

Newtonian noise studies for future generation gravitational-wave detectors

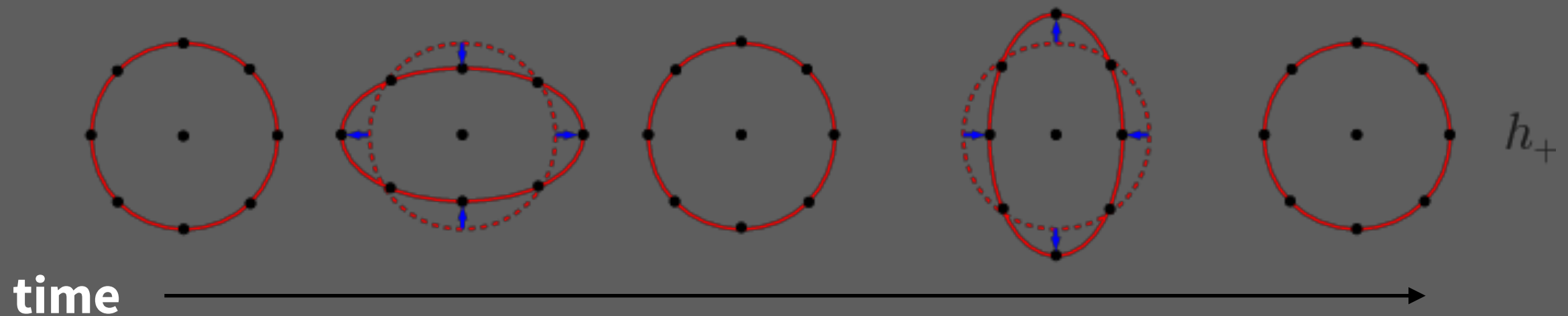
Pat Meyers
University of Melbourne
OzGrav
November 14th, 2018

Outline

- Gravitational waves (GWs) and LIGO
- The case for low frequency sensitivity
- Proposed low frequency detectors
- Newtonian noise
- Basic seismology
- Newtonian noise studies with Homestake 3D seismometer array
- Seismologically useful results

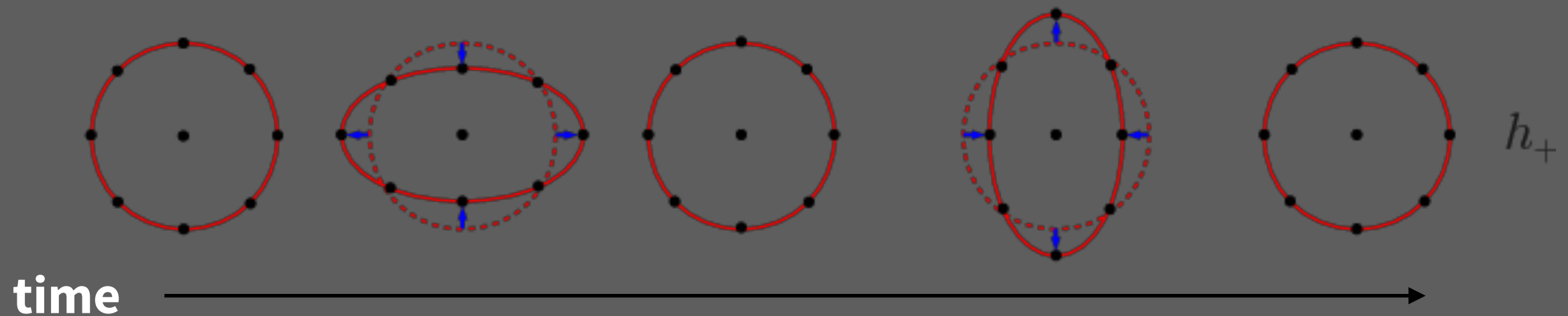
GW Polarization

“**Plus**” Polarization

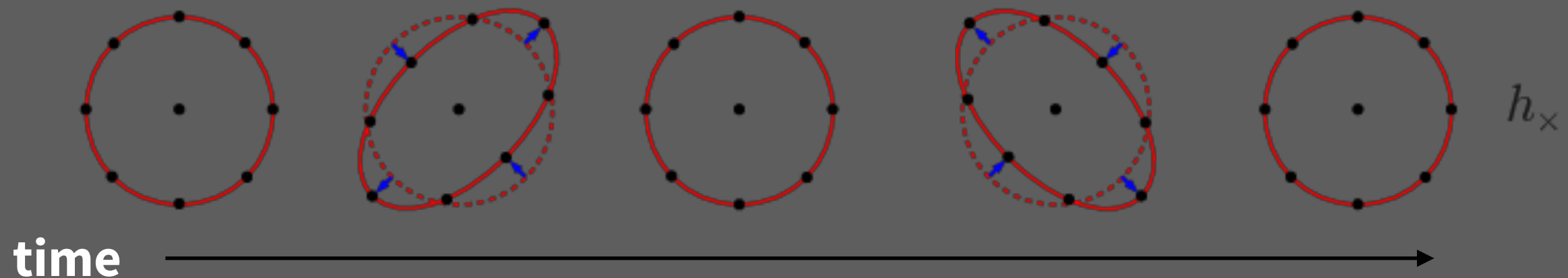


GW Polarization

“Plus” Polarization



“Cross” Polarization



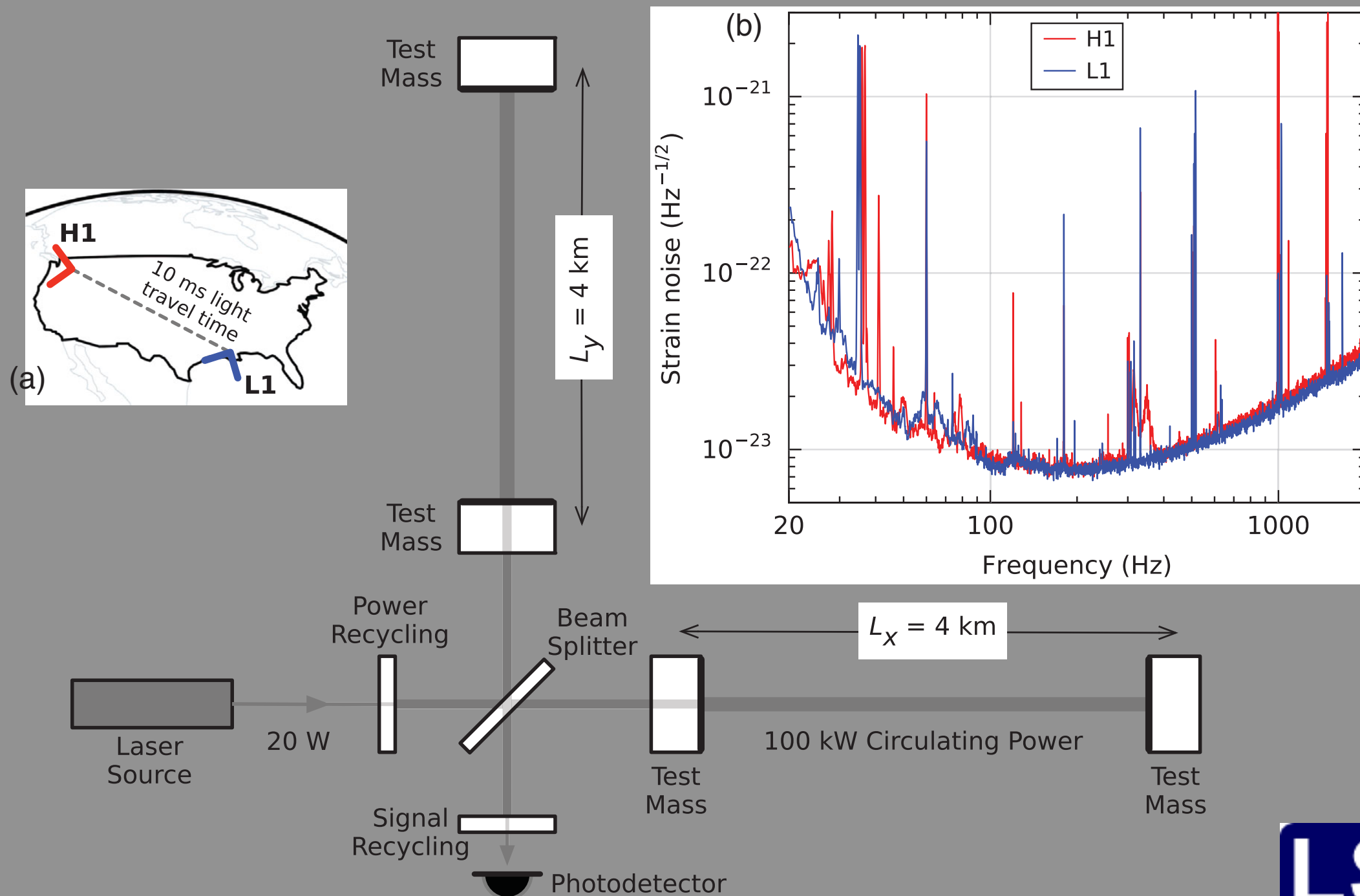
GW Amplitude

GW amplitude is measured in **strain**:

$$h = \Delta L / L$$

Blue arrows divided by radius of ring
(from previous slide)

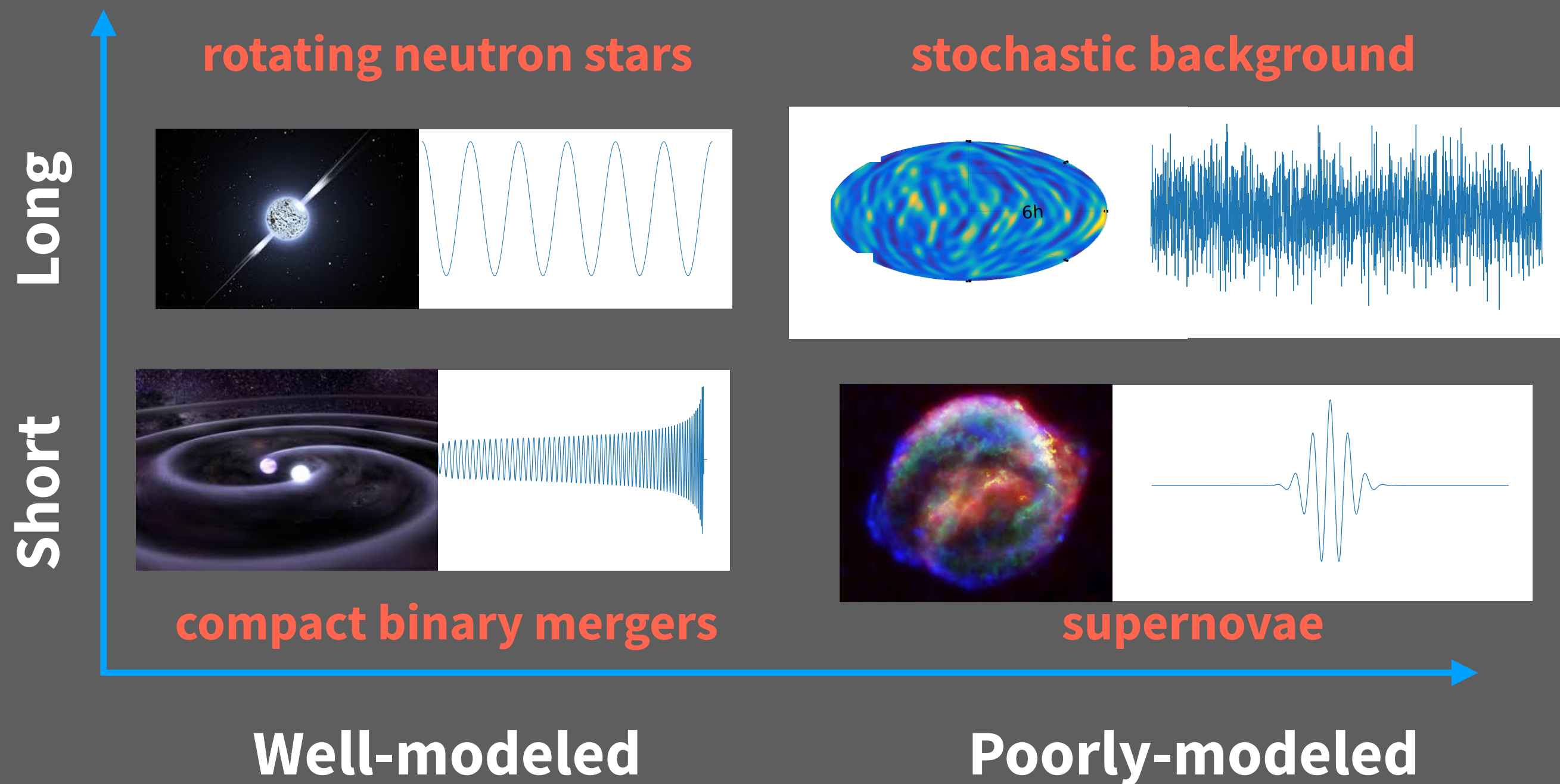
Advanced LIGO/Advanced Virgo



Martynov, D. V., et al. (2016). *Phys. Rev. D*, 93(11), 433. <http://doi.org/10.1103/PhysRevD.93.112004>

GW sources

targeted by LIGO



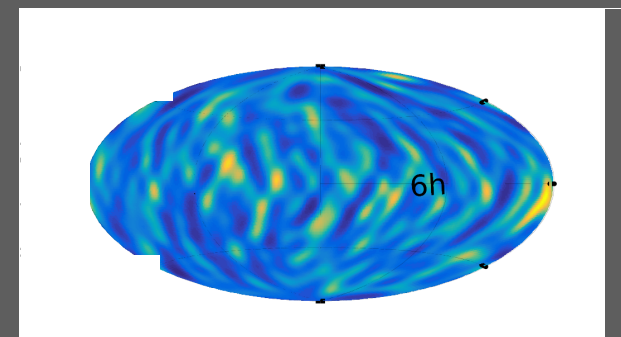
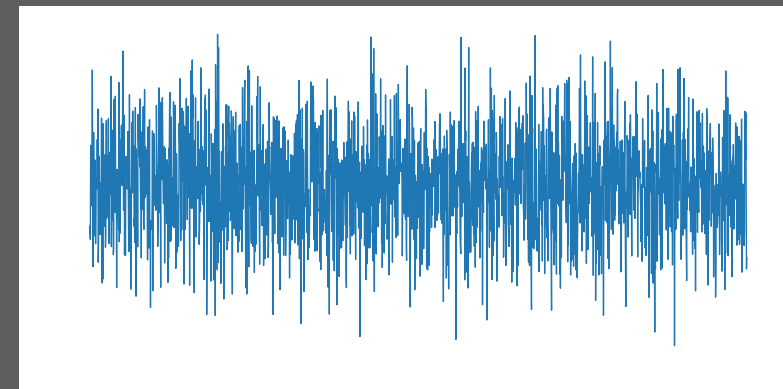
Stochastic Gravitational-wave Background

Sources

- Unresolved astrophysical sources
 - CBCs
 - Rotating neutron stars
- Early universe models
- cosmic strings

SGWB Properties/Assumptions

- Gaussian
- unpolarized
- stationary
- Isotropic
- In some searches (not discussed here) we relax one or more of these assumptions



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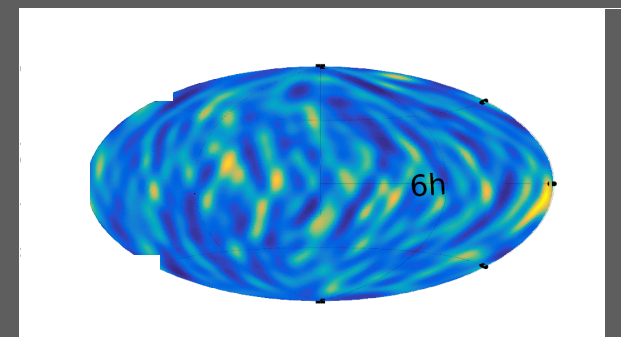
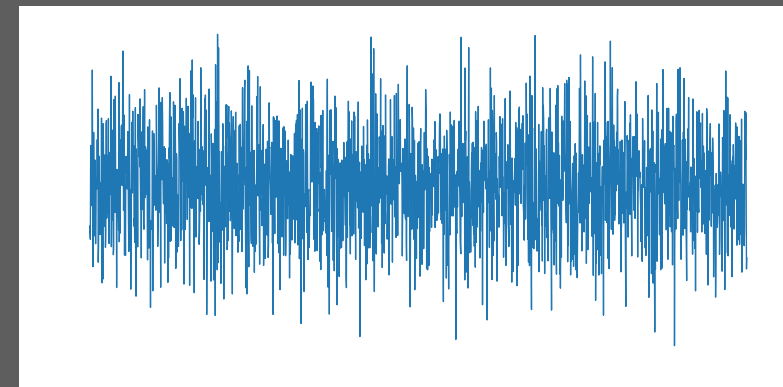
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$$\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df}$$

Energy density of GWs

Critical energy density
to close the universe



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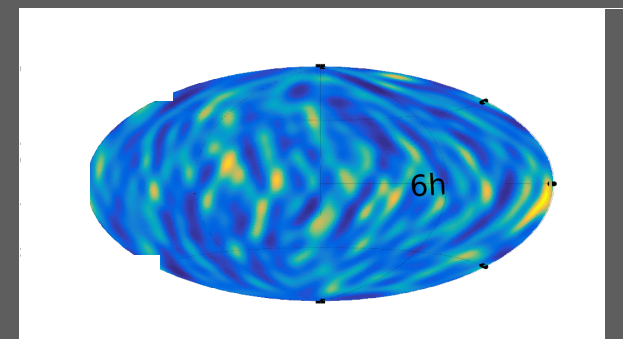
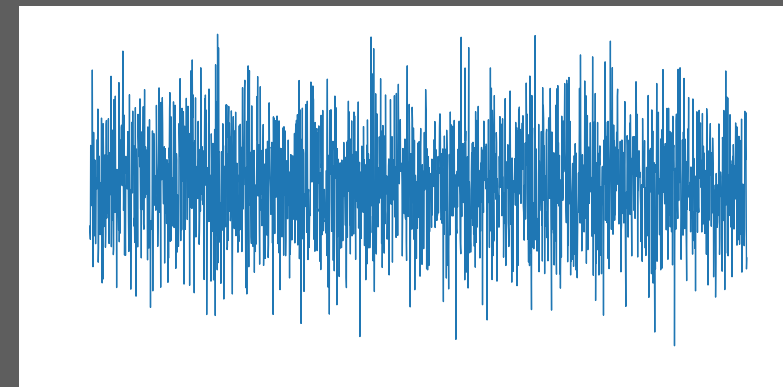
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Energy density of GWs

Critical energy density
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$$\rho_{\text{GW}} \propto \langle \dot{h} \dot{h} \rangle$$

$$\Rightarrow \Omega_{\text{GW}}(f) \propto f^3 \langle h h \rangle$$



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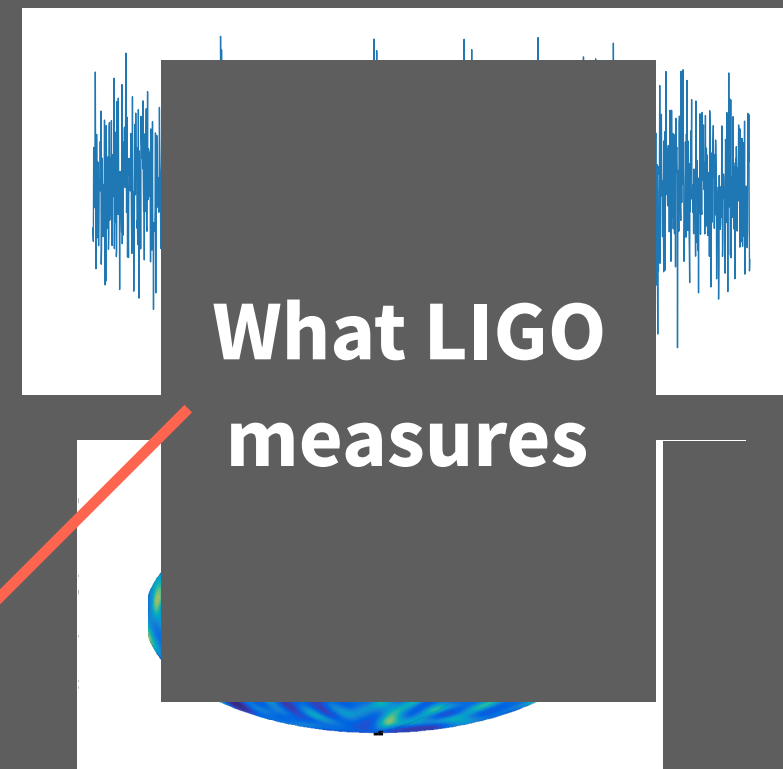
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Another case for low frequencies: BBH systems

