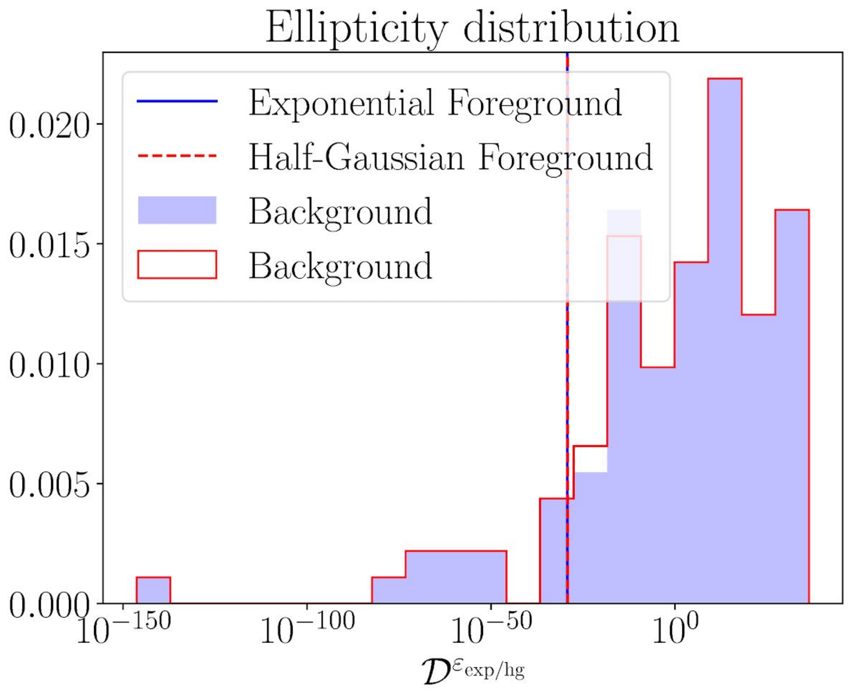
What about a different distribution? We can assume the distribution is a half-Gaussian (using the same prior on σ_ε as we had for μ_ε)and compare models

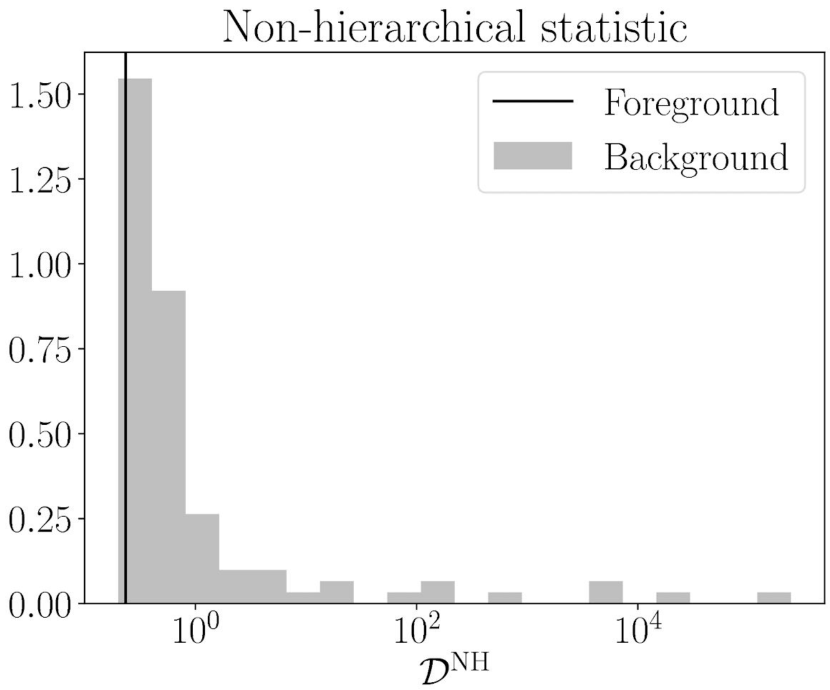
$$p(\varepsilon|\sigma_\varepsilon, I) = \frac{2}{\sqrt{2\pi}\sigma_\varepsilon} e^{-\frac{1}{2}(\varepsilon/\sigma_\varepsilon)^2}$$



S6 results

Using 92 pulsars from the S6 known pulsar search (Aasi *et al*, *ApJ*, 785 (2014)) we applied the analysis assuming a 20% Gaussian error on pulsar distances (and upper bound on ε of ~10^{-4}). **No detection** of an ensemble of sources.