- Higher-mass BBH systems coalesce at (much) lower frequencies
- Cosmological red shift \rightarrow further-away systems get redshifted
- Higher-mass \rightarrow inspiral faster at a given frequency (i.e. \dot{f} at a given f is higher for higher mass systems)
- Lower frequencies \rightarrow get more cycles \rightarrow more likely to see higher-mass (and more distance) black hole systems

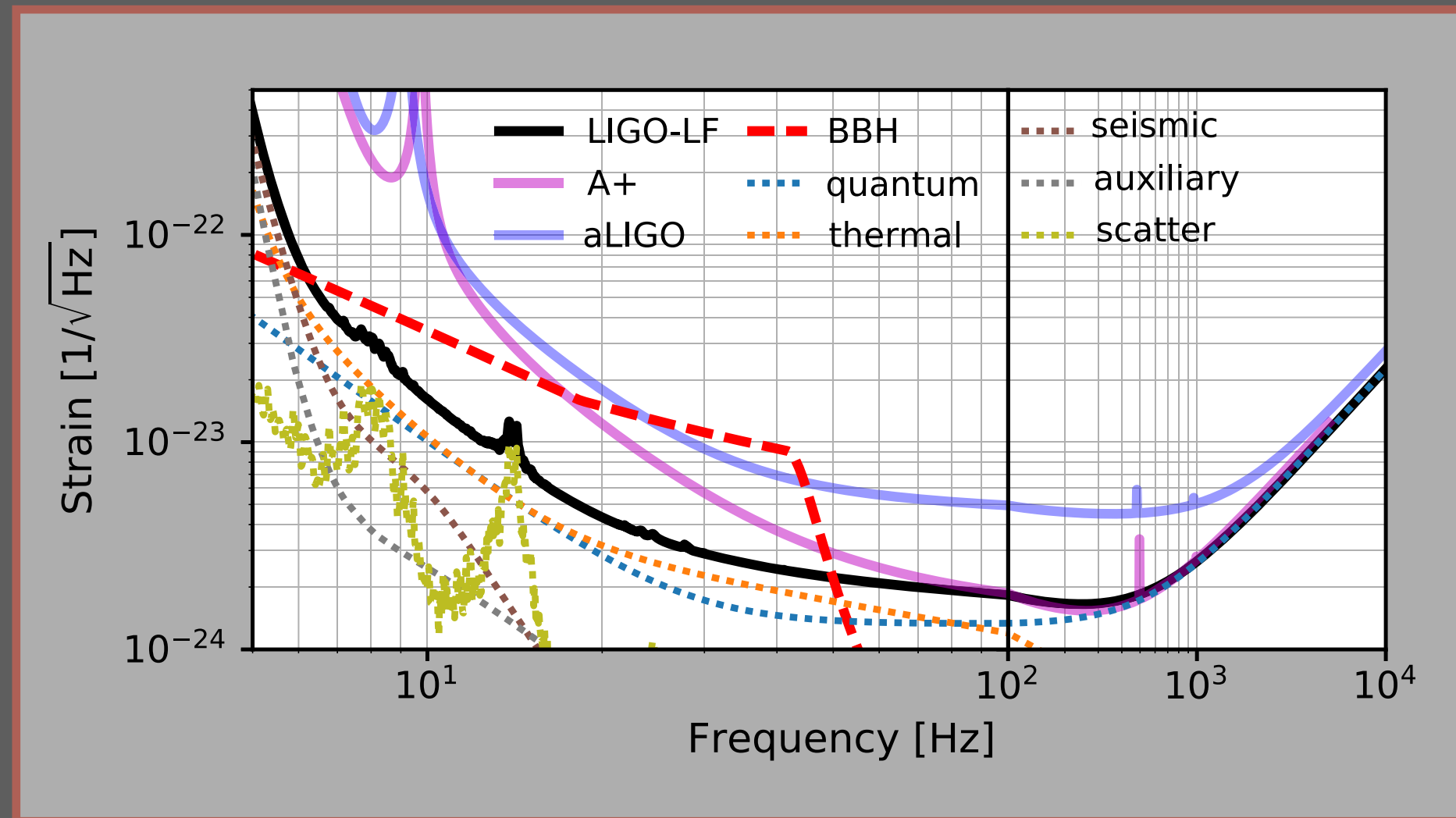
Abbott, B. P., et al. (2017). *Annalen Der Physik*, 529(1–2), 1600209.
<https://doi.org/10.1002/andp.201600209>

Proposed low frequency

current state

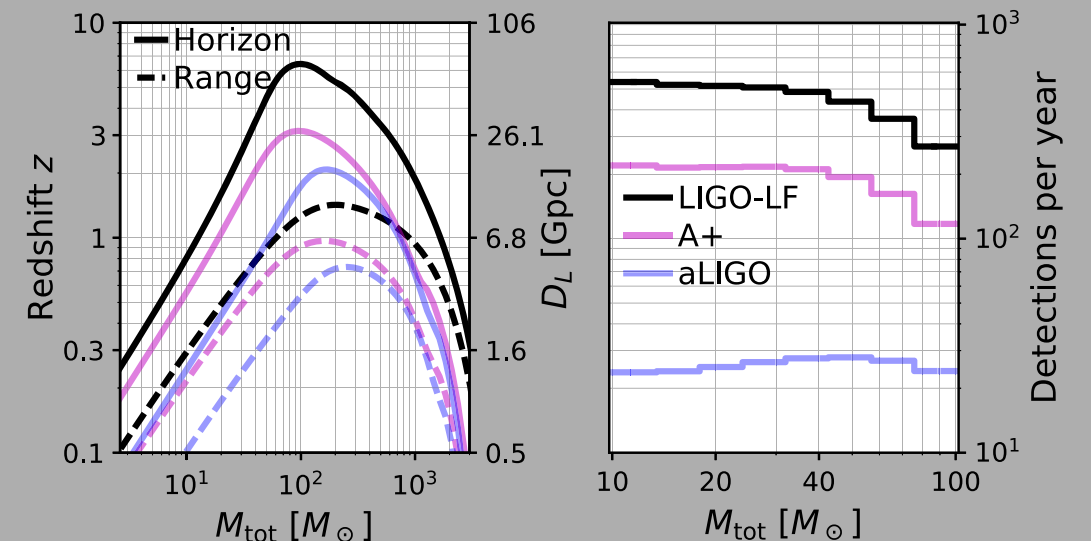
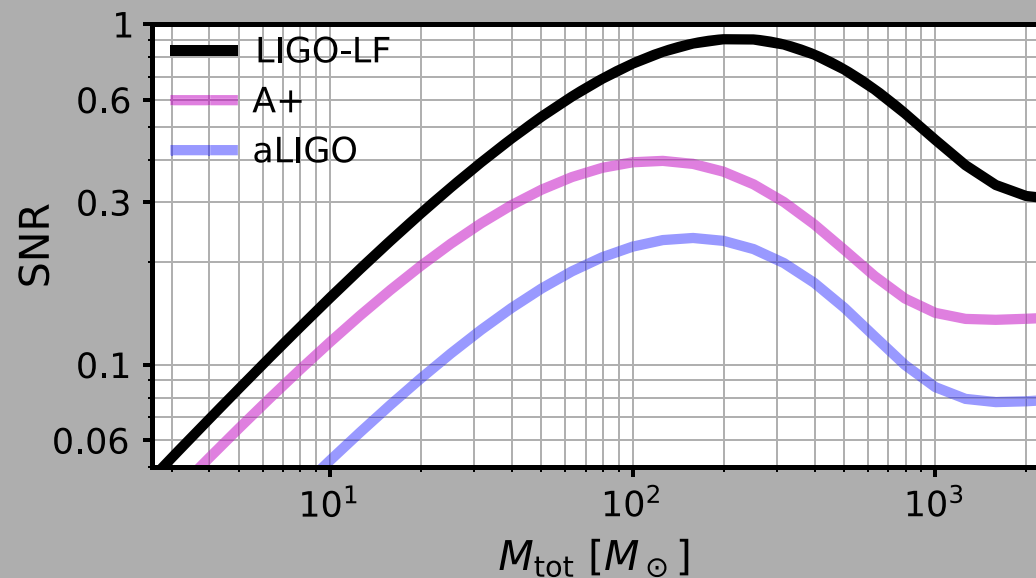
LIGO-LF

- “Use current facilities, but max out all of the current technology”
- Better auxiliary sensors → better damping/angular controls
- Double pendulum length in suspensions
- Increase fibre tensions
- Larger test masses
 - (40 kg → 200 kg)
- More squeezing

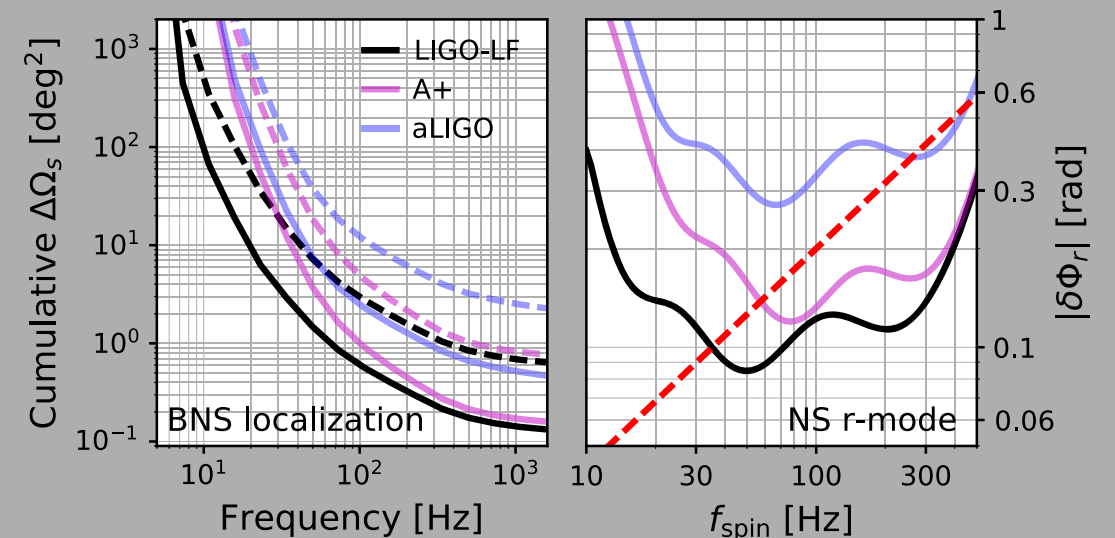


Yu, H., et al. (2018). <https://doi.org/10.1103/PhysRevLett.120.141102>

BBH improvements



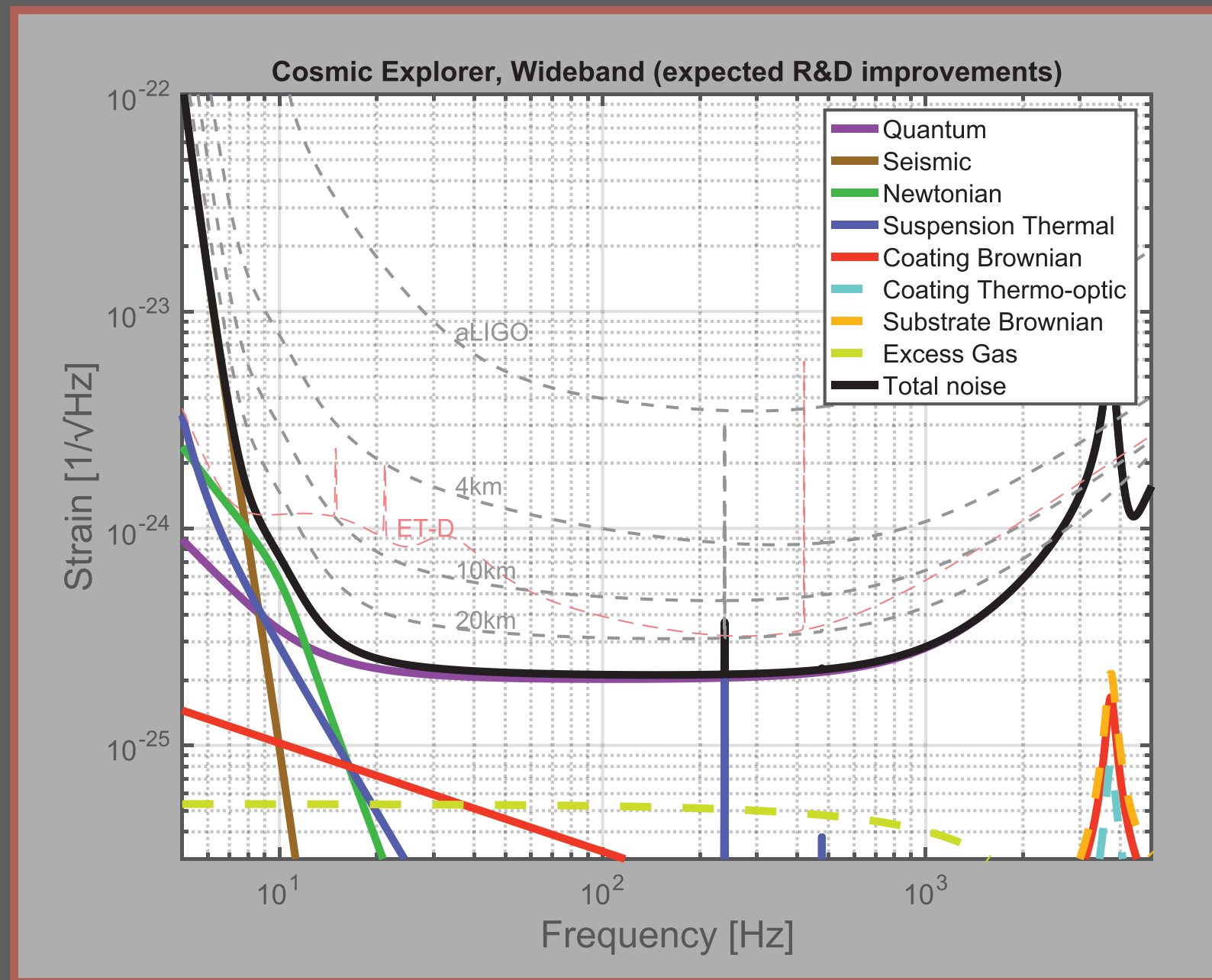
- Better sky localisation
- Sensitive to more distant objects
- Sensitive to higher-mass black holes



Yu, H., et al. (2018). <https://doi.org/10.1103/PhysRevLett.120.141102>

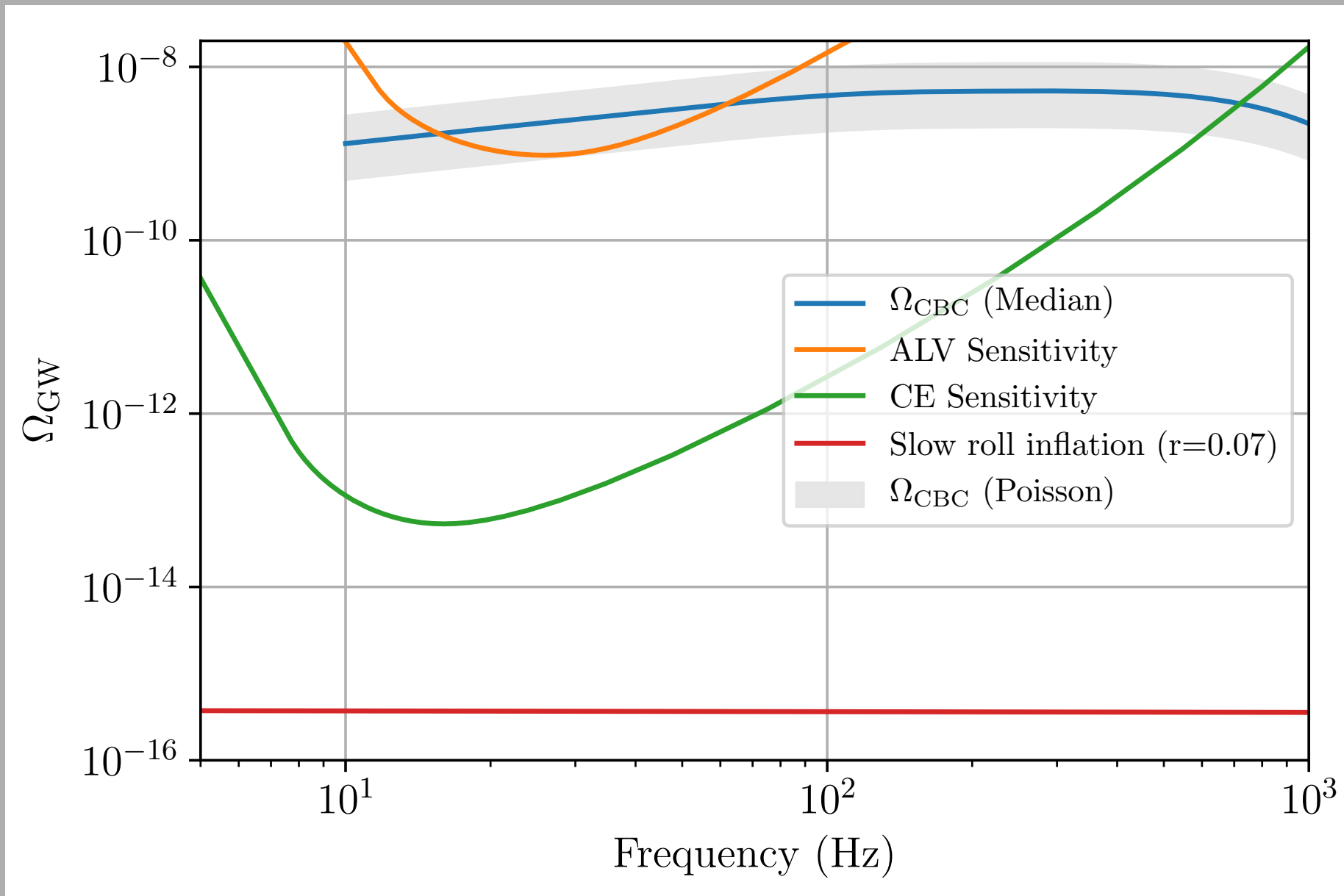
Cosmic Explorer

- 40 km arms
- $T = 123\text{ K}$
- $m_{\text{TM}} = 320\text{ kg}$
- $\lambda = 1550\text{ nm}$
- $P = 2\text{ MW}$



Abbott, B. P., et al. (2017). 34(4), 44001. <https://doi.org/10.1088/1361-6382/aa51f4>

CE SGWB Sensitivity



Plot courtesy of Andrew Matas

Other proposed (ground-based) detectors

- Einstein Telescope — 10 km triangular configuration (European proposal) ($0(1\text{ Hz})$ — $0(1\text{ kHz})$) [<http://www.et-gw.eu/>]
- **MANGO** — laser & atom interferometer ($.01\text{ Hz}$ — 1 Hz)
- **TOBA** — “Torsion bar antenna” ($1e-3\text{ Hz}$ — 10 Hz)
- **TorPeDO** — torsion bar, ANU, ($1e-3\text{ Hz}$ — 10 Hz)
- Could also be useful for directly measuring Newtonian Noise

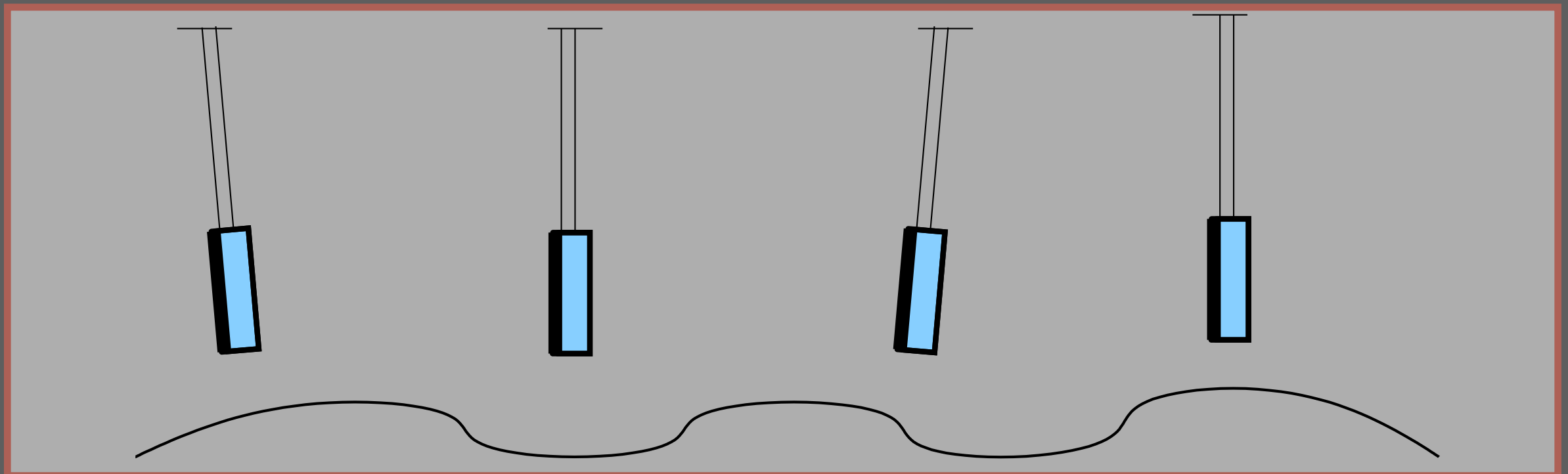
Harms, J., et al. (2013) <https://doi.org/10.1103/PhysRevD.88.122003>

McManus, D. J., et al. <https://doi.org/10.1088/1361-6382/aa7103>

Newtonian Noise

Newtonian Noise

- Gravitational fluctuations at the test mass
 - Density/temperature perturbations in atmosphere
 - Seismic waves
- Likely to become a limiting noise source for advanced detectors at lower frequencies



Newtonian Noise

from seismic fields

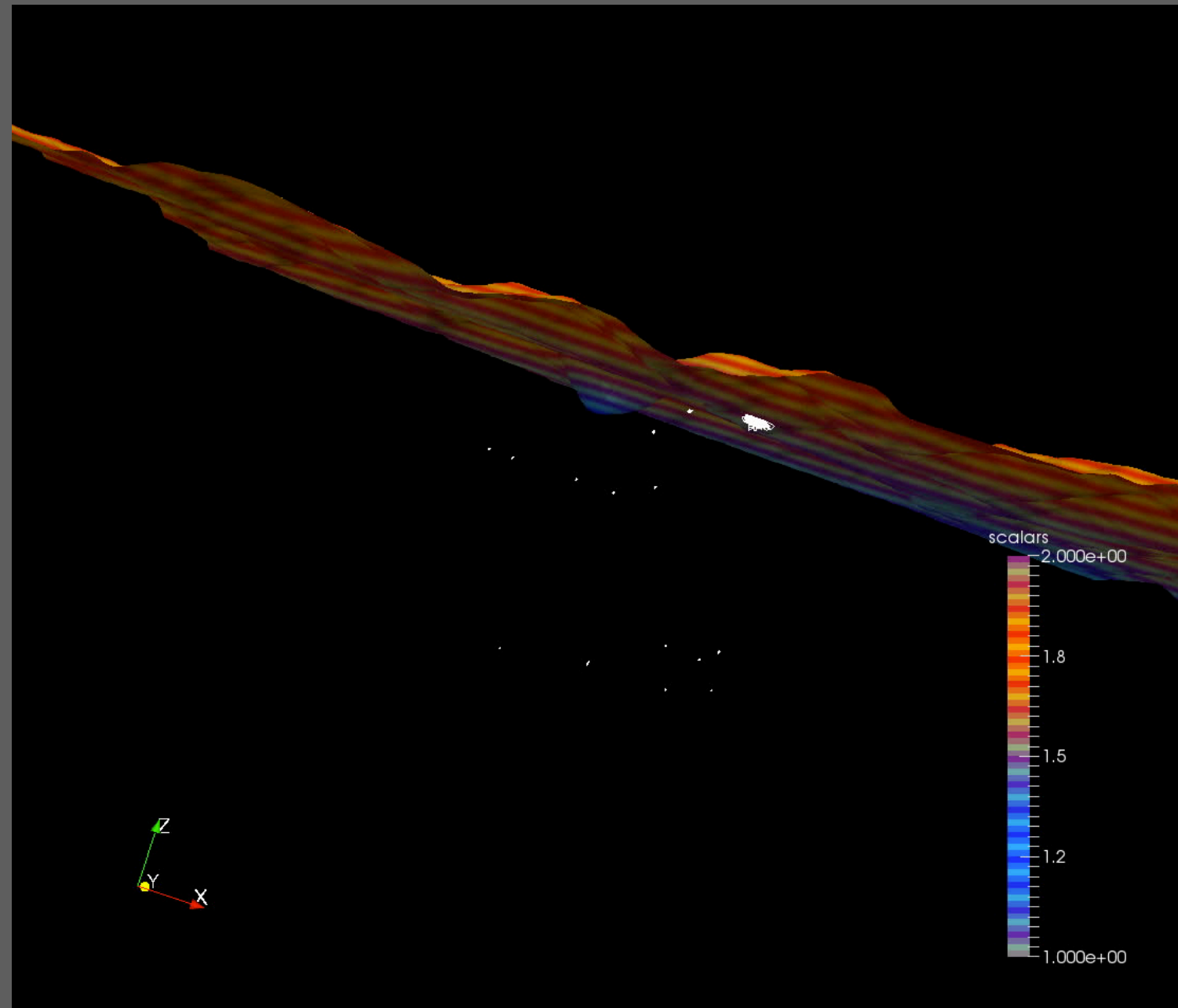
- Perturbation to gravitational field related by displacement field, ξ
- ξ is different for different types of seismic waves
- Name of the game: estimate ξ for different types of seismic waves

$$\delta \vec{a}(\vec{r}_0, t) = -G \int dV \rho(\vec{r}) (\vec{u}(\vec{r}, t) \cdot \nabla_0) \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3}$$

$$h_{\text{NN}} = \frac{\sqrt{2 (\delta a_{x,\text{rms}}^2 + \delta a_{y,\text{rms}}^2)}}{(2\pi f)^2 L}$$

Seismic waves

- **P**, **S**, and **R**-waves
 - **P** — “primary” or “pressure” waves. Longitudinal wave
 - **S** — “secondary” or “shear” waves. Transverse wave
 - **R** — “Rayleigh” waves. Superposition of **P** and **S** waves. Characterized by **retrograde** particle motion
- Visualization of transient is courtesy of Gary Pavlis at IU.
- [Link to video](#)



GOALS:

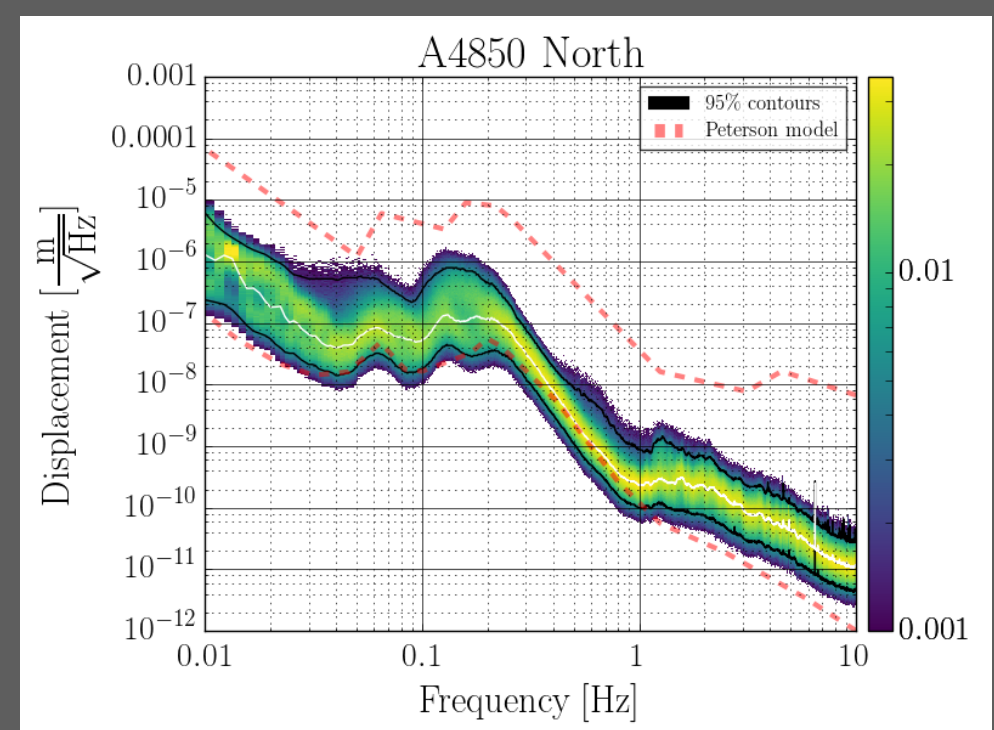
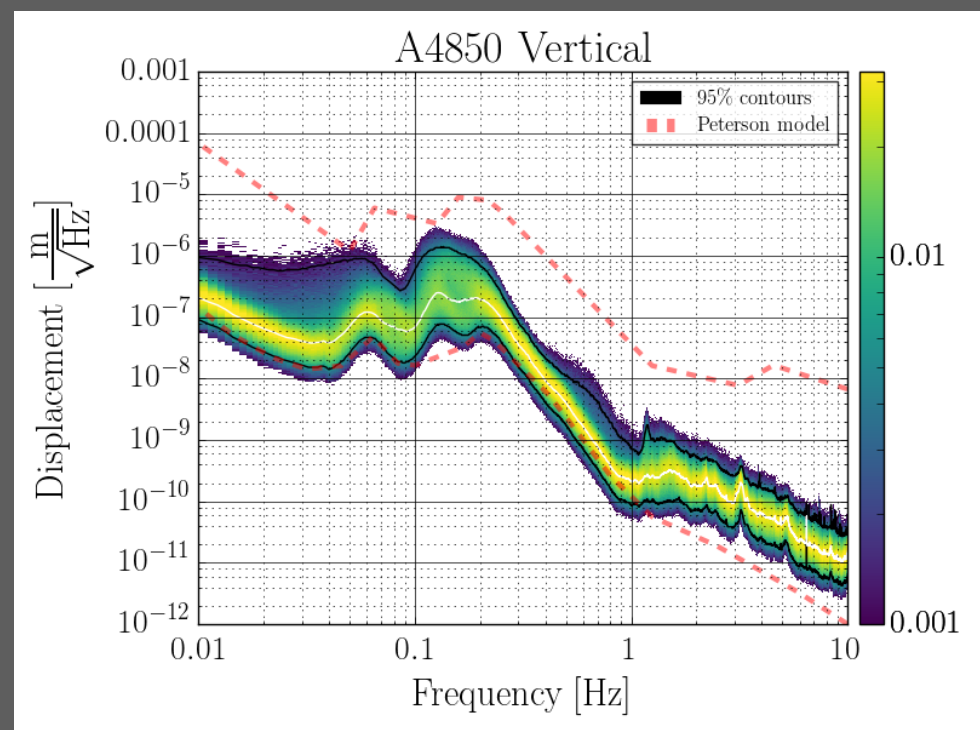
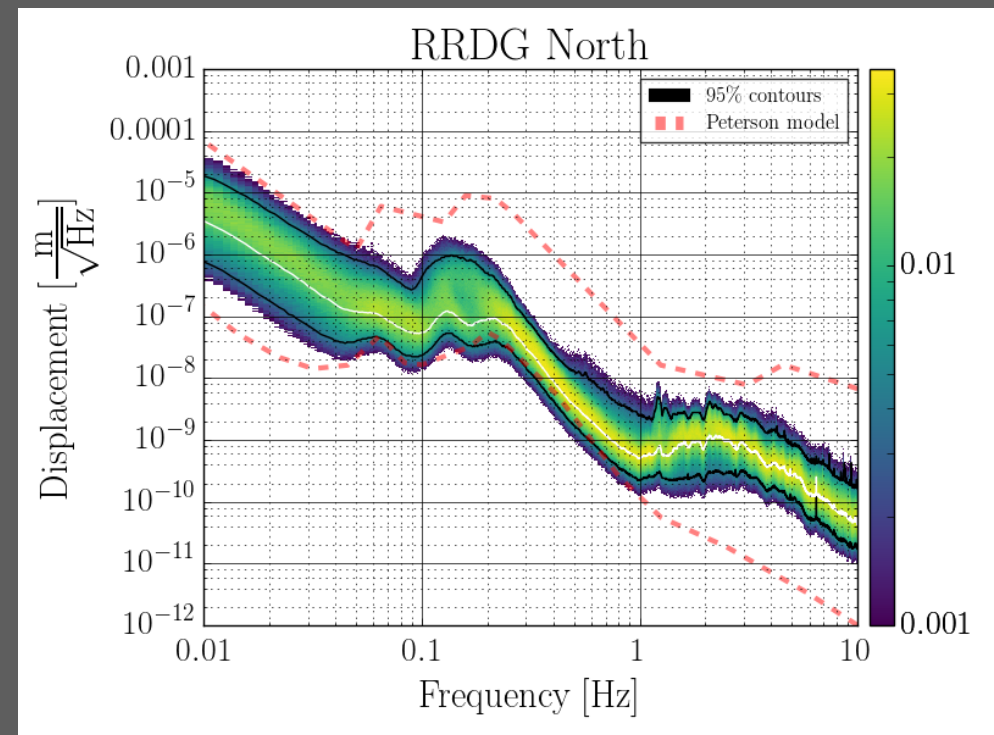
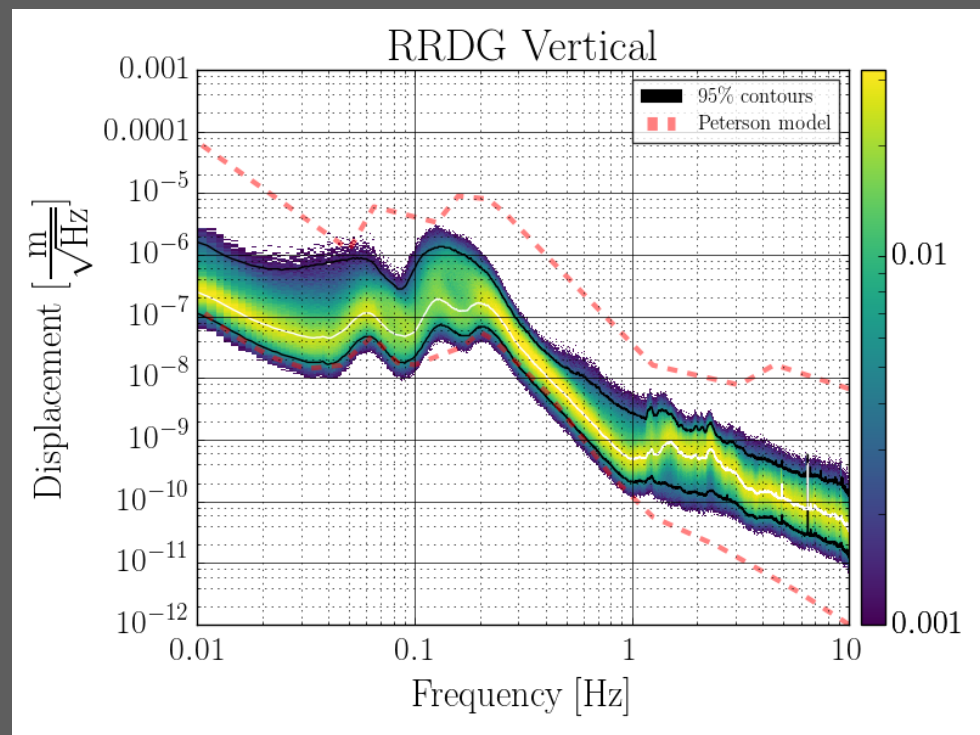
- Estimate amplitude of different seismic waves simultaneously
 - From this, estimate Newtonian Noise
-

Homestake array

- ▶ Located at Sanford Underground Research Facility in Lead, SD.
- ▶ Collaborators at CIT, Indiana University, University of Minnesota
- ▶ 24 seismometers
 - ▶ 15 underground
 - ▶ 9 surface
 - ▶ STS2 and Guralp 3T
- ▶ Ran from November 2014 - December 2016
- ▶ Roughly 1 cubic mile
- ▶ Data set is now public on IRIS



World class data



Seismic radiometer

- ▶ Try to decouple different wave types
- ▶ Reconstruct propagation direction and amplitude for each type

Correlation of i^{th} channel pair

Field amplitude in direction d (or basis element for sky decomposition)

$$\log(\mathcal{L}) \propto -\frac{1}{2} \left((Y_i^* - \gamma_{i,d}^* S_d) N^{-1} (Y_i - \gamma_{i,d} S_d) \right)$$

Direction, d : ϕ, θ

$$S_d = (\gamma^T \gamma)^{-1} \gamma^T Y_i$$

maximum likelihood estimator

- ▶ **Eventual goal:** Use maps to get NN estimates

Brief aside:

Gamma matrices

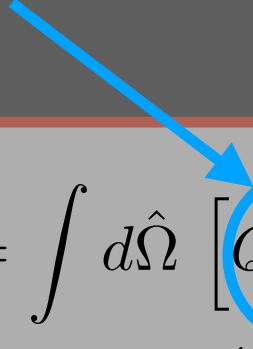
- ▶ α =channel of detector “i” (i.e., N, E, V)
- ▶ β =channel of detector “j”
- ▶ a =basis function label

$$\gamma_{R,a}^{i\alpha,j\beta} = \int d\hat{\Omega} \left[Q_a(\hat{\Omega}) \left(r_H(z) \hat{\Omega} \cdot \hat{\alpha} - e^{i\pi/2} r_V(z) \hat{z} \cdot \hat{\alpha} \right) \times \right. \\ \left. \left(r_H(z) \hat{\Omega} \cdot \hat{\beta} - e^{-i\pi/2} r_V(z) \hat{z} \cdot \hat{\beta} \right) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x} / v_R} \right].$$

Brief aside:

Gamma matrices

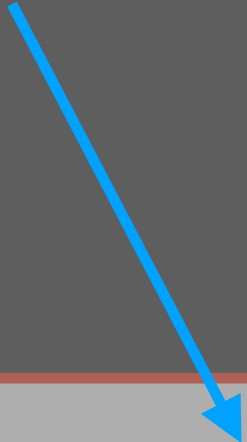
Basis function (delta functions)


$$\gamma_{R,a}^{i\alpha,j\beta} = \int d\hat{\Omega} \left[Q_a(\hat{\Omega}) \left(r_H(z) \hat{\Omega} \cdot \hat{\alpha} - e^{i\pi/2} r_V(z) \hat{z} \cdot \hat{\alpha} \right) \times \right. \\ \left. \left(r_H(z) \hat{\Omega} \cdot \hat{\beta} - e^{-i\pi/2} r_V(z) \hat{z} \cdot \hat{\beta} \right) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x} / v_R} \right].$$

Brief aside:

Gamma matrices

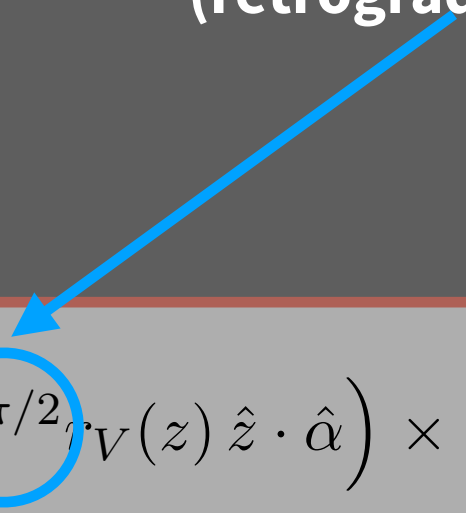
Propagation direction


$$\gamma_{R,a}^{i\alpha,j\beta} = \int d\hat{\Omega} \left[Q_a(\hat{\Omega}) \left(r_H(z) \hat{\Omega} \cdot \hat{\alpha} - e^{i\pi/2} r_V(z) \hat{z} \cdot \hat{\alpha} \right) \times \right. \\ \left. \left(r_H(z) \hat{\Omega} \cdot \hat{\beta} - e^{-i\pi/2} r_V(z) \hat{z} \cdot \hat{\beta} \right) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x} / v_R} \right].$$

Brief aside:

Gamma matrices

alpha/beta Phase difference
(retrograde motion)


$$\gamma_{R,a}^{i\alpha,j\beta} = \int d\hat{\Omega} \left[Q_a(\hat{\Omega}) \left(r_H(z) \hat{\Omega} \cdot \hat{\alpha} - e^{i\pi/2} r_V(z) \hat{z} \cdot \hat{\alpha} \right) \times \right. \\ \left. \left(r_H(z) \hat{\Omega} \cdot \hat{\beta} - e^{-i\pi/2} r_V(z) \hat{z} \cdot \hat{\beta} \right) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x} / v_R} \right].$$

Brief aside:

Gamma matrices

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Phase difference between I and j

Brief aside:

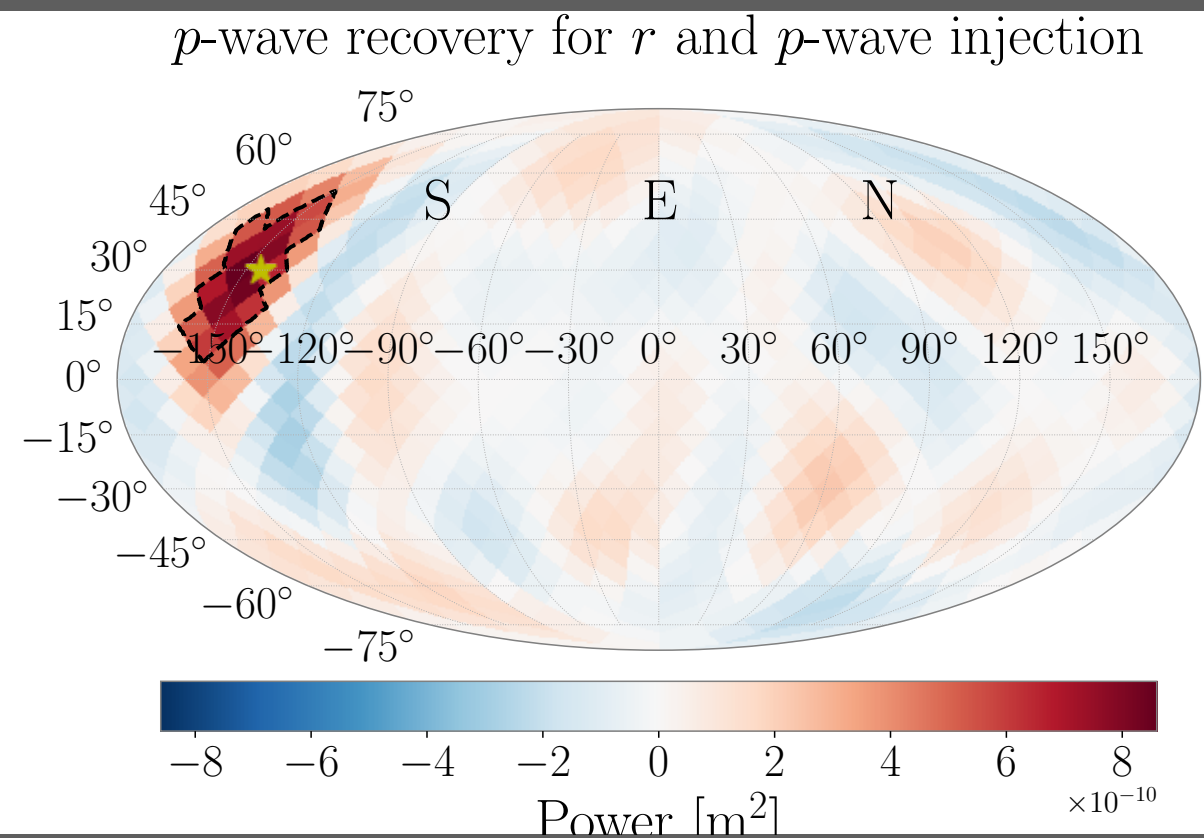
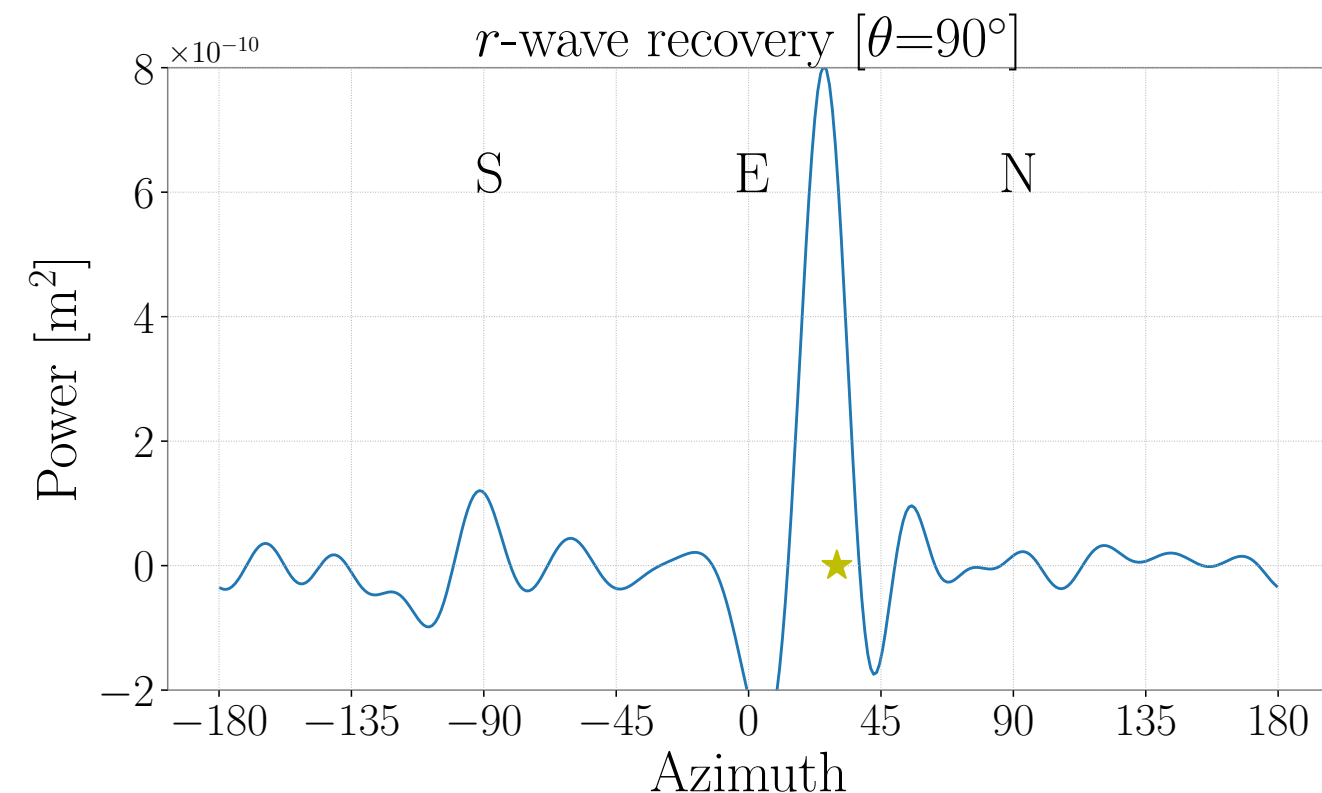
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**Govern how R-wave amp
fall with depth
(measured later)**

Seismic radiometer

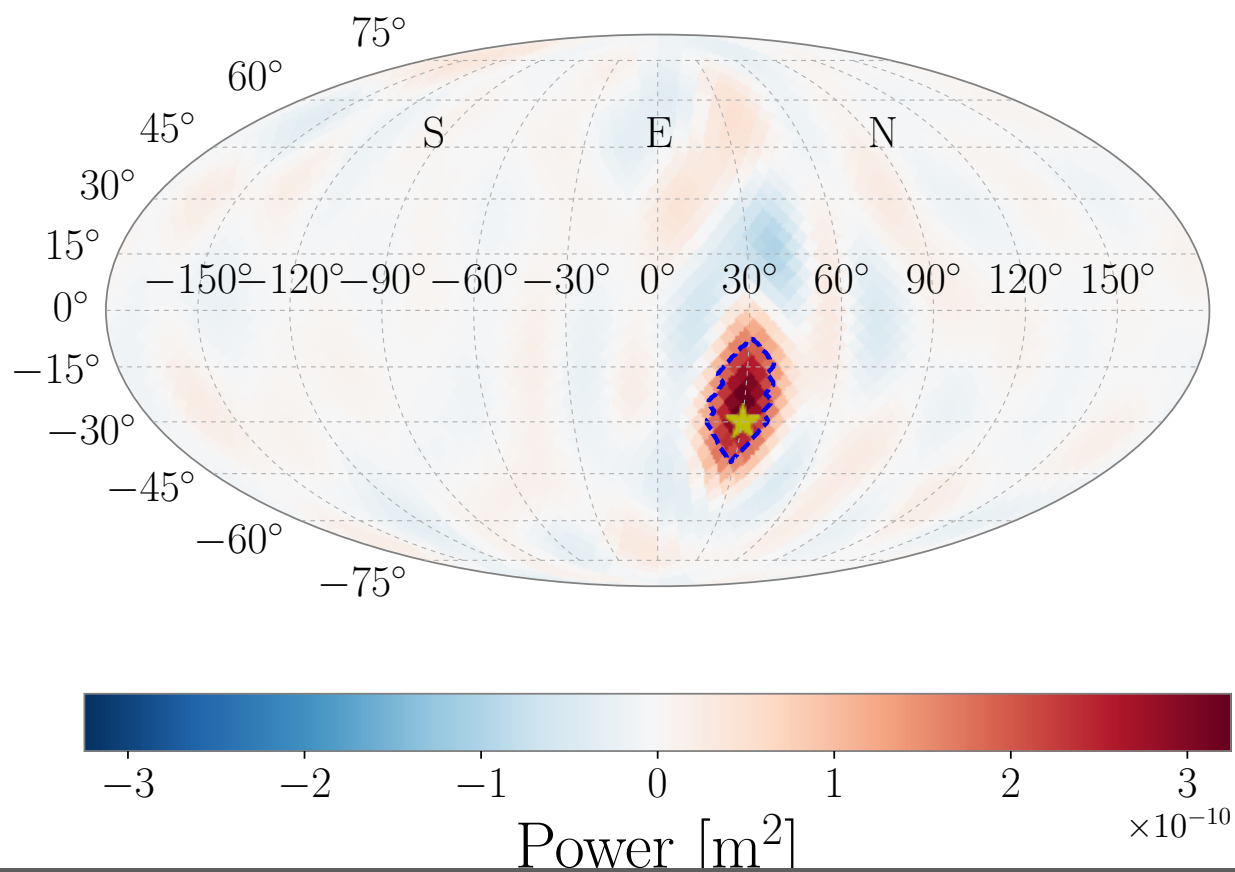
- ▶ Rayleigh and P-waves both injected
- ▶ Recovered in proper direction
- ▶ Amplitudes not quite right when 2 sources are present
 - ▶ (injections violate one assumption of search)



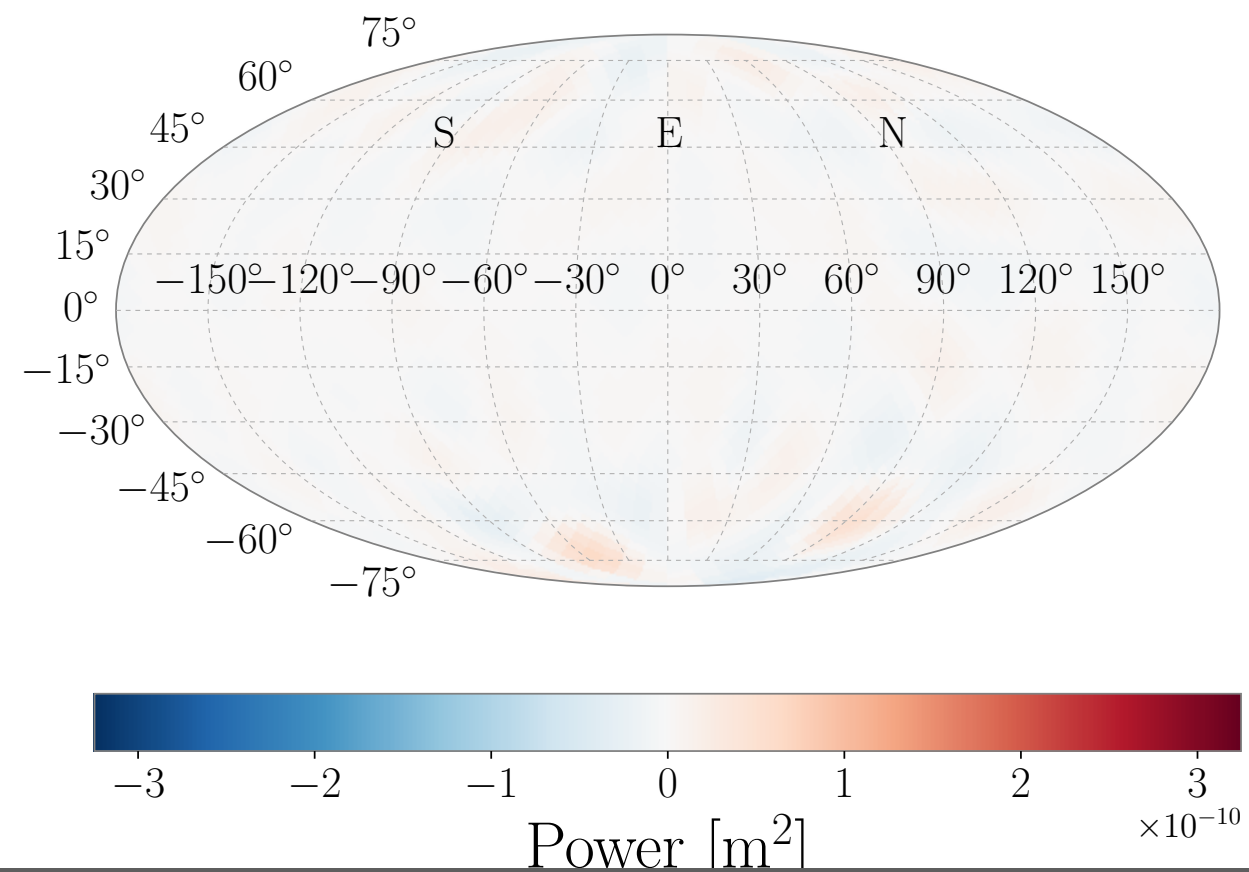
Seismic radiometer

► S-wave injection/recovery

s_h -wave recovery for s_h -wave injection



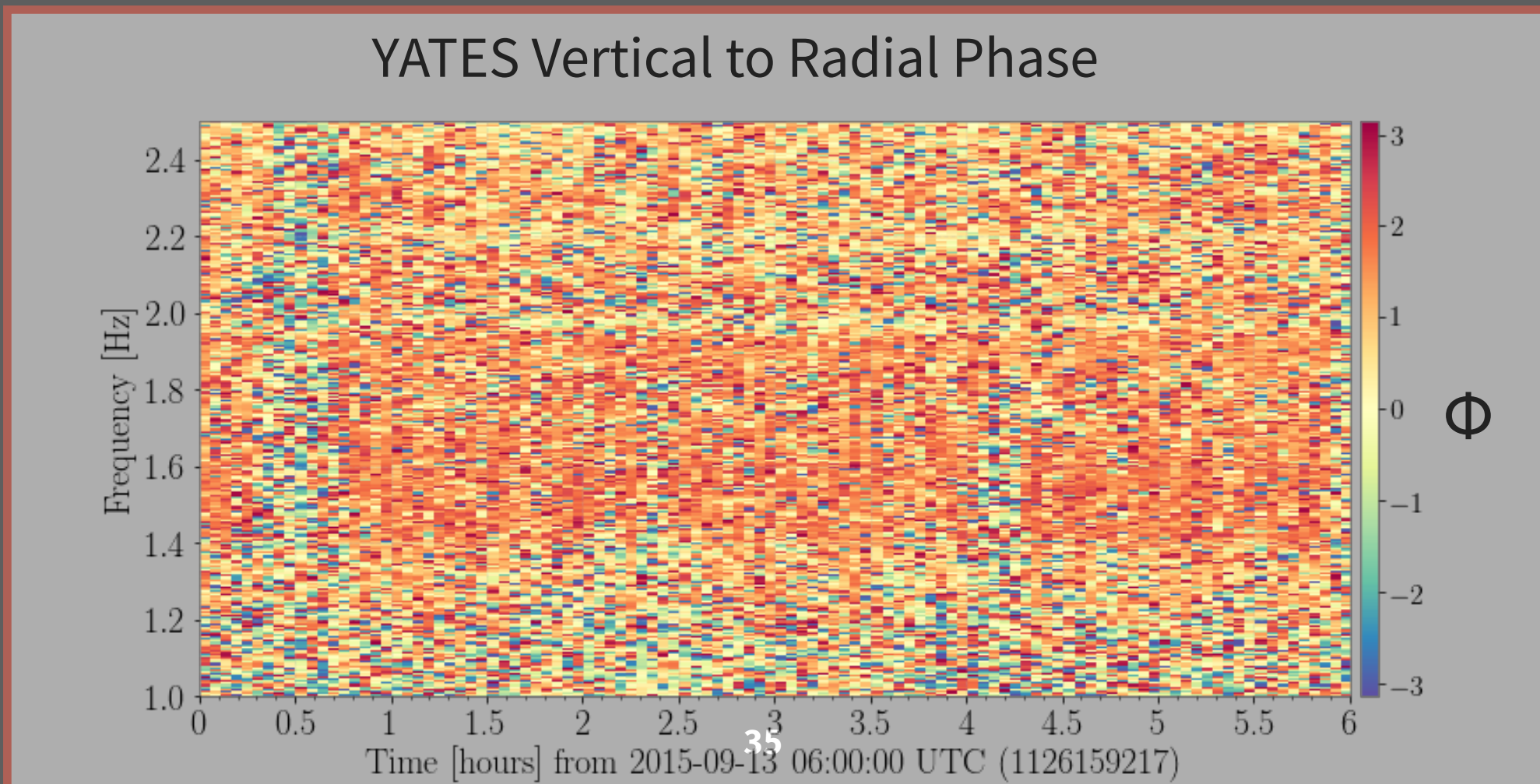
s_v -wave recovery for s_h -wave injection



Seismic Radiometer

1.5 - 2 Hz Source

- ▶ There is a source of (presumably) R-waves at 1.5 Hz
- ▶ Turns on and off at certain times of the day
- ▶ Would like to use Radiometer to resolve direction
- ▶ Will compare to a pure plane-wave model