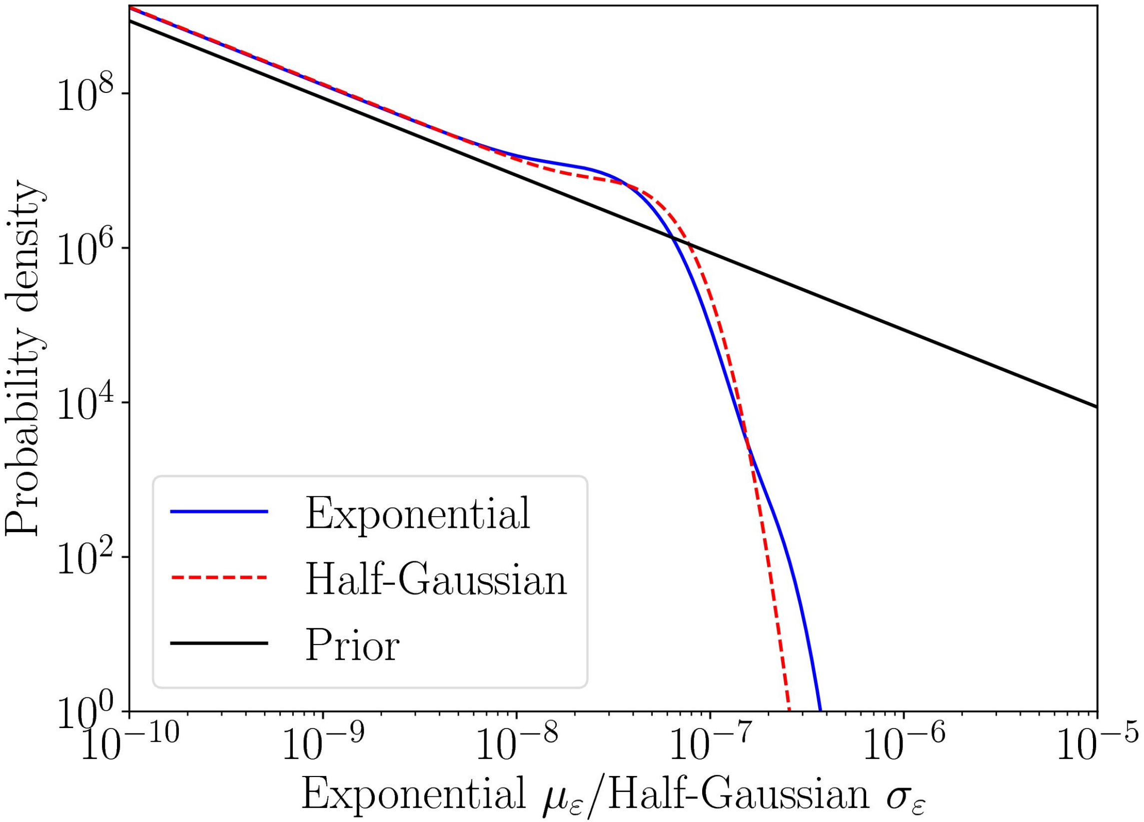




S6 results

Set 90% credible upper limits of:

- $\mu_\varepsilon < 3.9 \times 10^{-8}$ for an exponential distribution
- $\sigma_\varepsilon < 4.7 \times 10^{-8}$ for a half-Gaussian distribution