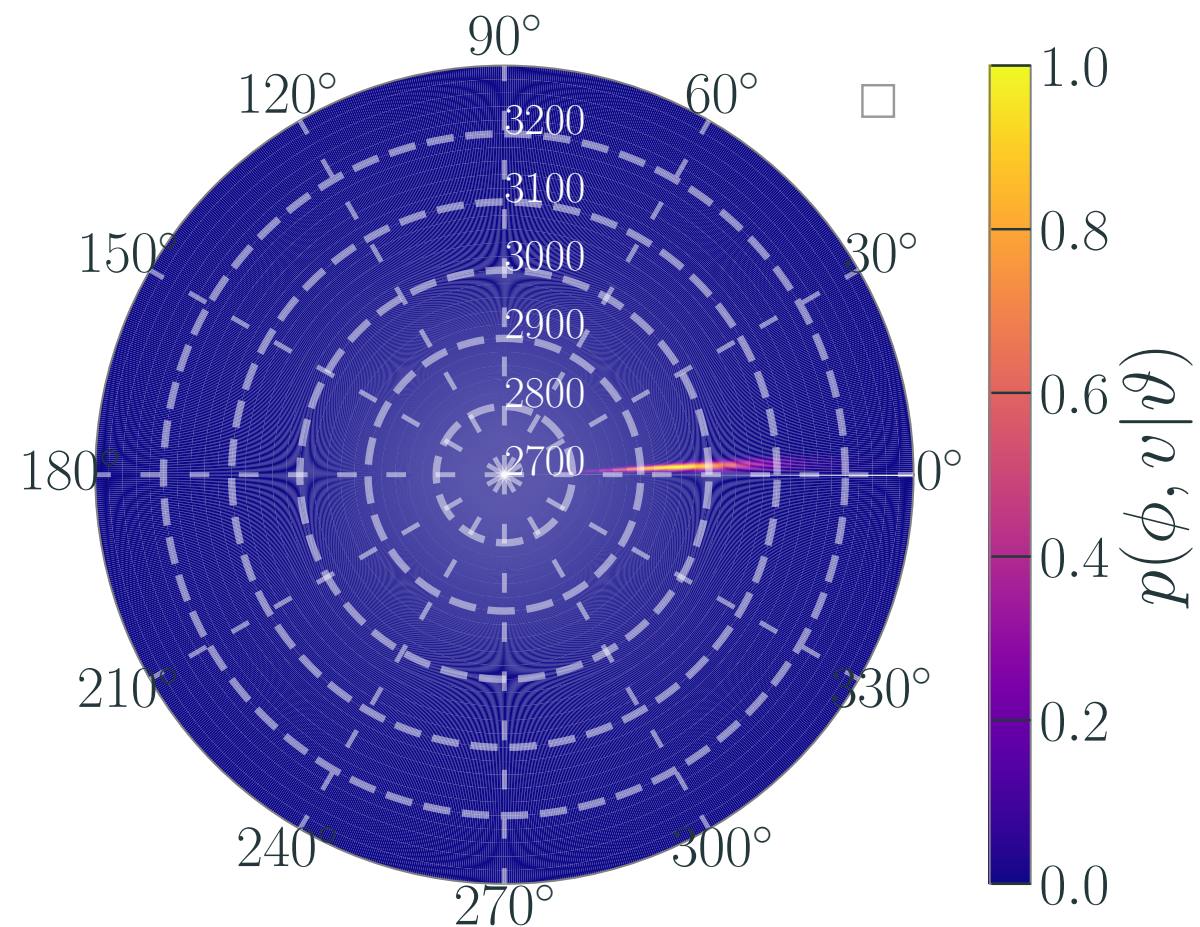
1.5 Hz source

timing-only analysis

- Use phase-delay between stations to estimate velocity, \mathbf{v} , and direction, $\hat{\Omega}$.
- Assuming pure-plane-wave, the phase delay should be:

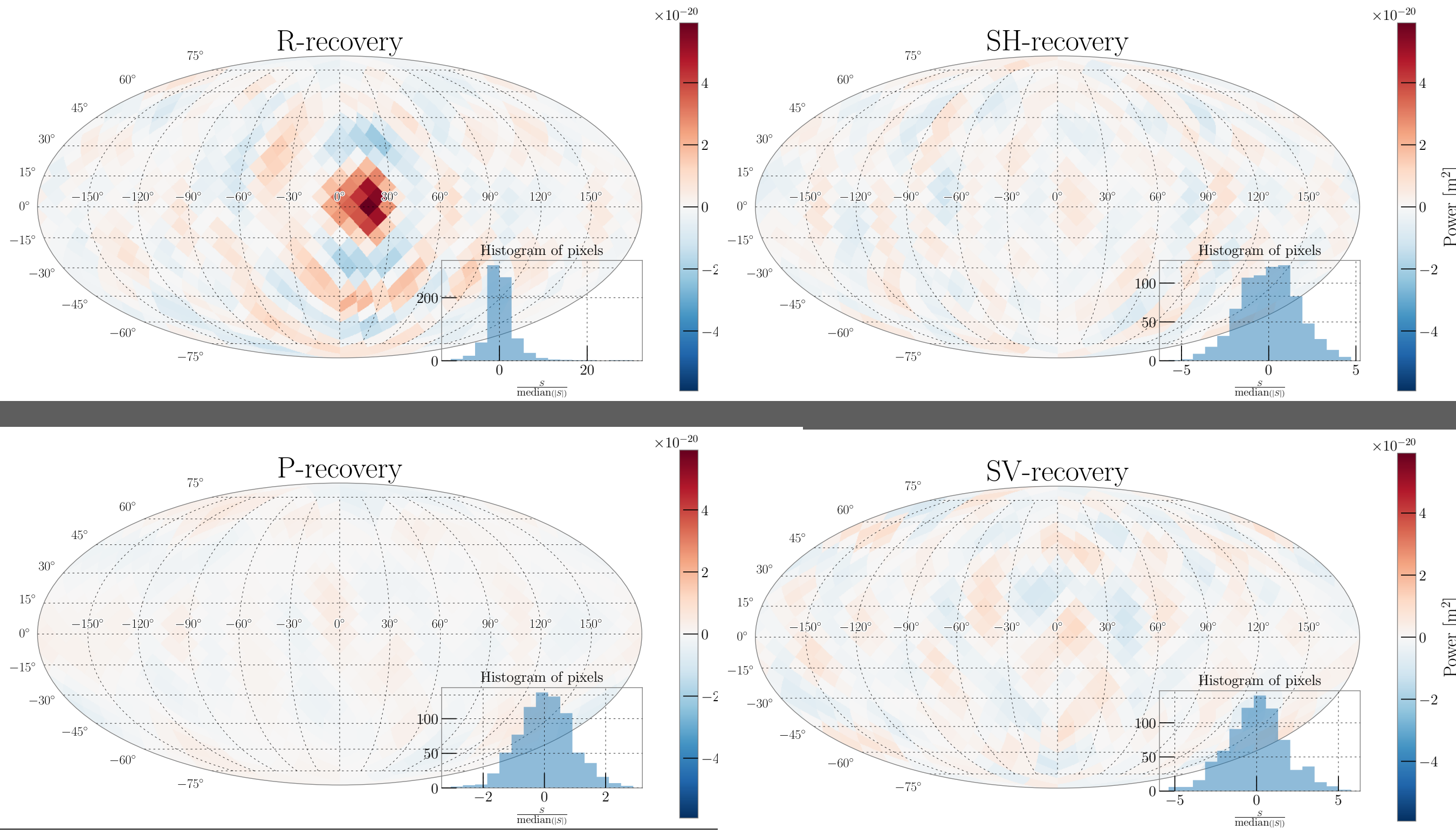
$$\vartheta_{ij} = \frac{2\pi f \hat{\Omega} \cdot \Delta \vec{x}_{ij}}{v}$$

Velocity and direction recovery: $\phi = 2.4^\circ, v = 2.94 \text{ km/s}$



1.5 Hz source

seismic radiometer



Estimate NN from maps

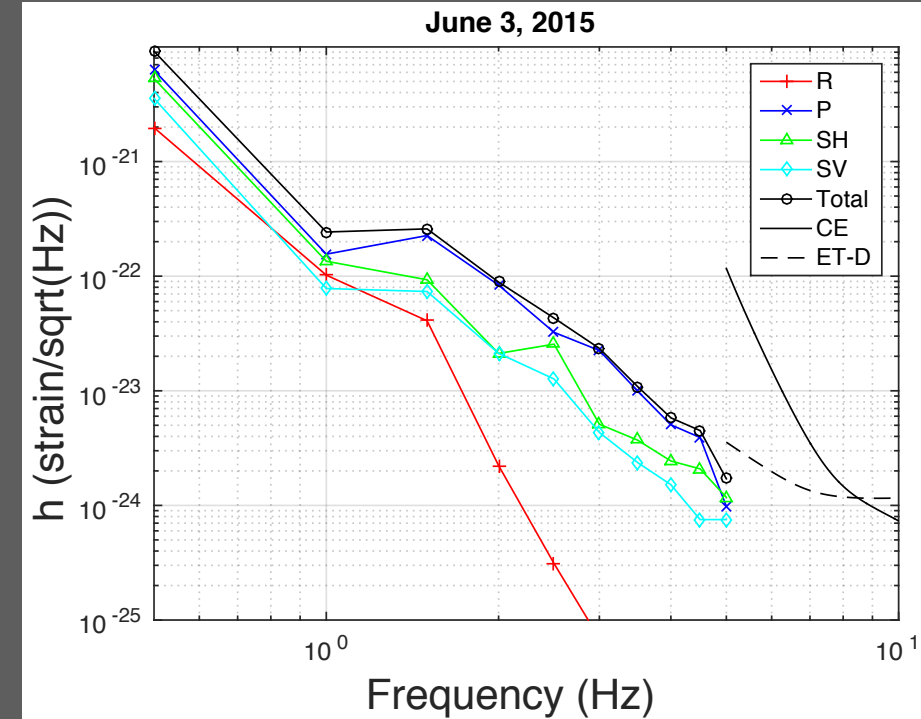
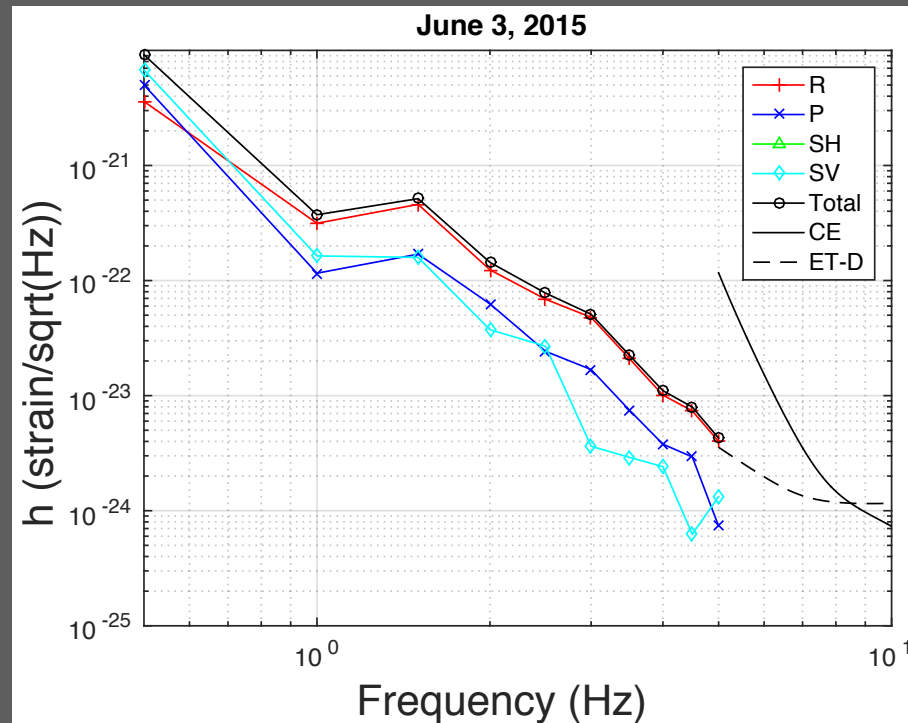
- We can now try to estimate the Newtonian noise from our recovery maps
- Run radiometer from **0.5 - 5 Hz** in **0.5 Hz** increments
- Assume “CE-like” detector (i.e. **40 km** arms)

Estimating Newtonian noise

Surface Detector

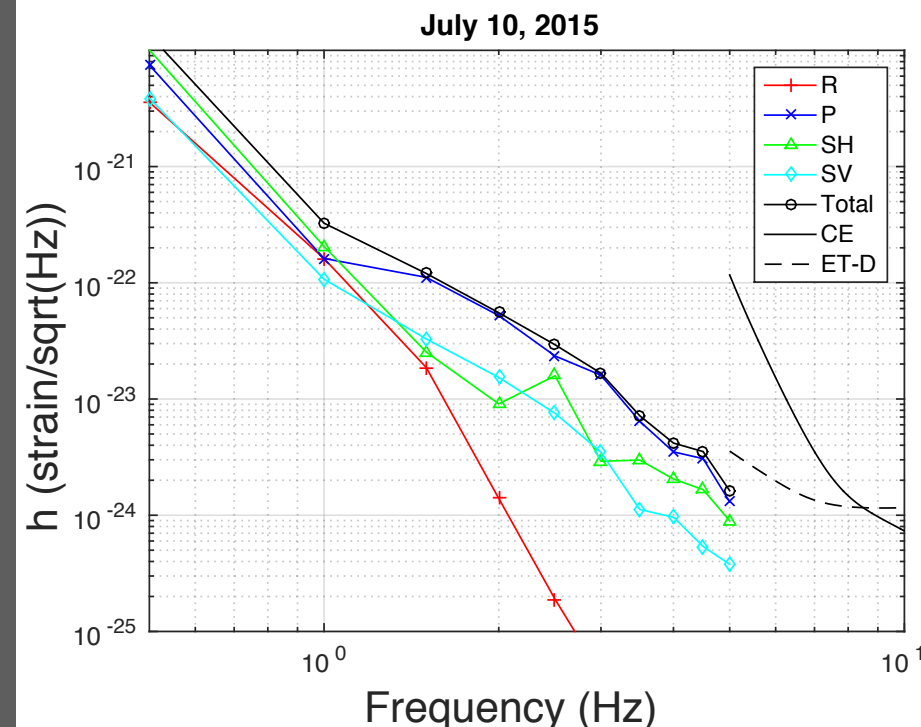
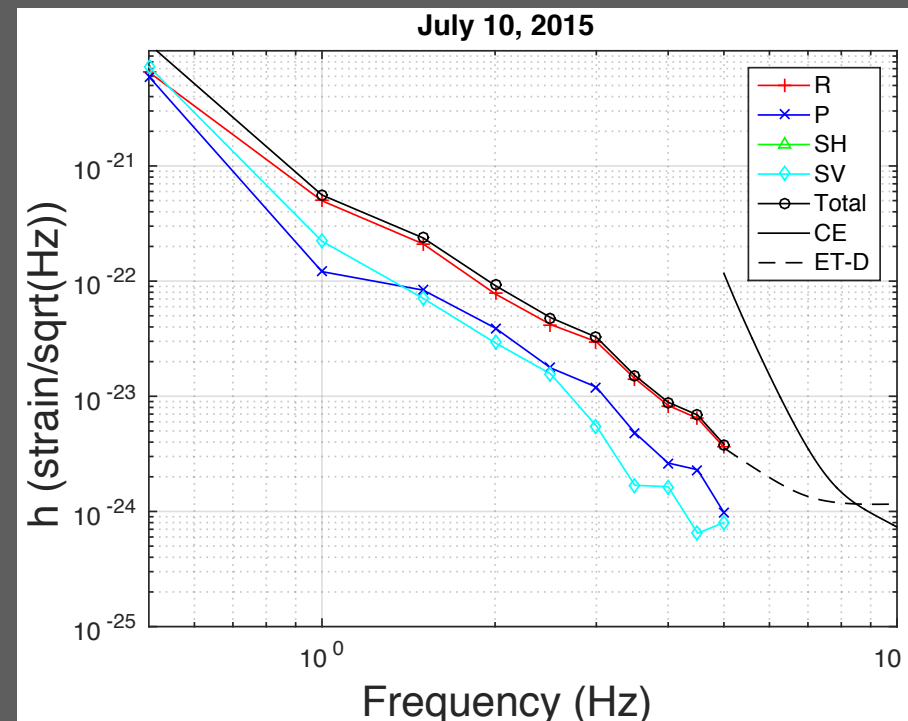
4850 ft. Detector

June 3, 2015



Plots made
by Andrew
Matas

July 10, 2015



Newtonian noise estimates

caveats

- Need better testing/assurance that recovered amplitudes make sense
- Currently we normalise the maps by the average total power in the surface stations across all three directions
- This should work “effectively” but is not ideal
- Need to figure out how to properly deal with “negative” power
- We’ve made some “estimates”

Dealing with Newtonian noise

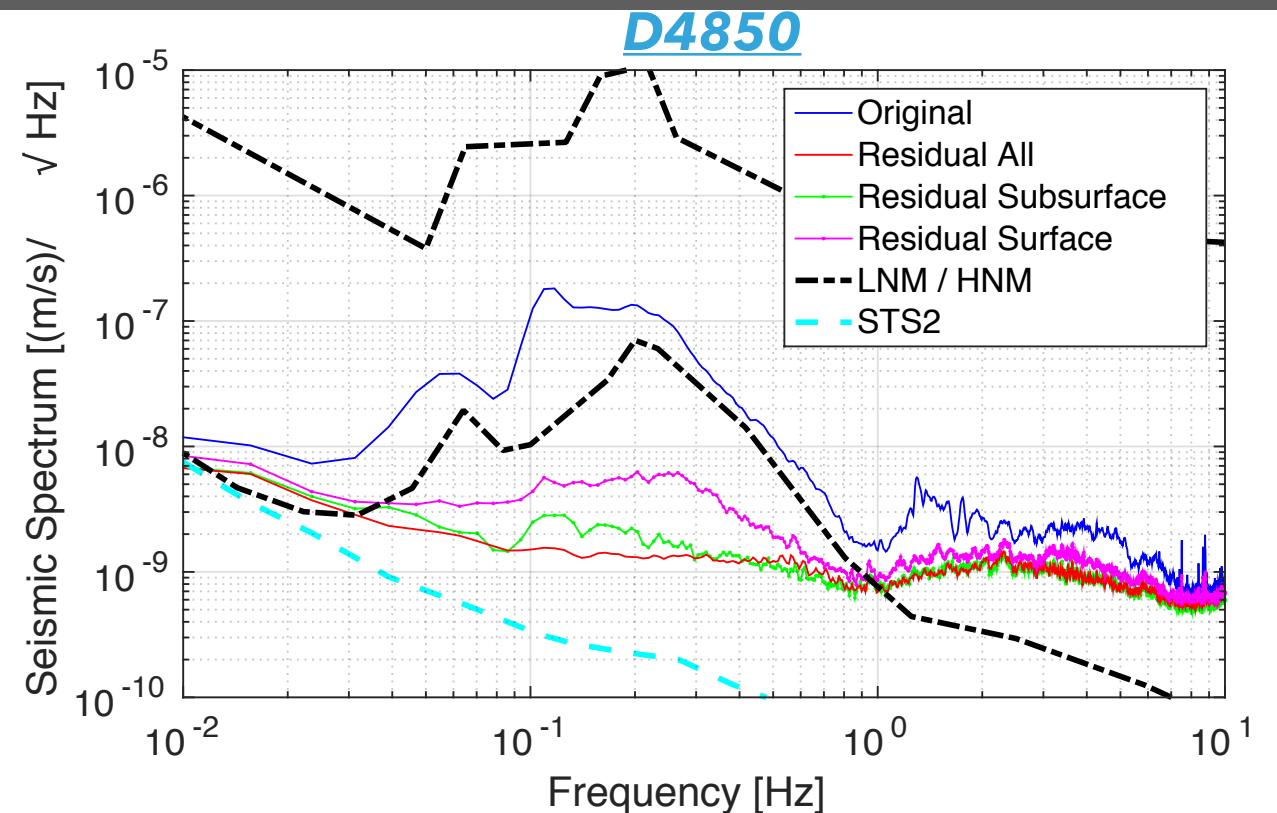
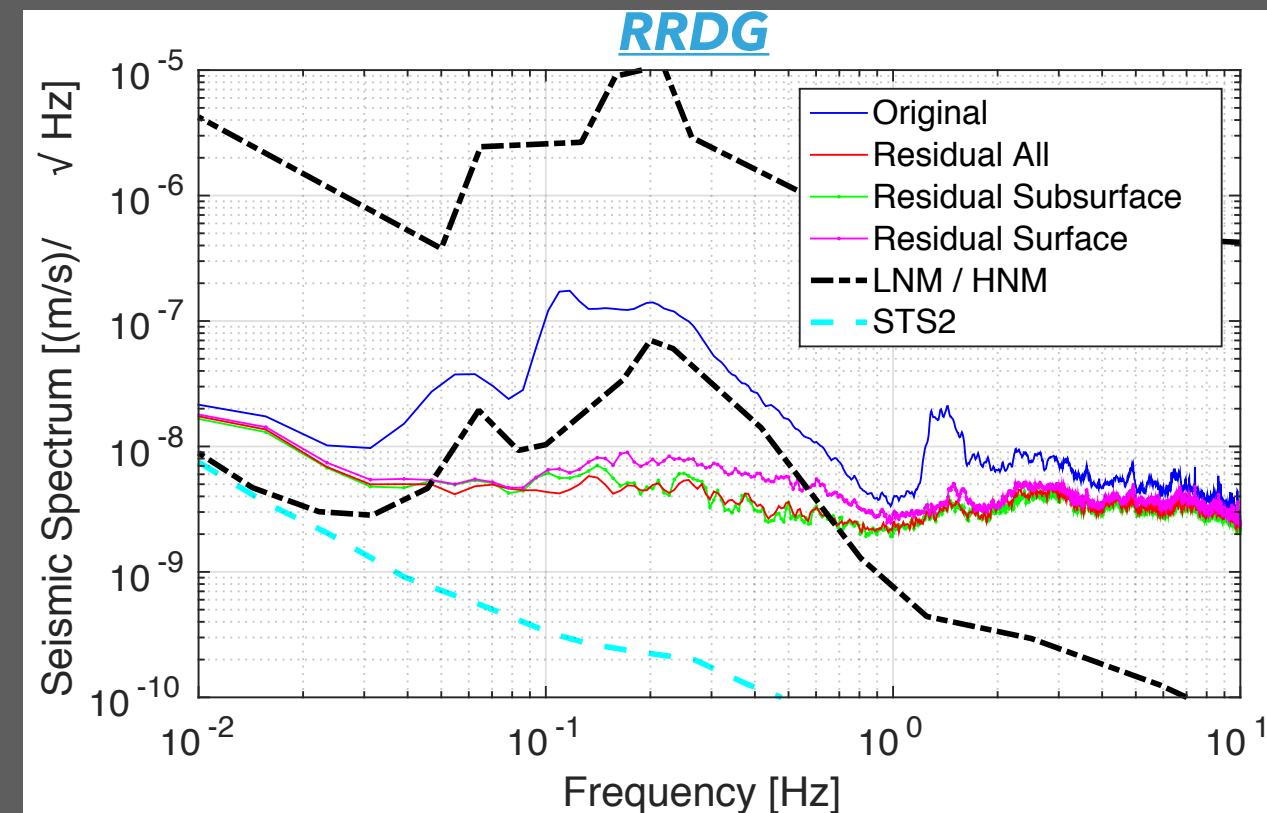
- Can't shield
- Wiener filter?
- Budget?
- Site selection?

Wiener filter

- Coughlin + Harms
 - Results for “cleaning” seismometer data using other seismometers is impressive
 - Goal is to eventually use something like this for NN
 - Geophone array and tilt-meters are installed at Virgo for explicit testing once we start to measure NN
 - Might be difficult because filters likely need to change as a function of time
 - Kalman filter?
 - Adaptive filter?

Wiener filter results

► Day 154 of 2015



Plots made by Michael
Coughlin/Jan Harms

Recap

Seismology sidetracks

REMEMBER THIS?!

$$\gamma_{R,a}^{i\alpha,j\beta} = \int d\hat{\Omega} \left[Q_a(\hat{\Omega}) \left(r_H(z) \hat{\Omega} \cdot \hat{\alpha} - e^{i\pi/2} r_V(z) \hat{z} \cdot \hat{\alpha} \right) \times \right. \\ \left. \left(r_H(z) \hat{\Omega} \cdot \hat{\beta} - e^{-i\pi/2} r_V(z) \hat{z} \cdot \hat{\beta} \right) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x} / v_R} \right].$$

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Seismology sidetracks

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Govern how R-wave amp
fall with depth
(measured later)

“Rayleigh-wave eigenfunctions”

Surface-wave eigenfunctions

- Two types of surface waves — “**Rayleigh**” and “**Love**”
- **Rayleigh**—
 - superposition of P- and S-waves
 - Travel along surface
 - Elliptical particle motion

Surface-wave eigenfunctions

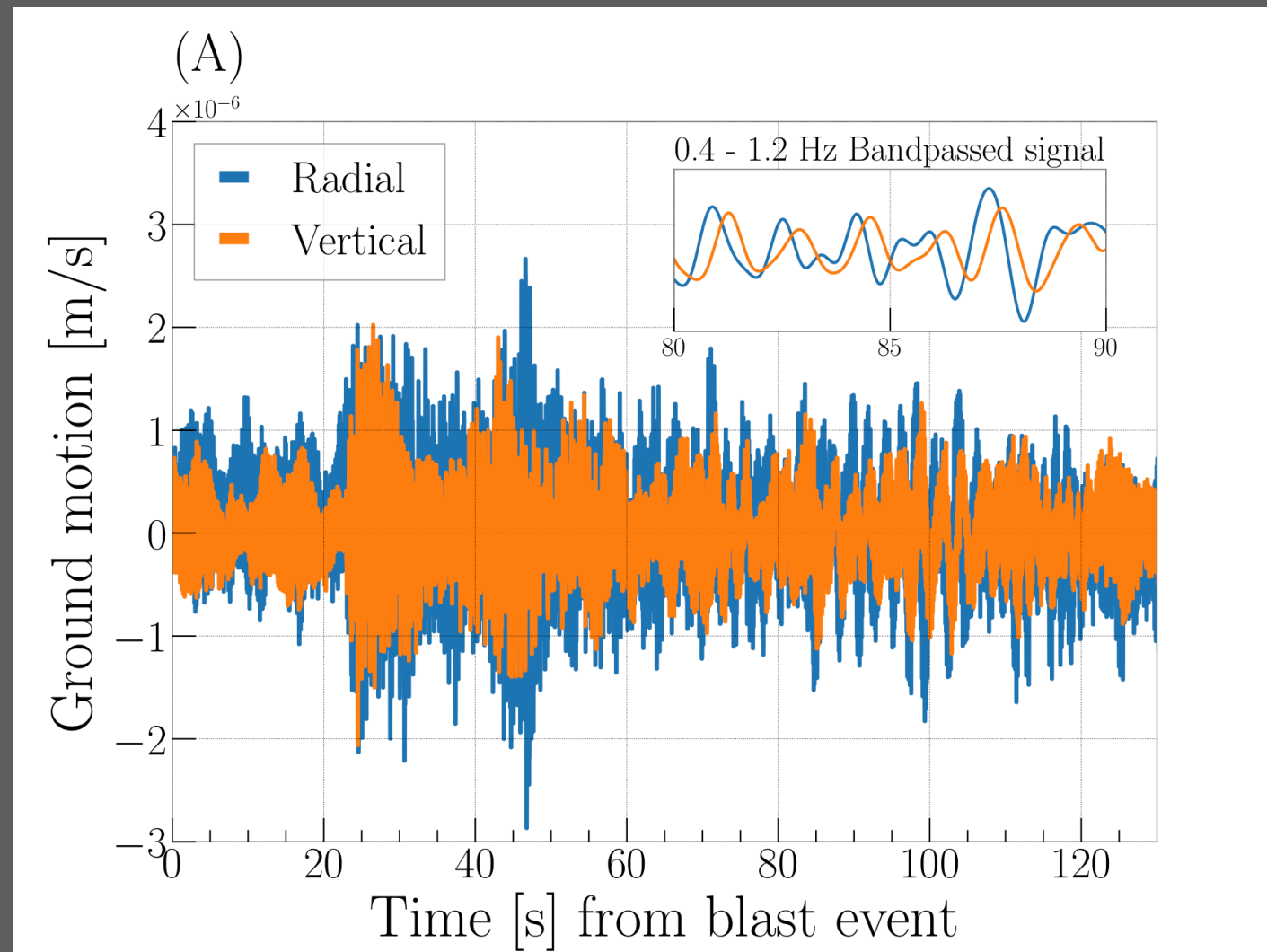
- Two types of surface waves — “**Rayleigh**” and “**Love**”
- **Love**—
 - Superposition of *multiply reflected S-waves*
 - Needs S-wave velocity profile that increases with depth
 - S-waves get refracted in this profile, turn around, and reflect off of the surface again
- Both types of surface wave have amplitudes that fall off with depth (“surface-wave eigenfunctions”)

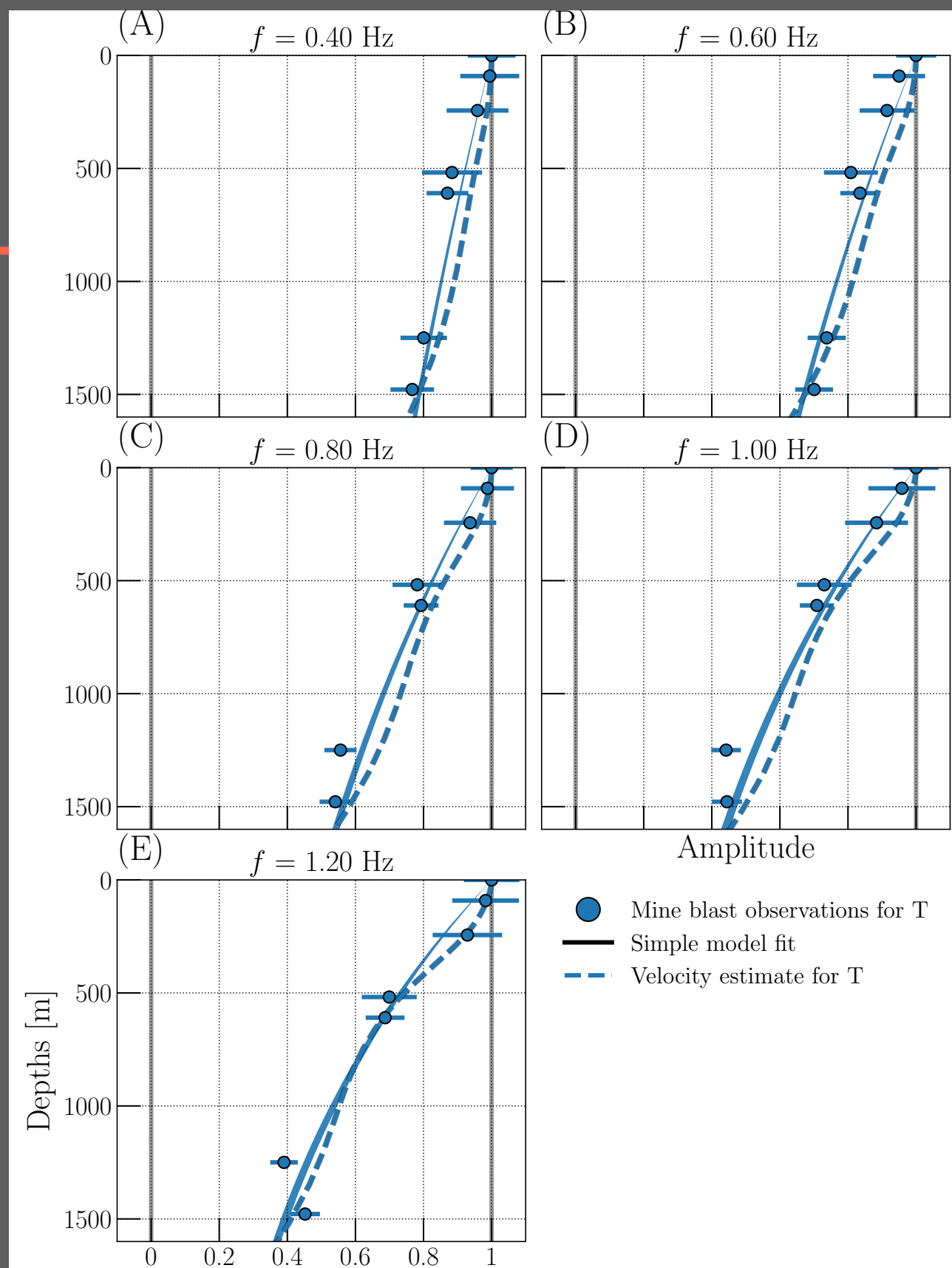
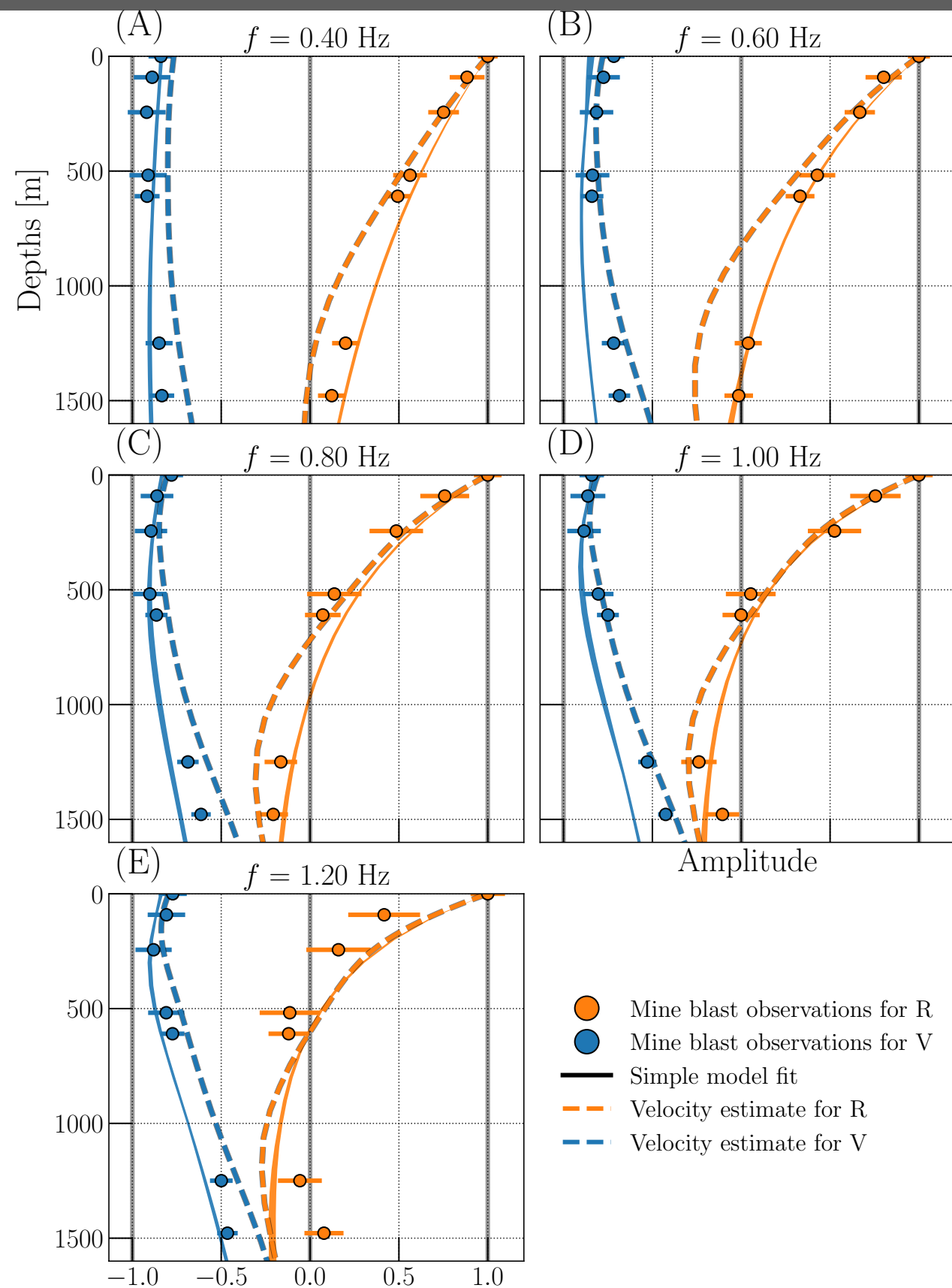
Homestake is special

- There are not many 3D arrays
 - There are borehole arrays — usually smaller scale, often use single-component sensors
- Usually experiments are confined to the surface
- **With Homestake we can *explicitly* measure how amplitude changes with depth for these waves — not something that has been done before**