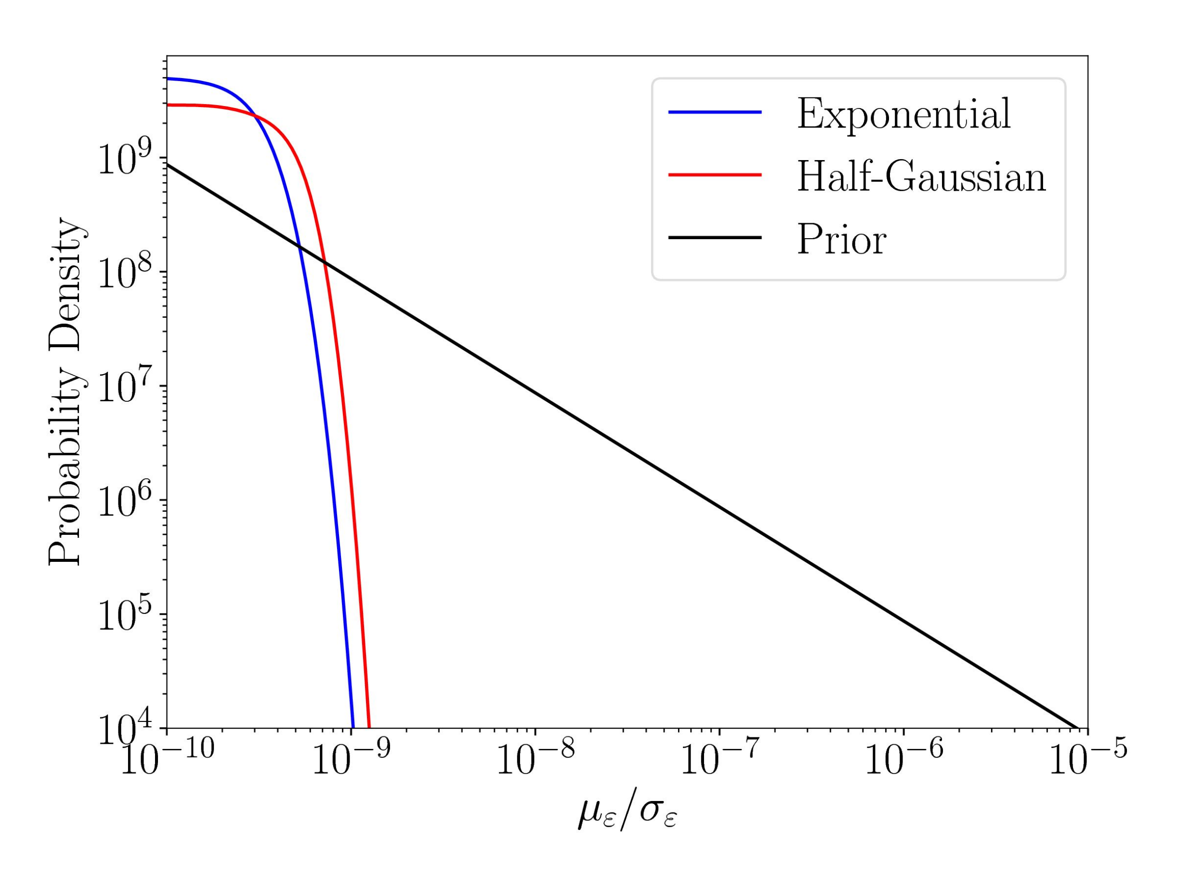


Spin-down limit results

Instead just use spin-down limits as likelihoods on ellipticity.

Set 90% credible upper limits of:

- $\mu_\varepsilon < 3.0 \times 10^{-10}$ for an exponential distribution
- $\sigma_\varepsilon < 4.1 \times 10^{-10}$ for a half-Gaussian distribution



Part III: Conclusions

Searches for known pulsars provide tight constraints on neutron star ellipticity that are starting to compete with *spin-down limits*

- We can apply the hierarchical Bayesian method to constrain (a parameterised model-dependent) ellipticity distribution using results from real searches.
- This provides a new detection statistic for the ensemble of pulsars rather than relying on detecting individual sources that can beat existing methods (see also, e.g., Cutler & Schutz, *PRD* 72, 063006 (2005), Fan, Chen & Messenger, PRD 94, 084029 (2016) & Smith & Thrane, arXiv:1712.00688).

Part III: Future work

What might be a good model for the underlying distribution?

- Exponential?
- Power law?
- Gaussian mixture?
- Non-parametric (i.e. histogram, IGMM)?

Should we fit separate distributions for MSPs & young pulsars / globular cluster & field pulsars / binary pulsars and isolated pulsars?

Should/could we also constrain model-dependent internal B-field distributions?

What distance priors are reasonable if using sources from blind searches?