Measuring eigenfunctions

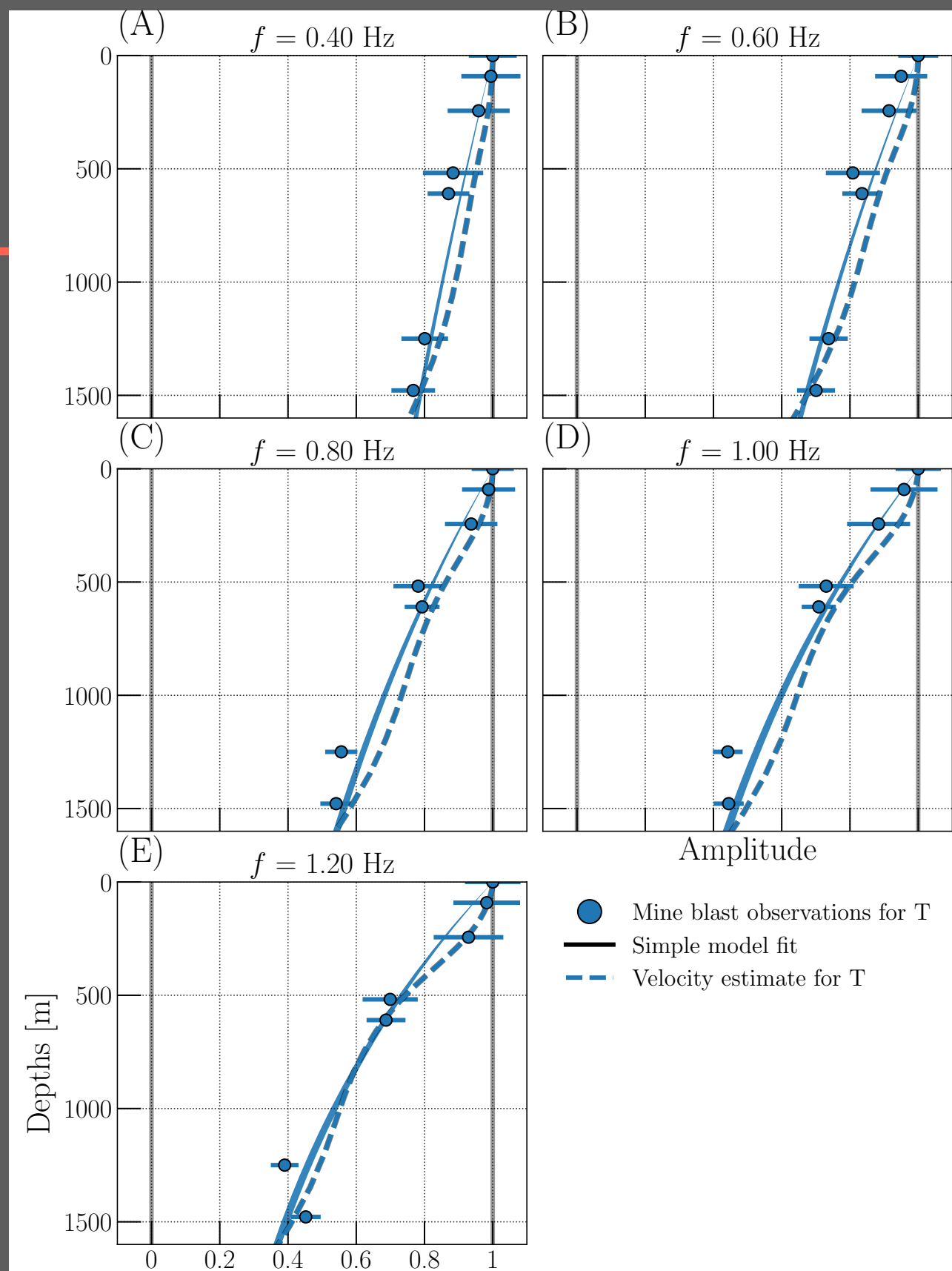
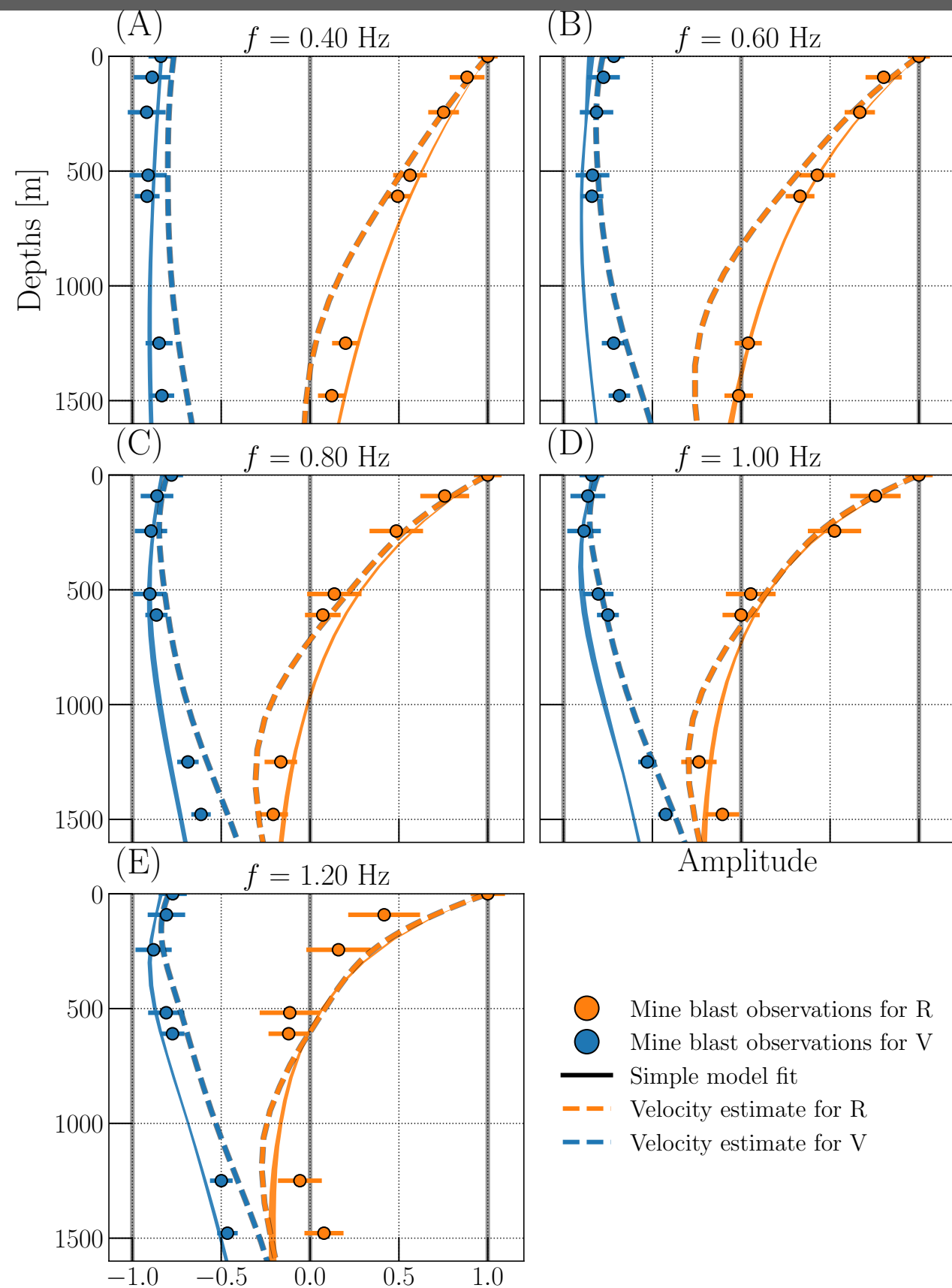
- Use the amplitude of the Rayleigh-wave part of a signal from mine blasts





Two fits — theoretical and empirical

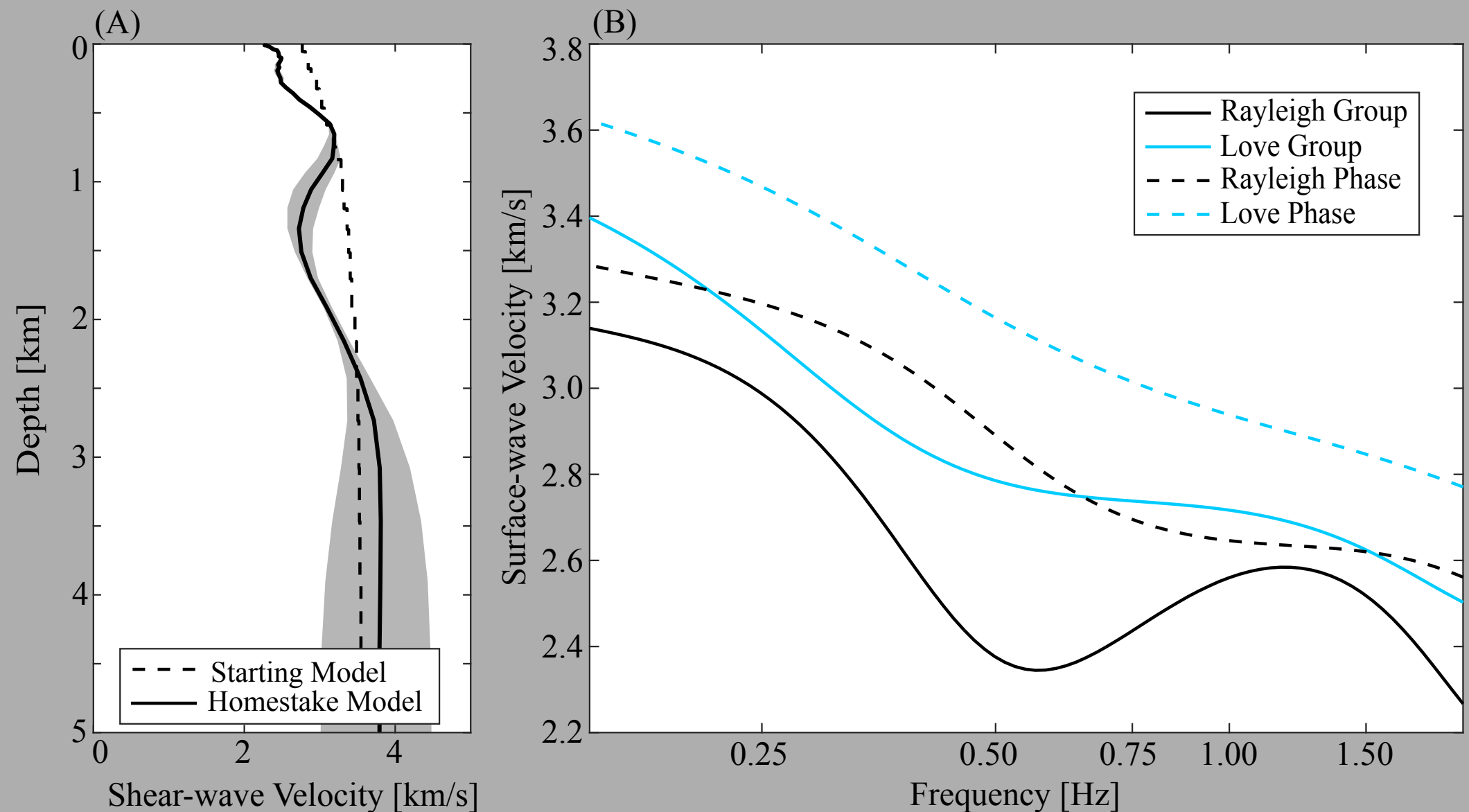
- On the results plot there are data points and two fits
- **FIT 1** — fits theory to the data points
 - Assume S-waves have a velocity depth profile that goes like a power law
 - This gives a functional form for R and L-wave eigenfunctions with some (frequency independent) free parameters
 - We fit those free parameters using a nested sampling technique



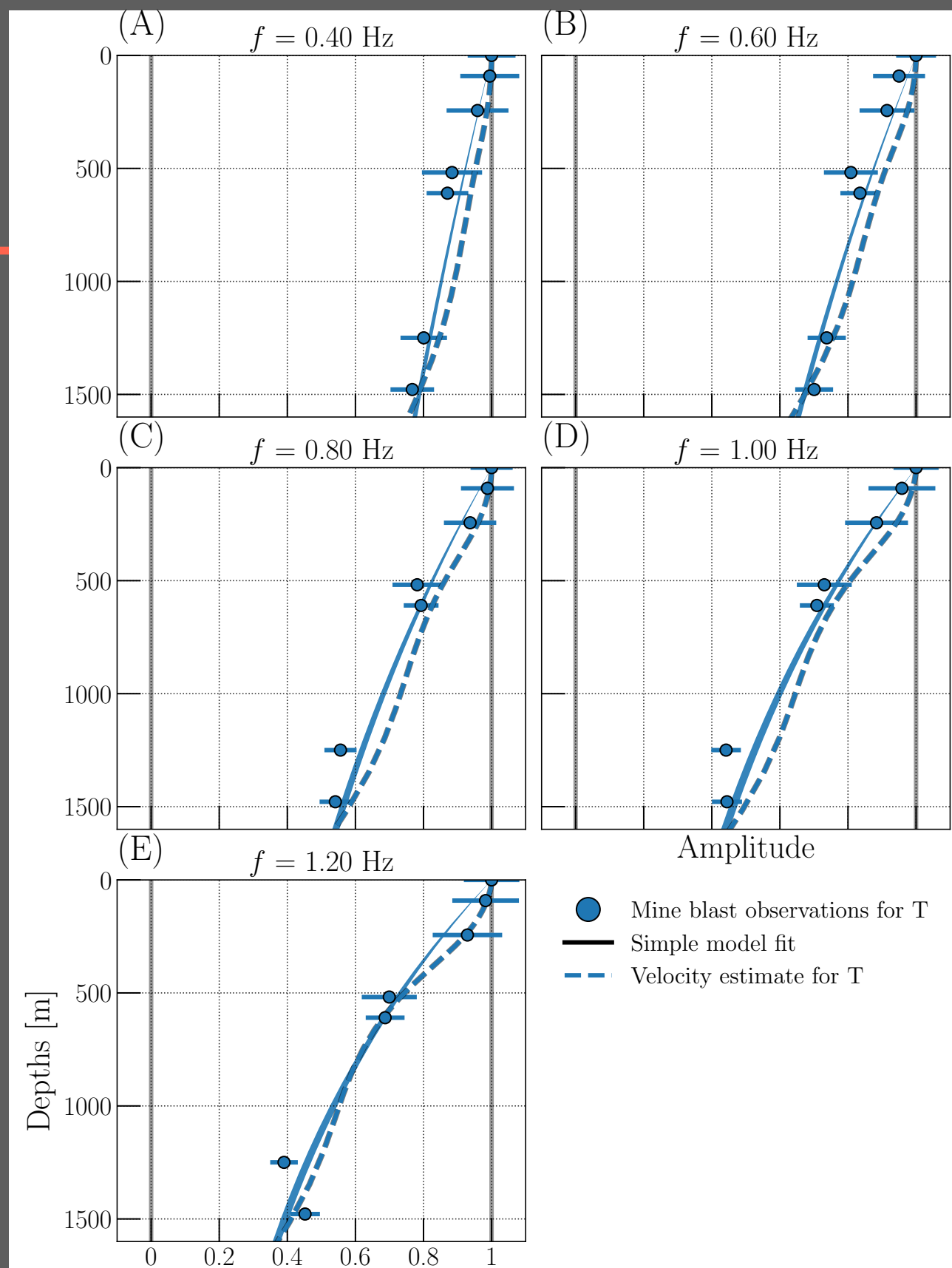
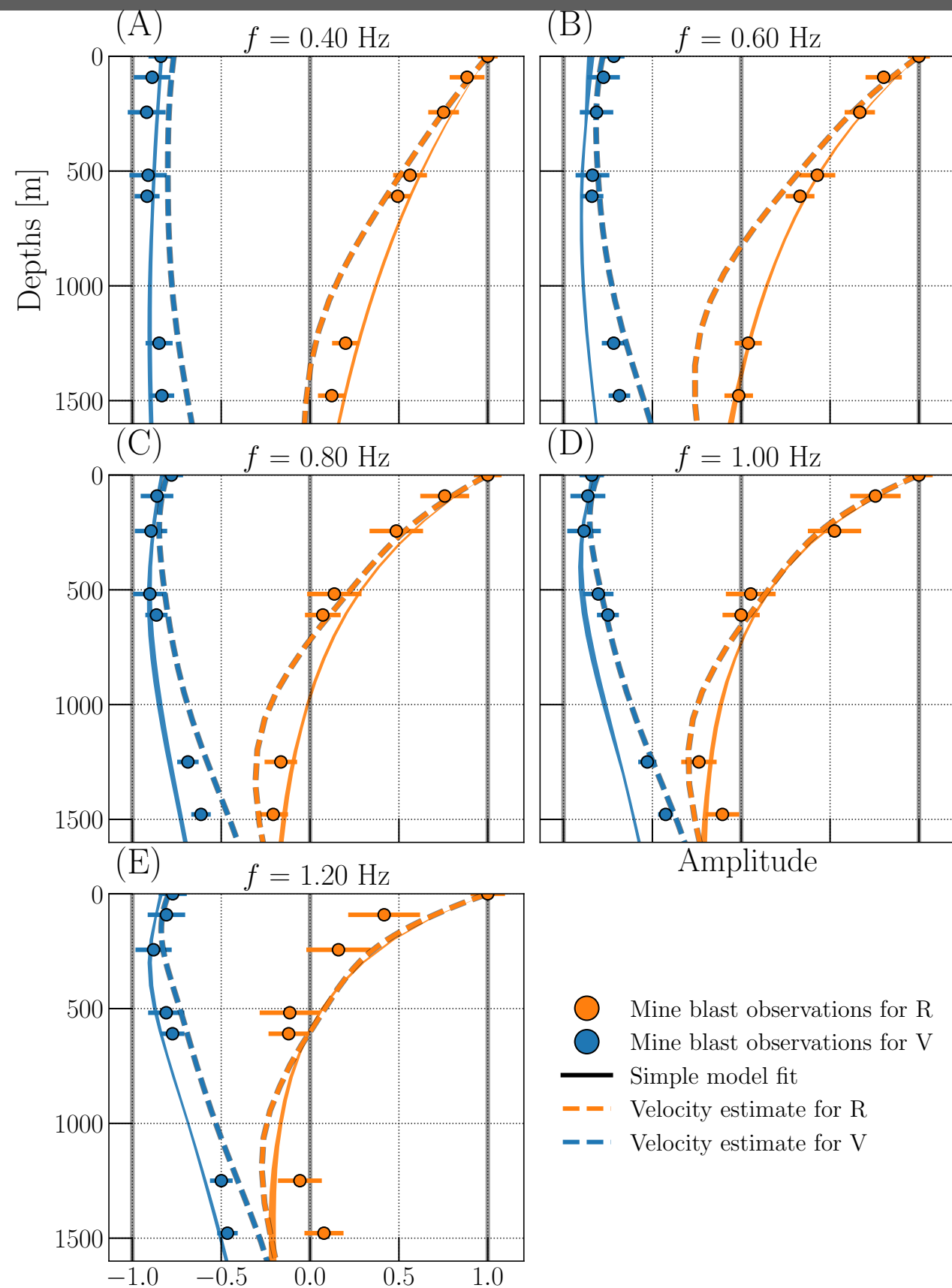
Two fits — theoretical and empirical

- On the results plot there are data points and two fits
- “**FIT**” 2 — independent analysis that produces results based on a model
 - Assume S-waves have a velocity depth profile that goes like a power law
 - Use velocity dispersion of R- and L-waves to infer an S-wave velocity depth profile
 - Use this inferred velocity depth profile to estimate the R and L-wave eigenfunctions

Dispersion + Depth profile estimates



Plots made by Daniel
Bowden



Conclusions

seismology

- Useful verification of a classical seismological result
- Result of cross-disciplinary cooperation
 - This project likely doesn't happen without GW application
 - Seismological applications make it very compelling as a project with several purposes

Conclusions

General

- Lower frequencies —> interesting GW sources, cosmology, and astrophysics
- Newtonian noise will likely be an issue
- Initial estimates of NN from seismology indicate it could potentially be an issue
- Interesting cross-disciplinary work comes out as a natural byproduct of working on areas of mutual interest

EXTRA

SGWB searches

Isotropic

- We construct an estimator for the energy density in each small frequency bin (and its variance):

$$\hat{\Omega}_{\text{GW}}(f) = \frac{10\pi^2}{3H_0^2} f^3 \frac{\text{Re}(\hat{C}(f))}{\gamma(f)}$$

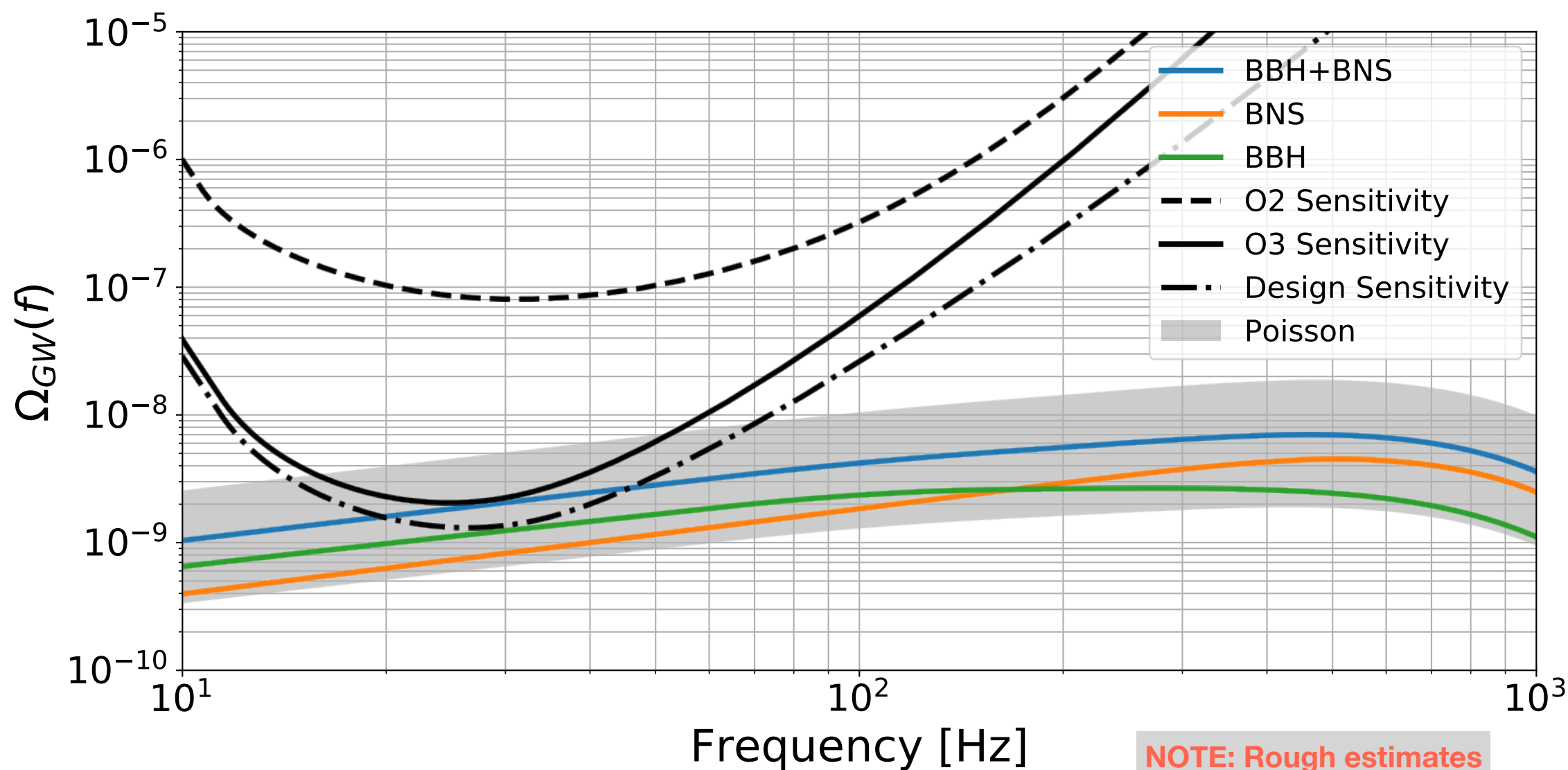
Assuming signal \ll typical noise level in the detector...

$$\sigma_{\hat{\Omega}}^2(f) = \frac{1}{2\tau\Delta f} \left(\frac{10\pi^2}{3H_0^2} \right)^2 f^6 \frac{P_1(f)P_2(f)}{|\gamma(f)|^2}$$

Power spectral
density of the
noise in each
detector

SGWB Sources

- **Largest contribution:** unresolved compact binary mergers
 - **~15** binary neutron stars in LIGO frequency band at any one time
 - **1** binary black hole coalescence every **~250 s**




GW Amplitude

$$h_{ij}^{\text{TT}}(t, \vec{x}) = \left(\frac{1}{r} \right) \left(\frac{2G}{c^4} \right) \ddot{Q}_{ij}^{\text{TT}}(t - r/c)$$

GW Amplitude

Falls off as r^{-1}


$$h_{ij}^{\text{TT}}(t, \vec{x}) = \left(\frac{1}{r} \right) \left(\frac{2G}{c^4} \right) \ddot{Q}_{ij}^{\text{TT}}(t - r/c)$$

GW Amplitude

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$$h_{ij}^{TT}(t, \vec{x}) = \left(\frac{1}{r}\right) \left(\frac{2G}{c^4}\right) \ddot{Q}_{ij}^{TT}(t - r/c)$$

10^{-44} N^{-1}

GW Amplitude

Falls off as r^{-1}

Quadrupole moment

$$h_{ij}^{TT}(t, \vec{x}) = \left(\frac{1}{r} \right) \left(\frac{2G}{c^4} \right) \ddot{Q}_{ij}^{TT}(t - r/c)$$

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GW Amplitude

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10^{-44} N^{-1}

For LIGO detectors:

Cataclysmic events

$$h \approx 10^{-21} \text{ for } \mathcal{O}(\text{seconds})$$

GW Amplitude

Falls off as r^{-1}

Quadrupole moment

$$h_{ij}^{TT}(t, \vec{x}) = \left(\frac{1}{r}\right) \left(\frac{2G}{c^4}\right) \ddot{Q}_{ij}^{TT}(t - r/c)$$

10^{-44} N^{-1}

For LIGO detectors:

Cataclysmic events

$$h \approx 10^{-21} \text{ for } \mathcal{O}(\text{seconds})$$

Look for a long time

$$h \approx 10^{-25} \text{ for } \mathcal{O}(\text{years})$$

SGWB searches

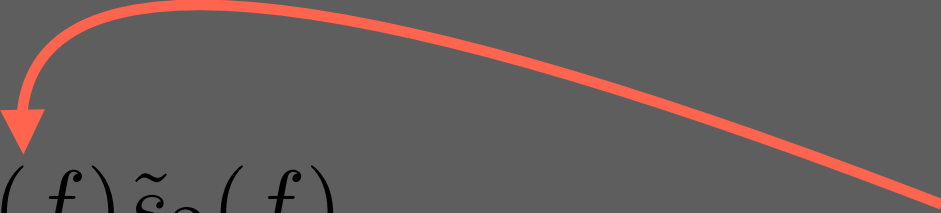
- There are not well-defined templates for the SGWB
 - So we **cross-correlate** data between detectors

$$\hat{C}(f) = \frac{2}{\tau} \tilde{s}_1^*(f) \tilde{s}_2(f)$$

SGWB searches

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$$\hat{C}(f) = \frac{2}{\tau} \tilde{s}_1^*(f) \tilde{s}_2(f)$$



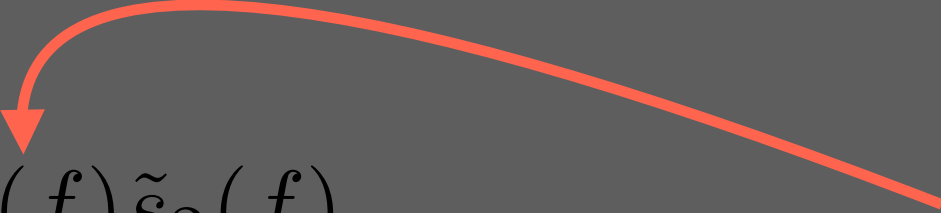
$$\tilde{s}_1(f) = \tilde{h}_1(f) + \tilde{n}_1(f)$$

GW signal **noise**

SGWB searches

- There are not well-defined signals for the SGWB
 - So we **cross-correlate** data between detectors

$$\hat{C}(f) = \frac{2}{\tau} \tilde{s}_1^*(f) \tilde{s}_2(f)$$

$$\tilde{s}_1(f) = \underbrace{\tilde{h}_1(f)}_{\text{GW signal}} + \underbrace{\tilde{n}_1(f)}_{\text{noise}}$$


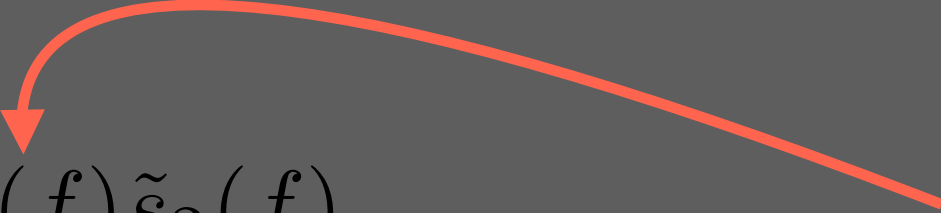
Substitute and take time average...

$$\langle \hat{C}(f) \rangle = \frac{2}{\tau} \left(\langle \tilde{h}_1^*(f) \tilde{h}_2(f) \rangle + \langle \tilde{n}_1^*(f) \tilde{h}_2(f) \rangle + \langle \tilde{h}_1^*(f) \tilde{n}_2(f) \rangle + \langle \tilde{n}_1^*(f) \tilde{n}_2(f) \rangle \right)$$

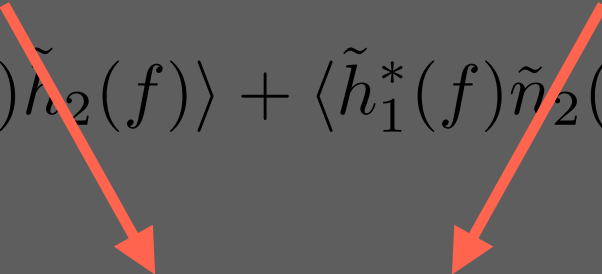
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ASSUMPTIONS!!!

Signal and noise are
uncorrelated

SGWB searches

- There are not well-defined signals for the SGWB
 - So we **cross-correlate** data between detectors

$$\hat{C}(f) = \frac{2}{\tau} \tilde{s}_1^*(f) \tilde{s}_2(f)$$

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Substitute and take time average...

$$\langle \hat{C}(f) \rangle = \frac{2}{\tau} \left(\langle \tilde{h}_1^*(f) \tilde{h}_2(f) \rangle + \langle \tilde{n}_1^*(f) \tilde{h}_2(f) \rangle + \langle \tilde{h}_1^*(f) \tilde{n}_2(f) \rangle + \langle \tilde{n}_1^*(f) \tilde{n}_2(f) \rangle \right)$$

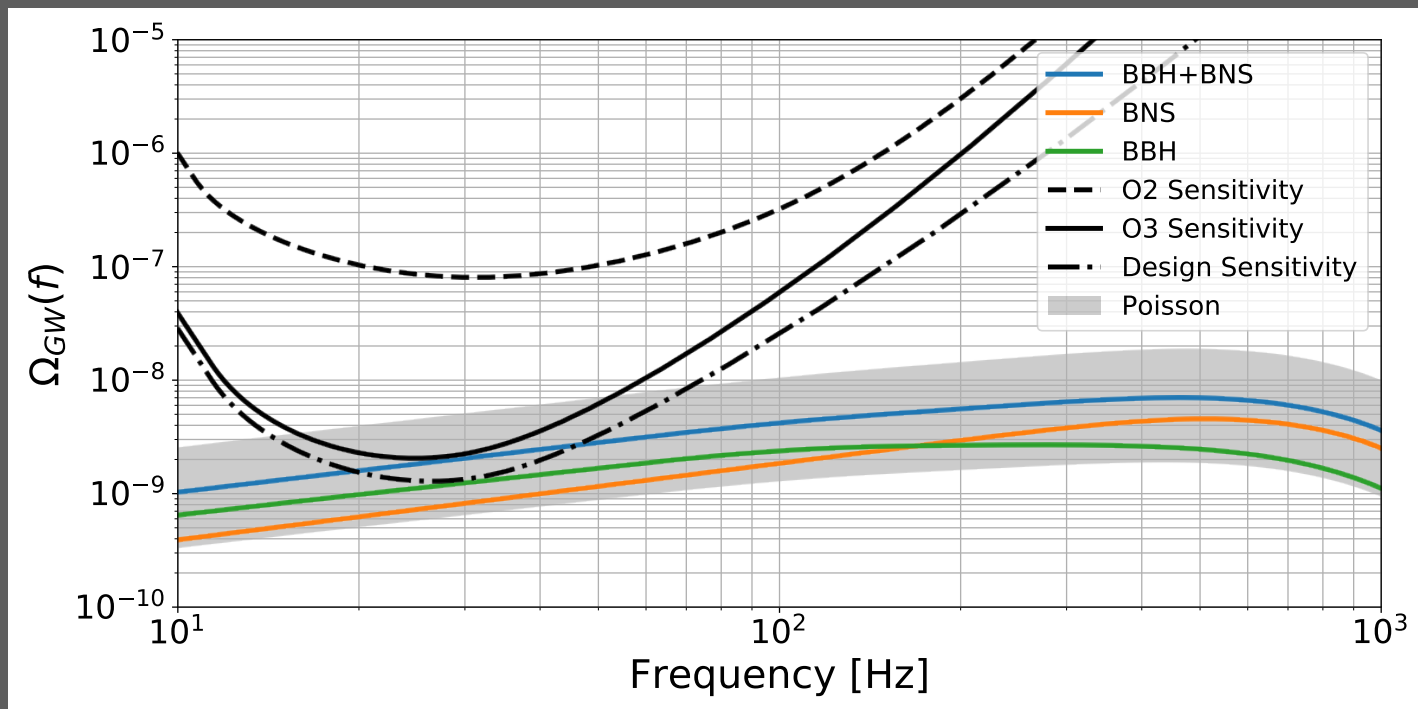
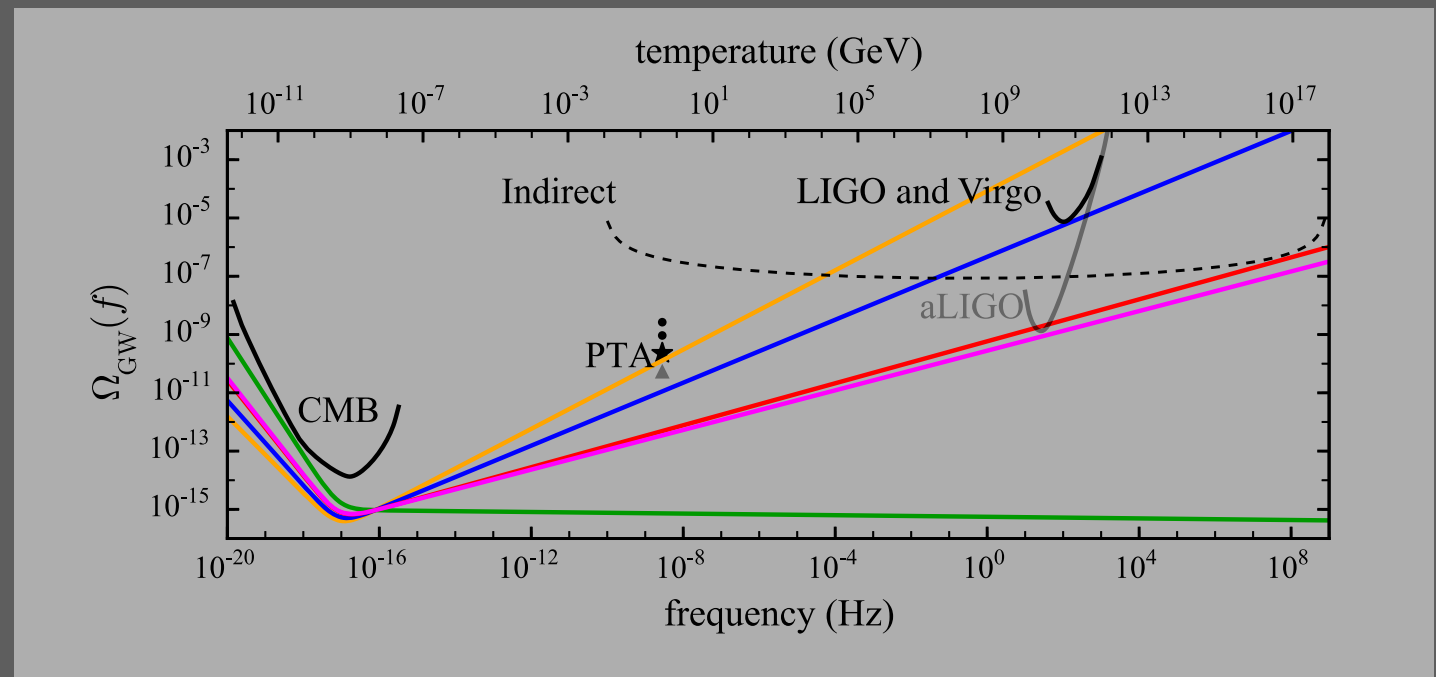
ASSUMPTIONS!!!

Signal and noise are
uncorrelated

noise is uncorrelated
between detectors

SGWB Sources/Detectors

Lasky, P. D., et al. (2016).
 PHYSICAL REVIEW X, 6(1),
 11035. <https://doi.org/10.1103/PhysRevX.6.011035>



Abbott, B. P., et al. (2018). *Phys Rev Lett*,
 120(9), 091101. <http://doi.org/10.1103/PhysRevLett.120.091101>

CE facts

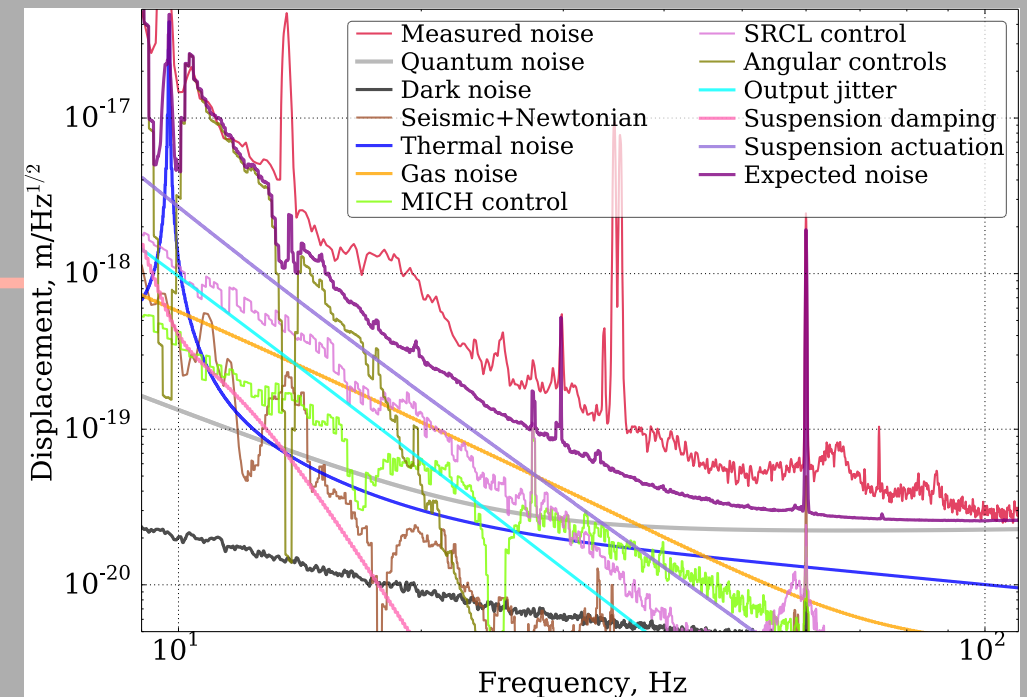
Table 1. Parameters used to produce the Cosmic Explorer (CE) target curve. The CE pessimistic and Einstein telescope, high- and low-frequency (HF and LF) parameters are included for comparison.

	CE	CE pess	ET-D (HF)	ET-D (LF)
L_{arm}	40 km	40 km	10 km	10 km
P_{arm}	2 MW	1.4 MW	3 MW	18kW
λ	1550 nm	1064 nm	1064 nm	1550 nm
r_{sqz}	3	3	3	3
m_{TM}	320 kg	320 kg	200 kg	200 kg
r_{beam}	14 cm	12 cm	9 cm	7 cm (LG ₃₃)
T	123 K	290 K	290 K	10 K
ϕ_{eff}	5×10^{-5}	1.2×10^{-4}	1.2×10^{-4}	1.3×10^{-4}

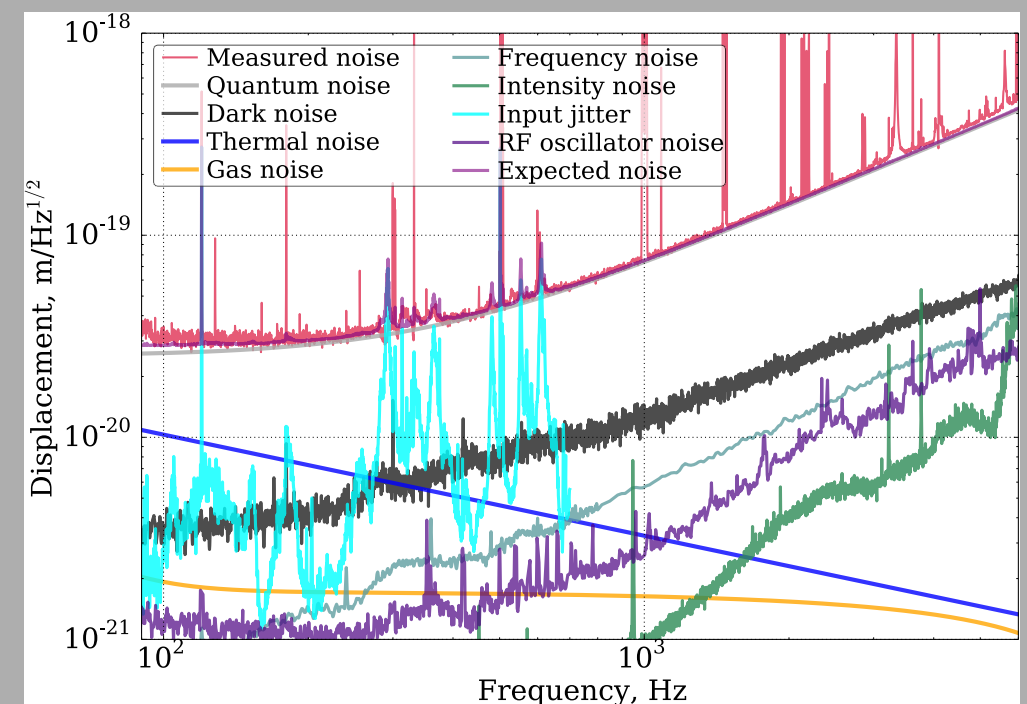
LIGO Noise Curve

Low Frequency Noise:

- **Seismic noise** — moving platforms that hold suspensions
- **Angular controls** — controlling angular degrees of freedom of mirrors
- **Quantum noise** — radiation pressure



(a) LIGO Livingston Observatory



(b) LIGO Hanford Observatory

Martynov, D. V., et al. (2016). *Phys. Rev. D*, 93(11), 433. <http://doi.org/10.1103/PhysRevD.93.112004>